ECLIPSING BINARY STARS : LIGHT CURVE MODELS AND SOFTWARE

P.G. Niarchos

Department of Astrophysics, Astronomy and Mechanics, University of Athens, GR 157 84 Zografos, Athens, Greece, *pniarcho@cc.uoa.gr*

ABSTRACT. The importance of studying Eclipsing Binary Stars for the determination of fundamental stellar data is sketched. The eclipsing binaries, originally classified phenomenologically, are now understood on the basis of much firmer physics and the improved understanding has led to a morphological basis of classification. The main light curve models developed since Russell's era are presented along with the respective programs for analysis (Kallrath & Milone 1999). These models are based on the geometric effects due to eclipses and on physical proximity effects between the components. Significant progress was made in the early 1970s, when models and programs were developed to compute (synthetic) light and velocity curves directly. Such models and programs were based on spherical stars, ellipsoidal geometry and on Roche geometry. During the last 25 years the analysis of photometric and spectroscopic data of eclipsing binaries has been performed by means of synthetic light curve programs based on Roche geometry. The synthetic light curve programs, based on physical models, have contributed to our understanding of physical processes in stars and have been used to solve several important astrophysical problems. A desirable feature of a light curve program is the ability to incorporate additional astrophysics. There is a continuing need to improve the model physics, as we become more aware of the observational properties of stars.

Key words: Stars: binary: eclipsing: light curve models: light curve analysis.

1. Introduction

There are several reasons why the study of Binary stars is so important:

- they are as common as single stars in the Universe. In the solar neighborhood the frequency is more than 50%
- they are the primary source of our knowledge of the fundamental properties of stars. Their study allows direct determination of stellar masses, stellar radii and stellar luminosities
- they are distant indicators for nearby galaxies
- the evolution of binary stars helps to explain a host of diverse and energetic phenomena such as : X-ray binaries, cataclysmic variables, novae, symbiotic stars and some types of supernovae
- these binaries are classic examples of the fundamental contribution that stellar astrophysics makes to our general understanding of physical processes in the universe.

2. Classification of Eclipsing Binaries and Importance of their Study

2.1 Phenomenological classification

2.1.1 EA Light Curves (Algol type)

- they show clearly defined eclipses with obvious start and end times
- the outside eclipses parts are flat-topped, suggesting that the effects due to proximity of the two components are small
- there is large difference between the depths of the two minima
- the reflection effect is pronounced.

2.1.2 EB Light Curves (β Lyrae type)

- they show well-defined eclipses
- the variation is continuous (ellipsoidal variation)
- they are characteristic of tidally distorted components
- the large difference in depths of minima indicates components of quite different surface brightness.

2.1.3 EW (or W UMa) Light Curves

- a continuous variation of brightness is present
- there is small difference in depths of the minima

- proximity effects are pronounced (mainly tidally distorted shapes of the stars)

Studies of eclipsing binaries show the value of treating all aspects of the light of the systems in light curve analysis, and not only their geometric characteristics. Eclipsing binary analysis is a formidable astrophysical task. The field includes radiation physics and sometimes hydrodynamics. It borrows methods from celestial mechanics, thermodynamics, and other branches of physics. Physical models are required for radiation transport in the component's atmospheres and for the dynamic forces controlling the stellar mass distribution.

2.2 Morphological classifification

The concept of the equipotentials in the Roche geometry allows us to separate eclipsing binaries into morphological classes :

- *detached systems*, if neither component fills its Roche lobe
- *Semidetached systems*, if one component fills its Roche lobe and the other does not
- *Over contact systems*, if both components exceed their Roche lobes and a common envelope is formed
- *Double contact system*, where each component fills its lobe exactly, and at least one rotates supersynchronously. The two components fill their limited lobes but do not touch each other (Wilson 1979).

There is a correspondence between the morphological classification and the phenomenological classification. The Algol-type light curves are produced by semidetached systems and the W UMa-type light curves by over contact systems.

The phenomenological classification of β Lyrae-type light curve has no morphological counterpart. Sometimes, β Lyrae-type light curves are produced by detached systems, sometimes by semi-detached systems and sometimes also by systems having marginal overcontact.

2.3 Why data derived from eclipsing binaries are important

A light curve can provide : the orbital inclination, the relative radii in units of the separation, the ratio of luminosities and the photometric mass ratio.

The radial velocity curves can provide : the mass ratio, the separation α in physical units (if inclination is known), the orbital dimensions (if inclination is known), the period P (also known from light curve).

The spectroscopic observations can provide: the spectral classification, the chemical composition and the temperatures of the two components.

The full determination of absolute eclipsing binary parameters requires both a light curve and a radial-velocity curve for each component. A combination of the above observations yields the fundamental source of information about sizes, masses, luminosities and distances or parallaxes of stars. This information provides the means : to test stellar structure and stellar evolution theories, to improve our understanding of such exotic objects as X-ray binaries, novae and Wolf-Rayet stars and to get a great wealth of knowledge from binaries in globular clusters.

3. Methods of Light Curve Analysis

3.1 The age of geometrical models

H.N. Russell (1887-1957) spoke about the Royal Road of Eclipses. Traveling this road entails the decoding of the messages encrypted in the light curves of eclipsing variables. As a rule, the light curve is determined by geometric effects due to eclipses and by physical proximity effects. The effects of ellipticity and reflection effect can be treated through the rectification procedure and the solution is made for spherical mode (Russell- Merrill method, 1952). Computer programs based on rectifiable models were developed by Jurkevich (1970)and Proctor and Linnell (1972). The solutions are possible only for well-separated detached systems, but not for semi-detached and over-contact systems.

3.2 The age of computational Astrophysics

Significant progress in the fiels was made in early 1970s. Models and programs were developed to compute (synthetic) light and velocity curves directly. Such programs are : the EBOP, based on spherical stars (Etzel 1993), the WINK, based on ellipsoidal geometry (Wood 1971), and models and programs based on Roche geometry developed by Lucy (1968), Hill and Hutchings (1970), Wilson and Devinney (1971) and Mochnacki and Doughty (1972). These new approaches permitted the computation of light curves on the basis of complex physical models describing the dynamic forces controlling the stellar mass distribution and the radiation transport in the component's atmospheres.

Physical models based on equipotentials and Roche geometry were implemented in the Wilson-Devinney program (Wilson & Devinney 1971, Wilson 1979) and in LIGHT2 program (Hill & Rucinski 1993). Physical models and programs led to least-squares determinations of the light curve parameters (Wilson & Devinney 1971, 1972, 1973) and Lucy (1973). The computational implementation of Roche models coupled with least-squares analyses really started the age of computational astrophysics in eclipsing binary research. Other programs based on physical models are those of Hill & Rucinski (1993), Linnell (1984), Hadrava (1997) and others.

3.3 Astrophysical problems solved by Light Curve Methods

- the Algol paradox
- the structure of W UMa-type stars
- bolometric albedos of convective envelopes
- undersized subgiants (Wilson 1994)
- understanding intriguing binaries such as ϵ Aurigae and β Lyrae by including gas streams and discs in the light curve modelling
- understanding of physical processes in stars
- successful modelling of W UMa stars as overcontact systems
- understanding evolution subsequent to mass transfer episodes through observations of binaries with major circumstellar mass flows. Examples : β Lyrae, V356 Sge, KU Cyg, AX Mon and many symbiotic stars

4. Models for computing eclipsing binary observables and Software

4.1 A general approach to modeling Eclipsing Binaries

We define an eclipsing binary observable curve O

$O := [(t_k, o_k) \mid 1 \leq k \leq n]$

a set of n elements in which each element is a pair, *(t, o)*, where *t* represents an independent, time-related quantity and *o* is the corresponding observable (Kallrath & Milone 1999). The quantity t may either represent the *time* or the *photometric phase* Φ (traditionally used). The term light is usually used in the abstract sense and may represent not only the photometric brightness but any observable, such as :

- the light at a given wavelength
- the radial velocity
- polarisation
- photospheric spectral line profile
- spectral distributions due to circumstellar flows, and
- any other quantity associated with the phase, but also other quantities independent of phase which we call systemic observables.

It may represent a measured value of an observable, and/or a value derived from a light curve program.

Thus an 'observable curve' may be, for instance:

- an observed light curve O^{obs}
- a calculated light curve O^{cal}
- a wavelength-dependent light curve O^{λ}
- a radial velocity curve O^{vel}
- a polarization curve O^{pol}
- a set of output arrival times O^{pul}

Before 1970 an observed light curve O^{obs} of an eclipsing binary was analyzed following rectification procedures. However, the underlying physical models were relatively simple and neglected effects which later turned out to be relevant. Photometric and spectroscopic data were analyzed separately and with different methods. Today's methods permit analysis of photometric, spectroscopic, and other data simultaneously.

4.2 Direct problem

If the vector $\mathbf{x}$ represents all relevant eclipsing binary parameters, then for a given set of phases a whole observable curve $O^{cal}(\mathbf{x})$, or a set of several curves $O^{cal}(\mathbf{x}*)$ can be computed. Such a computation consists of three major parts:

1. The physics and geometry of orbits and components
2. The computation of local radiative surface intensity as a function of local gravity, temperature, chemical composition, and direction. The proper formulation of the radiative physics requires the use of accurate model atmospheres
3. The computation of the integrated flux in the direction of the observer. This computation must take eclipses into account. The inclusion of other effects such as circumstellar matter, i.e., gas streams, disks, attenuating clouds, etc., may be desirable.

4.3 Inverse problem

Determine a set of parameters $\mathbf{x}*$ from a set of eclipsing binary observations by the condition that the set of curves $O^{cal}(\mathbf{x}*)$ best fits a set of observed curves O^{cal}. The system parameters x are modified according to an iterative procedure until the deviation between the observed curves O^{obs} and the calculated curves $O^{cal}(\mathbf{x}*)$ becomes minimal in a well- defined sense. The system parameters $\mathbf{x}*$, corresponding to the observed curves O^{obs}, are ordinarily regarded as the solution of nonlinear least-squares problem. The least squares techniques used in Eclipsing Binary Data Analysis are:

1. The classical Differential Corrections
2. Multiple Subset Method and Interactive Branching
3. Damped Differential Corrections
4. The Simplex Algorithm
5. Other approaches.

4.4 Distinction Between Models and Programs

A *model* is a set of mathematical and physical relations which enables the mapping of a set of eclipsing binary parameters $\mathbf{x}$ to a light curve L^{cal} for a given set of phase values, while a *light curve code or program* is the software implementation of such a model.

While the model is abstract and generic and relates a stellar system's physical attributes (gravitational potential, eclipse conditions, etc.), the program requires a choice of coordinates, integration or summation procedures and matrix inversion routines. The degree of realism of the model fundamentally determines the reliability of the predicted light curve. However the program itself constrains the accuracy of the result as well as the efficiency with which the result is reached. Therefore, it seems reasonable that those who develop light curve models maintain close contact with those who write and upgrade the program.

A desirable feature of the a light curve program is an ability to incorporate additional astrophysics. There is a continuing need to improve the model physics, as we become more aware of the observational properties of stars. It should also be structurally well defined but especially expandable.

5. Synthetic Light Curve Models

The Russell-Merrill Model and Technique

- Basic assumptions : spherical stars and in subsequent versions ellipsoidal in shape. The technique is described in Russell and Merrill (1952).
- The geometrical model for distorted stars is a triaxial ellipsoid.
- Light curves of stars showing evidence of tidal distortion and reflection are transformed by rectification process into those of spherical stars.
- The physics is limited to Planckian radiation, linear limb darkening law, gravity darkening and simple treatment of reflection effect.
- Manual techniques continued to be used till 1970.
- Computerized versions were developed by Jurkevich (1970) and Proctor and Linnell (1972).

The eclipsing Binary Orbit Program EBOP

- A FORTRAN program based on spheroidal model (Etzel 1981), known as NDE model (Nelson and Davis 1972).
- Suitable for for detached systems, not for deformed components.
- Provides options to implement further physics.
- Makes use of spheroidal stars moving in circular or eccentric orbits.
- Linear limb darkening law is used.

The Wood model and the WINK program

- Assumes the components of the binary system to be triaxial ellipsoids (Wood 1971, 1972).
- Suitable for systems with moderate oblateness and reflection effect.
- It is not in use any more.

PHYSICAL MODELS :

ROCHE GEOMETRY BASED PROGRAMS

Binnendijk's Model

- Synthetic light curve program for contact binaries, based on Roche model with cylindrical coordinates (Nagy 1975, Binnendijk 1977).
- Good exposition of the physical principles behind the model.
- Binnendijk stressed the importance of surface brightnes as a radiation parameter and emphasized the importance of radial velocity, line profile, and spectrophotometric data and therefore he forsaw many of the modern advances that have been made.

Hadrava's Program FOTEL

- Suitable for simultaneous solution of light curves and radial velocity curves of eclipsing binaries (Hadrava 1997).
- It uses a very simplified model of physics and geometry for the flux calculations.
- Designed to handle a large number of original observational data and to run on a PC.
- It can take into account a third body or component.

- The code uses a Simplex algorithm compined with a direct least-squares solution of multiplicative parameters.
- Errors and cross-correlations of all fitted parameters are included.

Hill's Model and its Program LIGHT2

- Based on Roche model;
- Black-body, color-index based, and theoretical atmosphere fluxes;
- Differential corrections based on Marquardt method;
- Multiple (≤ 10), elliptical spot structures;
- Line profiles are calculated;
- For complete review and current capabilities see Hill and Rucinski (1993).

Linnell's model

- Based on Roche geometry (Linnell 1984, 1993);
- Assumes circular and eccentric orbits;
- The software package consists of a series of programs;
- Monochromatic light curves are calculated;
- Calculates theoretical light curves at the times of observations;
- Permits placement of dark (bright) spots on the component photosphers;
- Synthetic spectra are calculated (Linnell et al. 1998);
- Can handle multicolor light curves;
- Impractical implementation of the whole synthetic photometry program except for the division of the entire project into separate programs.

Rucinski's Model

- Free of systematic errors related to uneven distribution of integration points;
- Roche model;
- Differential corrections via Rucinski's least-squares program;
- No spots are modelled;
- Line profiles are calculated;
- Extreme simplicity and modular structure permit easy modification;
- Input through a separate file;
- The code (in FORTRAN) is not supported generally and has restricted distribution.

The Russian school's Models

- Strong focus on eclipsing binaries with extended atmospheres (Wolf-Rayet binaries and X-ray binaries) (Cherepashchuk 1966, 1975);
- Spherical models with enhanced features for atmospheric eclipses and disks;
- Methods used are based on the regularization algorithms by Tikhonov (1963); Strong mathematical background;
- Unique solutions are determinable only in certain cases;
- X-ray systems are among the principal binaries studied by the Russian school (Bochkarev & Karitskaya 1983, Bochkarev et al. 1975, Goncharsky et al. 1991, Antokhina et al. 1992. 1993).

Wilson-Devinney's Model and Program

- Extended in many publications and software releases (Wilson and Devinney 1971, Wilson 1979, 1990, 1993).
- Supports elliptical orbits with eccentricity and non-synchronous rotation of components;
- The most recent version includes Kurucz stellar atmospheres models;
- Simple reflection effect is considered;
- Different laws of limb-darkening can be used;
- Several modes are provided to specify the geometry of a binary system or to add constraints or relations between parameters;
- Gravity darkening and albedos can be treated either as fixed parameters or adjusted ones;
- It consists of two main programs named LC and DC, where two dozen subroutines are utilized.
- Relative dimensions of the components are derived and many quantities and parameters in the model are dimensionless.

Other approaches

Budding's Eclipsing Binary Model

- Spherical model, circular orbits, circular spots;
- Original light curves are cleaned from the effects of spots.

Kopal's Frequency Domain Method

- Kopal's contributions to the field of modern light curve analysis were crucial;
- The basic aspects of the technique are described in Kopal (1979, 1990);
- Methods in great mathematical depth are used for the representation of the fractional loss of light in symmetric eclipsing binary light curves in terms of integral transforms, especially Hankel and Fourier transforms, and discuss the asymptotic properties of finite sums of the latter;
- Corrections are applied for limb- and gravity-darkened tidally distorted non-spherical stars with light curve perturbations;
- Superiority of the method, since allows direct determination of the parameters from the observations;
- Main disadvantages:
  1. components are assumed to be spheres, or corrected spheres
  2. it is doubtful whether a few Fourier coefficients can contain all the information in a light curve
  3. the angle of the external tangency must be specified
  4. it is not easy to introduce additional physical effects into the model in this analysis.
  5. the method becomes complicated in the case of partial eclipses.

Mochnacki's General Synthesis Code, GENSYN

- GENSYN was developed by Mochnacki and Doughty in the early 1970s and further improved around 1983. It was intended to do both light curve and line profile synthesis;
- Uses cylindrical coordinate scheme;
- It was the first Roche-geometry based light curve code to incorporate full mutual irradiation by mapping each surface element to all those illuminating it.

Collier-Mochnacki-Hendry Spotted General Synthesis Code

- Combination of the programs SPOTTY (Collier) and GENSYN (Mochnacki) in 1987 to analyze spotted eclipsing systems (Hendry and Mochnacki 1992);
- Allows determination of the most likely spot distribution making use of the maximum entropy algorithm (Hendry et al. 1992);
- More recently, Hendry wrote a program to fit both orbital elements and spot distributions to photometric and spectroscopic data.

THE WILSON-DEVINNEY PROGRAM

The Wilson-Devinney program is the most widely used of all the synthetic and analytic light curve modeling codes. It is appropriate to describe its features, capabilities, and continuing developments in some detail. It has seen continual improvements and the 1998 version provides powerful features.

Current Capabilities of the WD program

- Full functionality of the WD-Program (version distributed before 1995);
- Kurucz atmospheres;
- Initial parameter search with the Simplex algorithm;
- Automatic differential corrections;
- Damped Levenberg-Marquardt scheme;
- Spectral line profiles can be computed; they can be associated with specific regions of the stars, permitting an analysis of chromospheric fluorescence and also lines from spots;
- Either time or phase can be independent variable;
- Output data can be used as input to any commercial or private plotting program;
- The program is now entirely in double precision;
- The LC input/output format is determined by a control integer allowing certain decisions about what will be computed;
- The parameter error estimates are now standard deviations;
- Shell-based I/O interface.

Several improvements await incorporation

1. Polarimetry is a ripe field for exploitation in the light curve analysis. The existing data are scarce but the means to incorporate them into analytical methods could encourage further observations in this demanding field;
2. Atmospheric eclipses for components with extended atmospheres;
3. Improved accuracy.

Atmospheric Options

1. Kurucz new stellar atmospheres incorporate computed opacities from a comprehensive list of 58 million lines and for 56 temperatures in the range 2000 to 200,000 K.
2. The models range from 3500 K to 50,000 K and over a large range in log g, but not over the entire range of opacities;
3. The atmospheres were integrated over the standard passbands UBVRJIJ, RCIC, and uvby, the nominal extended Johnson infrared passbands JHKLMN, the improved infrared bands, iz, iJ, iH, iK, iL, iL',iM, iN, in, and iQ, and for a range of narrow, square-edged passbands centered on wavelengths in the far-ultraviolet, appropriate for IUE, HSP, or other space platforms with far-ultraviolet detectors.

Applications and Extensions

The intention is to provide hints about what can be achieved and how various extensions can be used to derive astrophysical results;

1. The eclipsing X-ray Binary HD 77581 / Vela X-1
    - The binary contains an X-ray pulsar, while the ellipsoidal variable HD 77581 / GP Velorum is the optical counterpart of the pulsed, eclipsing X-ray source Vela X-1
    - A BO.5 supergiant and a neutron star move in an eccentric orbit of about e=0.1 and a period of 8.96 days
    - The pulse arrival times are an additional observable used in a simultaneous analysis
    - B, V light curves, optical radial velocities, pulse arrival times and estimations of the X-ray eclipse duration are used in a simultaneous analysis to ensure a self-consistent solution (Wilson & Terrell 1998).
2. Eclipsing binaries in Clusters

    The objective is to analyze eclipsing binary systems in well studied clusters in a boot-strap program to increase knowledge of both binaries and clusters and test evolution models.
3. Fitting of Line Profiles

    Mukherjee et al. (1996) combined the theory of stellar line broadening for local profiles with the WD program and used it to estimate rotation rates of Algol binaries by fitting line profiles to observed data.

6. The Structure of the Future Light Programs

The aspects of a general light curve program include:

- the light curve models (provided by astronomers);
- capabilities of the least-squares solvers (could be implemented by astronomers, mathematicians, computer scientists);
- user-friently front-ends (the development requires the expertise of a software engineer rather than an astronomer or mathematician. Of course many gifted individuals are capable of fulfilling more than one of these roles).
- the above three components need to be linked appropriately. Such a program is expected to emerge in the future.

Prospects and expectations

- Modern light curve analysis was born early in the twentieth century with the pioneering work of Henry Norris Russell;
- Thanks first to the theoretical investigations of Zdenek Kopal beginning in the 1940s, and second to a number of workers who developed practical computer applications, two decades later;
- The WD program has become the light curve analysis tool of choice by the majority of the community;
- The number of observed light curves will continue to exceed the number analyzed (use of CCDs etc.);
- New techniques of analyzing data should be invented (Neural network techniques);
- The quality of light curve analyses can also be expected to improve;
- Phenomena of extended atmospheres, semitransparent atmospheric clouds, variable thickness disks, and gas streams will be treated by such programs;

- Targets of modeling work : besides the determination of orbits, stellar sizes and masses, it seems likely that the detailed physics of stellar surfaces, including those arising from activity cycles will be included.

Close binary research might initiate projects involving complicated physics and requiring sophisticated mathematics and numerical methods.

Recommendations to Observers

1. Mid-range to long-period binaries ($P > 5d$, but especially $P > 50d$) need observations of all kinds. The longer period binaries are nearly unexlpored territory;
2. Infrared light curves are especially needed for binaries with large temperature differences between the two components. Simultaneous observations in the optical and infrared are critically important;
3. Individual observations should be published or archived with easy access to them;
4. Binaries with active mass flow need to be followed continuously over at least several orbits. Suitable targets for APTs.
5. Polarimetric observations are equally important. Although light curves are nearly periodic, polarization curves mainly show transient events. Absolute time of observations should be given;
6. X-ray binaries are a major and important new area. Again absolute time should be published. X-ray binaries provide excellent data for simultaneous fitting of multiband light curves, optically determined radial velocities, and pulse arrival times.
7. Spectrophotometry provides an even greater potential bounty, and, in principle, thousands of light curves, if proper star spectra are taken (not easy and seldom done).

References

Antokhina E.A., Seyfina E.V., Cherepashchuk A.M.: 1992, *Sov. Astron.*, **36**, 143.

Antokhina E.A., Pavlenko E.P., Cherepashchuk A.M., Shugarov S.Y.: 1993, *Astron. Rep.*, **37**, 407.

Binnendijk L.: 1977, *Vistas*, **21**, 359.

Bochkarev N.G., Karitskaya E.A., Shakura N.I.: 1975, *Sov. Astron. Lett.*, **1**, 118.

Bochkarev N.G., Karitskaya E.A.: 1983, *Sov. Astron.*, **27**, 421.

Cherepashchuk A.M.: 1966, *Sov. Astron.*, **10(3)**, 227.

Cherepashchuk A.M.: 1975, *Sov. Astron.*, **19(1)**, 47.

Etzel P.B.: 1981, in E.B. Carling and Z. Kopal (eds.), *Photometric and Spectroscopic Binary Systems,* 111, D. Reidel, Dordrecht, Holland.

Goncharsky A.V., Romanov S.Y., Cherepashchuk A.M.: 1991, *Finite-Number Parametric Inverse Problems of Astrophysics*, Moscow University Press, Moscow, Russia (In Russian).

Hadrava P.: 1997, *FOTEL 3 - User's Guide*, Technical report, Astronomical Institute of the Academy of Sciences of the Czech Republic, 25165 Ondrejov, Czech Republic.

Hendry P.D., Mochnacki S.W.: 1992, *ApJ*, **338**, 603.

Hendry P.D., Mochnacki S.W., Cameron A.C.: 1992, *ApJ*, **399**, 246.

Hill G., Rucinski S.M.: 1993, in E.F. Milone (ed.), *Light Curve Modeling of Eclipsing Binary Stars*, 135.

Jurkevich I.: 1970, *The Henry Norris Russell Memorial Volume*, **Vol. 12** of *Vistas*, 63, Pergamon Press, Oxford, UK.

Kallrath J., Milone E.F.: 1999, *Eclipsing Binary Stars*, Springer-Verlag, New York, Inc.

Kopal Z.: 1979, *Language of the Stars*, D. Reidel, Dordrecht, Holland.

Kopal Z.: 1990, *Mathematical Theory of Stellar Eclipses*, Kluwer Academic Publishers, Dordrecht, Holland.

Linnell A.P.: 1984, *ApJ Suppl.*, **54**, 17.

Linnel A.P.: 1993, in E.F. Milone (ed.), *Light Curve Modeling of Eclipsing Binary Stars*, 103, Springer, New York.

Linnell A.P., Etzel P.B., Hubeny I., Olson E.C.: 1998, *ApJ*, **494**, 773.

Mukherjee J.D., Peters G.J., Wilson R.E.: 1996, *MNRAS*, **283**, 613.

Nagy T.E.: 1975, *Bull. Amer. Astr. Soc.*, **7**, 533.

Nelson R.H., Davis W.D.: 1972, *ApJ*, **174**, 617.

Proctor D.D., Linnell A.P.: 1972, *ApJ Suppl.*, **24**, 449.

Russell H.N., Merrill J.E.: 1952, *Princeton Obs. Contr.*, **26**, 1.

Tikhonov A.N.: 1963, *Dokl. Akad. Nauk USSR*, **153**, 49.

Wilson R.E.: 1979, *ApJ*, **234**, 1054.

Wilson R.E.: 1990, *ApJ*, **356**, 613.

Wilson R.E.: 1993, in J.C. Leung and I.-S. Nha (eds.), *New Frontiers in Binary Star Research*, **Vol. 38** of ASP Conference Series, 91, Astronomical Society of the Pacific, San Francisco, CA.

Wilson R.E., Devinney E.J.: 1971, *ApJ*, **166**, 605.

Wilson R.E., Terrell D.: 1998, *MNRAS*, **296**, 33.

Wood D.B.: 1971, *AJ*, **76**, 701.

Wood D.B.: 1972, *A Computer Program for Modeling Non-Spherical Eclipsing Binary Star Systems,* Technical Report X-110-72-473, GSFC, Greenbelt, MD.