THE STABILITY OF CEPHEID PULSATIONS

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ABSTRACT. Analyses of Cepheid period changes using *O–C* diagrams are used to establish the direction of evolution of individual stars through the instability strip. The superposition of random fluctuations in period on *O–C* data is less easy to interpret. Such chaotic behavior is a standard feature of Mira and RV Tauri variables according to the work of Percy, but has not been examined in detail for Cepheids. Unlike the situation for cooler classes of pulsators, most Cepheids do not exhibit random fluctuations in period. Binarity plays only a minor role in the scatter in *O–C* data.

Key words: Stars: variable: pulsating: Cepheids.

1. Introduction

Stellar evolutionary tracks associate classical Cepheids with stars in a variety of instability strip crossing modes during advanced stages of nuclear fuel consumption. In the very rapid first crossing of the strip the energy originates from hydrogen burning in a thin shell surrounding a helium-rich core. In the slower second and third crossings energy is generated primarily from helium burning in the core, while during somewhat faster fourth and fifth crossings energy is generated from helium burning in a thin shell surrounding the core. During intermediate evolutionary stages as a red supergiant, large scale convection is capable of dredging up processed material from the core with abundance patterns reflecting previous stages of nuclear processing that have taken place in the core: depleted C and O abundance and enhanced N abundance reflecting prior stages of core hydrogen burning through CNO processing, and enhanced C and O abundance reflecting prior stages of helium burning (e.g. Luck & Lambert 1981). The neon-sodium cycle is responsible for enhanced Na abundances in such stars (Sasselov 1986; Luck 1994; Denissenkov 1994). Careful abundance analyses of Cepheids can therefore be used to establish likely evolutionary status on the basis of such patterns.

Alternatively, each instability strip crossing for a Cepheid is accompanied by gradual changes in overall dimensions and pulsation periods, P, as they evolve: increasing mean radius and P during evolution towards the cool side of the HR diagram (first, third, and fifth crossings), and decreasing mean radius and P during evolution towards the hot side of the HR diagram (second and fourth crossings). Since each crossing occurs at a different pace and at a different luminosity, the rate of period change is closely related to strip crossing mode, within possible constraints imposed by variations in chemical composition and pulsation mode (Berdnikov et al. 1997; Turner et al. 1999). The changes are revealed by parabolic trends in *O–C* data, where each datum represents the difference between Observed and Computed times of light maximum calculated from a linear ephemeris. The changes amount to mere seconds or minutes per year in pulsation periods of days to months, but the effects are cumulative. The observed offsets from established ephemerides are therefore significant and measurable as differences from the predicted epochs of light maximum amounting to several hours or more — in some cases as offsets of several days.

Two other mechanisms can generate systematic trends in *O–C* data. One is binarity, which produces light travel time differences in times of light maximum resembling cyclical changes in pulsation period as a Cepheid orbits the system's center of mass. The other is a meandering trend observed for some Cepheids that apparently originates from random changes in the period of pulsation. Of the three effects, evolution, light travel time effects, and chaotic fluctuations, evolution is the most obvious over long time baselines, while light travel time effects are usually marginally detectable at best. Chaotic period fluctuations are conspicuous relative to evolutionary trends and observational scatter for some Cepheids, but their origin is unexplained. Such behaviour is obvious in Cepheids such as S Vul, SV Vul and SZ Tau for example (Berdnikov 1994; Berdnikov & Pastukhova 1995), but is not observed in many

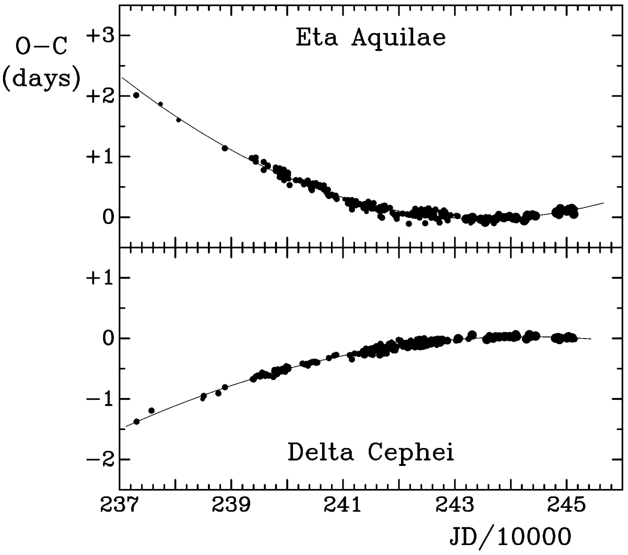

Figure 1: O–C data and calculated trends for the Cepheids η Aql (upper) and δ Cep (lower).

others, such as TX Cyg, CD Cyg, and V1726 Cyg (Turner et al. 2001).

2. Evolutionary Effects

Figs. 1 and 2 illustrate evolutionary effects in the O–C data for four Population I Cepheids: η Aql and δ Cep (Fig. 1), and T Ant and SV Vul (Fig. 2). The time baselines of O–C data for η Aql and δ Cep span more than two centuries and the evolutionary trends are very well established: period increase for η Aql and period decrease for δ Cep. The same is true for T Ant and SV Vul (period increase for T Ant and period decrease for SV Vul), although the data for the two stars span only a century. Turner et al. (2001) recently demonstrated that evolutionary trends are detectable even in V1726 Cyg, a Cepheid discovered only two decades ago. Archival data from the Harvard Observatory plate collection were essential for augmenting the small amount of O–C data for the star.

Least squares fitting of a parabola to O–C data provides a rate of period change (in seconds per annum) that can be compared directly with predictions from stellar evolutionary models. In most instances the observed rates of period change agree closely with predictions and empirical expectations for different instability strip crossings (Fig. 3), although some ambiguous cases may warrant additional O–C data or more careful analyses of existing data. The O–C trends for the Cepheids in Figs. 1 and 2 indicate a third crossing for η Aql, a second crossing for δ Cep, a third crossing for T Ant, and either a second or fourth crossing for SV Vul (the rates of negative period change are about the same for Cepheids with periods of 40–80 days).

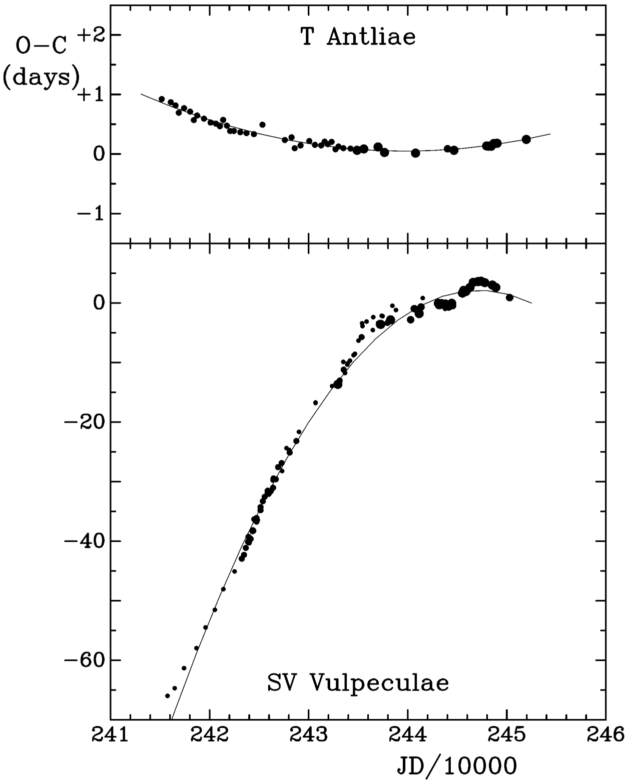

Figure 2: O–C data and calculated trends for the Cepheids T Ant (upper) and SV Vul (lower).

3. Random Fluctuations in Period

Random fluctuations in period are a standard feature of long-period pulsators, as revealed by the work of Percy and his collaborators (Percy et al. 1997; Percy & Hale 1998; Percy & Colivas 1999). They have examined the O–C data for Mira variables and RV Tauri stars using a test devised by Eddington & Plakidis (1929), in which one calculates the average accumulated delays between light maxima separated by x cycles, $\langle u(x) \rangle$, without regard to sign. The data should follow a trend line represented by the relation:

$$\langle u(x) \rangle^2 \;=\; 2a^2 + xe^2,$$

where a represents the magnitude of the random errors in measured times of light maximum and e represents the magnitude of any random fluctuations in period.

The "e" parameter, as a measure of randomness in pulsating stars, appears to increase roughly linearly with period (Percy et al. 1997; Percy & Hale 1998; Percy & Colivas 1999; Percy 2001). The available data, plotted in Fig. 4, when analyzed by non-parametric techniques (to avoid undue influence from outliers), are described by:

$$e \;=\; -0.421(\pm 0.146) + 0.015(\pm 0.001)\,P.$$

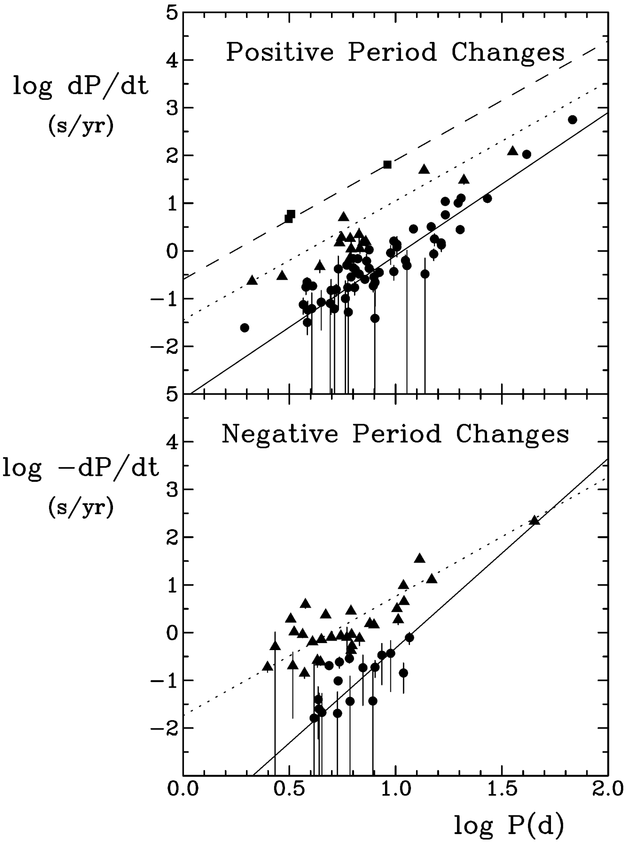

Figure 3: Observed rates of period change (with calculated uncertainties) are compared with predictions from stellar evolutionary models and empirical calculations (from top to bottom, the lines indicate first, fifth, and third crossings, upper; fourth and second crossings, lower). Different symbols indicate the assessment of likely crossing mode to individual Cepheids.

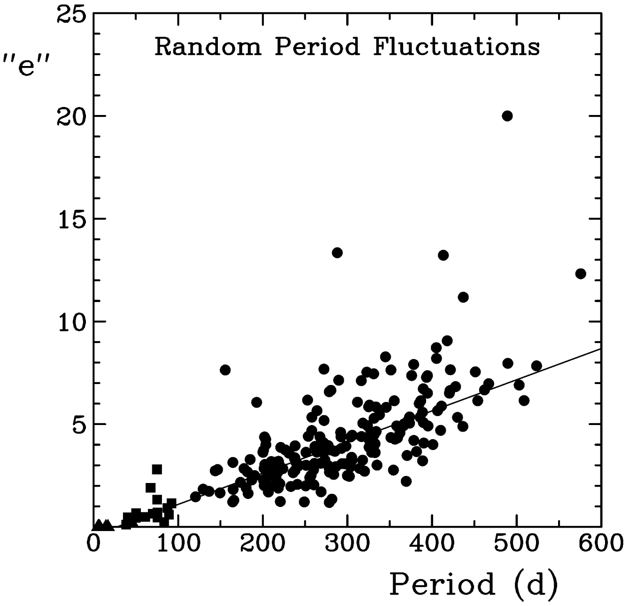

Figure 4: The dependence of the parameter e as a function of pulsation period for Cepheids (triangles), RV Tauri variables (squares), and Miras (circles).

The possibility arises that the scatter in Fig. 4 originates largely from chemical composition differences between stars, but only stellar atmosphere studies can test that properly.

The situation for Cepheids is relatively unstudied, which is where *O–C* data are useful, once evolutionary trends are removed. The situation is less straightforward than for Miras and RV Tauri stars, since very few light maxima are ever observed directly, except possibly for a few long period Cepheids. An *O–C* datum usually represents a value obtained from fitting a light curve derived from observations over several adjacent cycles to a standard light curve. Eddington & Plakidis tests for the four Cepheids of §2 are illustrated in Fig. 5, and reveal no signature of random fluctuations in period for η Aql, δ Cep, and T Ant, but a strong positive signature for SV Vul. The case for V1726 Cyg is described by Turner et al. (2001). According to the results of Fig. 4, chaotic fluctuations in period should be relatively uncommon in Cepheids, except possibly for those of long period.

The cases for T Ant and SV Vul are rather curious, given that the residuals for both stars appear to display similar signatures (Fig. 6). The alternative possibility to random fluctuations in period is to consider light travel time effects in a binary system. Such an alternative fails for SV Vul since it produces unreasonable parameters for the system ($P \simeq 57.5\,\mathrm{yrs}, a_1 \sin i \simeq 329$ A.U., $M_1 \geq 10^4\ M_\odot$). It *is* a possibility for T Ant, where the derived parameters are more reasonable ($P \simeq 42.4\,\mathrm{yrs}, a_1 \sin i \simeq 10.8$ A.U., $M_1 \geq 0.7\ M_\odot$).

4. Discussion

Few correlations exist between deductions about instability strip crossing mode for Cepheids made from atmospheric abundance analyses and the modes inferred from rates of period change. Sometimes the latter are ignored entirely, as was the case for SV Vul in the recent study by Luck et al. (2001). Luck et al. argue that SV Vul seems likely to be in the first crossing of the instability strip on the basis of C, O, and Na abundances that appear to be representative of its original composition. As demonstrated in §2, however, SV Vul is crossing the instability strip for the second or fourth time. The possible discrepancy with the abundance data is now rather interesting. In order to resolve the two seemingly discordant results, it is necessary to consider other possibilities, such as (i) SV Vul is in the fourth crossing of the instability strip and its abundance pattern reflects a partial second dredge-up,

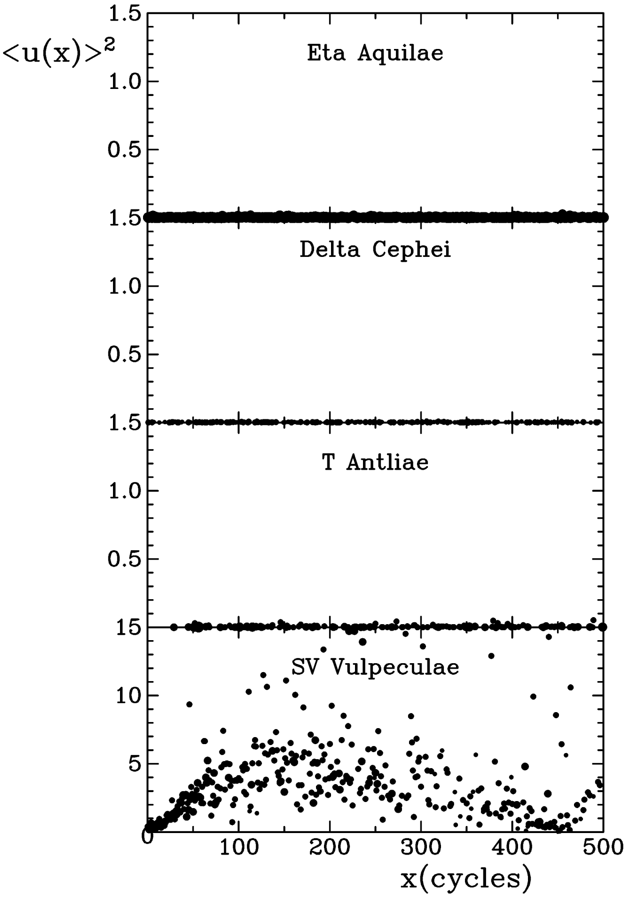

Figure 5: Randomness diagrams (plots of $\langle u(x) \rangle^2$ versus cycle count difference x) derived from residuals in the *O–C* data for η Aql (upper), δ Cep (upper middle), T Ant (lower middle), and SV Vul (lower). Symbol size increases with increasing weight for the data.

or (ii) SV Vul merged with a close companion prior to reaching the first dredge-up and is presently in a second crossing of the strip without displaying the abundance anomalies of other second crossing Cepheids.

Examination of abundance patterns in the Cepheids studied by Luck & Lambert (1981) suggests that the second scenario proposed above is unnecessary. All Cepheids in the fourth crossing of the strip (DT Cyg, RT Aur, and ζ Gem) or fifth crossing (X Sgr) share similar abundance anomalies to SV Vul, namely solar or mildly underabundant C and O and overabundant N. If the analogy extends to Cepheids with long periods like SV Vul, one can conclude that the second dredge-up is not as efficient as the first in bringing core-processed material to the stellar surface.

Of the other Cepheids studied by Luck & Lambert (1981) which have derived rates of period change, five (SU Cas, η Aql, X Cyg, T Mon, and RS Pup) are indicated to be in the third crossing of the strip and two (T Vul and δ Cep) are indicated to be in the second crossing. They exhibit the underabundance of C and O and overabundance of N expected for such stars.

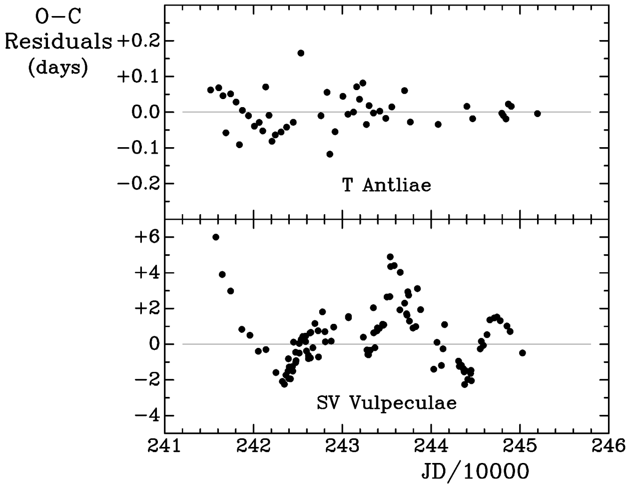

Figure 6: The residuals from evolutionary trends for T Ant (upper) and SV Vul (lower) plotted as a function of Julian date.

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