DETERMINATION OF CHARACTERISTIC TIME SCALES IN SEMI-REGULAR STARS: COMPARISON OF DIFFERENT METHODS

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ABSTRACT. Results of the analysis of the charactersistic time scales using the periodogram, scalegram and the wavelet analysis are compared for 173 stars most actively observed by the members of the AFOEV and VSOLJ. The periodogram analysis is more effective for stars with stable nearly coherent pulsations, whereas the wavelet analysis is a better tools for stars with switching pulsational modes, e.g. AF Cyg. The scalegram analysis is effective for non-harmonic and chaotic-like variations. The diagrams of the effective time scales obtained using these methods are analyzed for making additional classification of the stars within subclasses. The use of the test functions for automatic classification is discussed. An example analysis is made for S Aql. Particularly, the ephemeris is determined: $Max.JD = 2443578.9(3) + 146.^{d}693(6) \cdot E$.

Key words: Stars: variable: semi-regular: S Aql

1. Introduction

Semi-regular variables represent a highly interesting class of long-period pulsating stars, which exhibit a variety of types of observational appearance. Present paper continues the series of articles dedicated to the time series analysis of the patrol visual observations of the members of AFOEV:
ftp://cdsarc.u-strasbg.fr/pub/afoev/aql/s
and VSOLJ:
ftp.kusastro.kyoto-u.ac.jp/pub/vsnet/VSOLJ/database /stars/aql_s.jd .

The catalogues of the characteristics of 53 long–period (mainly Mira–type) stars have been published by Marsakova and Andronov (1998, 2000). The catalogue of characteristics of 173 semi-regular variables was recently published by Andronov and Chinarova (2000).

Here we present time series analysis using various methods, using S Aql as an example of data. The complete data set contains 4537 observations in the time interval JD 2422579.98-2451549.20 since 1920.

2. Methods for time series analysis

Taking into account non-stable character of the light curve, it is necessary to use few supplementary methods, which should be optimized for various types of variability. Extensions of the existing methods for astronomical signals with (generally) irregularly spaced arguments are described e.g. in the monographs by Terebizh (1992) and Andronov (2001b).

2.1. Mono-, multi- periodic and multiharmonic variations

The method for determining parameters of harmonic signals is obviously the sine fit

$$m(t) = C_1 - r_1 \cos(\omega(t - T_{01})), \qquad (1)$$

where C_1 is the zero-point coinciding with the signal continuously averaged over the complete phase interval. $\omega = 2\pi/P$ is angular frequency, and P is the period of variations, and T_{01} is initial epoch for the maximum (i.e. minimum of the magnitude). For such an analysis, we use the program *Four* by Andronov (1994).

The periodogram for S Aql (Fig.1) shows a highest peak at the periodogram at $P = 146.^{d}6934 \pm 0.^{d}0057$, semi-amplitude $r_1 = 0.^{m}884 \pm 0.^{m}012$, initial epoch for maximum $T_{01} = 2443578.89 \pm 0.^{d}31$ and zero-point $C_1 = 10.^{m}337 \pm 0.^{m}008$. This value differs from the sample mean for the data $\langle m \rangle = 10.^{m}284 \pm 0.^{m}012$ because the data cover the phase interval not homogeneously. The scatter of the data at the phase curve is $\sigma_0 = 0.^{m}550$, whereas the standard deviation of the data from the mean is $0.^{m}829$. The test function $S(f)$, which is equal to the ratio of the variance of the signal to the variance of "signal+noise", is equal to 0.544 in our case, i.e. the "noise" only a little bit smaller than the "signal".

There are two other relatively high peaks at the periodogram corresponding to the periods $P_2 = 104.^{d}64$ and $P_3 = 245.^{d}23$. It is noticeable, that $P_2/P = 5/7$

and $P_3/P = 5/3$ within few tenth of per cent. So these waves may be formally interpreted as harmonics of the main period $P_4 = 734^d$: $P = P_4/5$, $_2 = P_4/7$ and $P_3 = P_4/3$. Another formal explanation is that P_2 and P_3 are biases of the primary wave with $P = P_4/5$ with a wave with a period $P_5 = P_4/2 = 367^d = 1$ year.

Thus the star is an excellent example of possibility of misinterpreting the results of the periodogram analysis as a presence of multiple waves with rational period ratio. The cause is that the photometric period of the star deviates from $\frac{2}{5}$ yr by only 0.4% !.

There are no high peaks corresponding to harmonics at periods P/j, where j is an integer, thus one may conclude that the mean light curve is nearly sinusoidal. Otherwise, one could use the program *Fdcn* for determination of the parameters of the trigonometric polynomial, including the statistically significant value of it's order s. Another program was elaborated to determine parameters of the multi-periodic model, including unknown periods (Andronov, 1994).

2.2. Wavelet analysis

To study nearly-periodic and multiperiodic processes with variable shape of the signal, the wavelet analysis is used. Foster (1996) has extended the "Morlet wavelet" method for general case of signals with irregularly spaced arguments. One may name it as a "weighted periodogram analysis with a running window". Andronov (1998, 1999) generalized this approach, adding possibilities to compute weighted mean test functions and to make wavelet smoothing.

In Fig. 2, the test functions used for the wavelet analysis, are shown: *WWZ, WWT*, S and the wavelet (semi-)amplitude R (see Andronov, 1998, 1999 for a complete description). The main peak occurs at $145^d.6$, $146^d.7$, $145^d.3$, $145^d.4$, respectively, corresponding to the test function $S = 0.74$, which is much larger (and closer to unity), than its analog for periodogram analysis (0.54). This shows an advantage of the wavelet analysis for signals with variable shape. Similarly, the mean wavelet amplitude $R = 0^m.98$ is much larger than $0^m.88$ for a coherent sinusoid.

The peak at P_3 is distinguishable, and it corresponds to $R = 0^m.58$. Another intersting feature is maximum at $P = 3520^d$ with $R = 0^m.19$. This period is located between the values $3831^d \pm 20^d$ and $3246^d \pm 29^d$ corresponding to the peaks at the periodogram. Obviously, the peaks so close in frequency may not be separated by the wavelet analysis, as well as that of 104^d and 146^d.

The "wavelet skeleton" is shown in Fig. 3. One may note a large number of low peaks with apparently changing periods. However, the main peak has a relatively smaller scatter in period and a better separation from the low peaks. However, at the time intervals with gaps in the observations, the best fit appar-

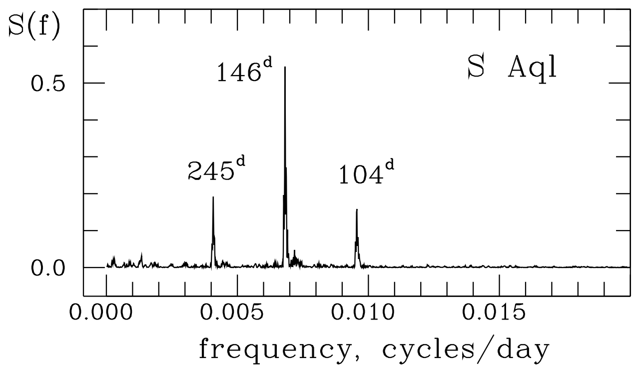

Figure 1: Periodogram of the observations of S Aql.

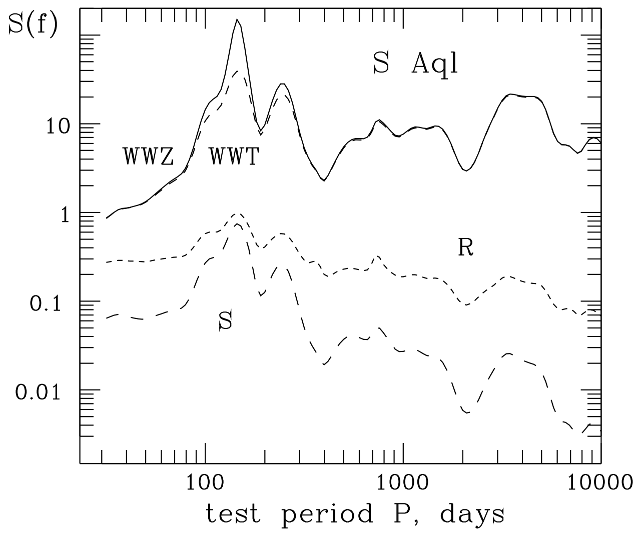

Figure 2: The main test functions for the wavelet analysis averaged over time as described by Andronov (1999).

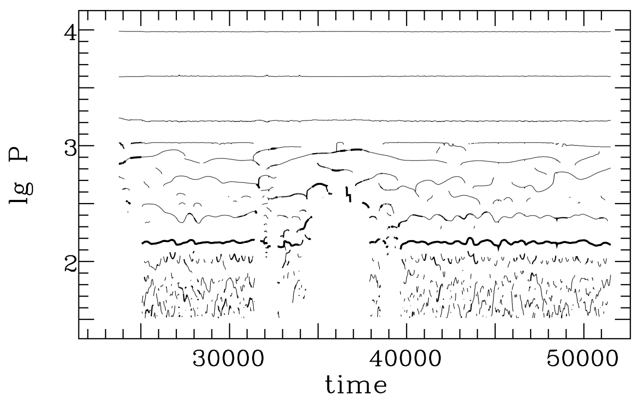

Figure 3: The "wavelet skeleton", i.e. dependence of periods corresponding to local peaks at the wavelet periodogram computed at a mean time t. The highest (at this t) peak is shown by most thick line, the peaks higher than 50% of the highest peak, are shown by an intermediate line, and lower peaks - by a thin line. For the main period value, $\lg P = 2.17$.

ently "switches" to larger values, increasing the effective width of the time filter.

Another effective method intermediate between the periodogram and wavelet analysis is the method of "running sines", where the weight function (time filter) is rectangular, and the period is constant. In this case, the fit (1) is computed for a local interval from $t - \Delta t$ to $t + \Delta t$, where Δt is called "the filter half-width". The parameters zero-point C_1, semi-amplitude r_1 and the phase (corresponding to T_{01}) are determined as functions of time. Such a method has advantages over the periodogram analysis (owing to study signals of low coherence) and the wavelet analysis (as the phase variations allow to study period variations of much smaller amplitude). An example of application of such a method for variations of UV Aur (symbiotic binary with a pulsationg component) is presented by Chinarova (1998). Statistical properties of model parameters are discussed by Andronov (1999).

2.3. Scalegram analysis using "running parabolae"

This method was proposed by Andronov (1997). It may be classified as the Morlet-type wavelet method with $\Delta t/P \ll 1$. Instead of making slow computations of trigonometric functions (few hours of computer time for a typical number of data of few dozens thousand), the polynomials are computed, and the weight function is non-zero only in the vicinities of the center.

In Figure 4, the test functions are shown, i.e. an unbiased error estimate σ_3 of the r.m.s. deviation of the observations from the fit; $\sigma[x_C]-$ the mean squared error estimate of the smoothing function at the arguments of observations; $S/N = \sigma_C/\sigma[x_C]-$ the amplitude signal-to-noise ratio, where σ_C^2 is the variance of the fit values.

As achieved, at small Δt, there is a standstill at the dependence $\sigma_3(\Delta t)$, showing no significant regular variability at high frequencies. The value $\sigma_3 = 0.^{\rm m}24$ is much smaller than $\sigma_0 = 0.^{\rm m}55$ corresponding to the harmonic sine fit, and is typical for the scatter of the visual observations from the AFOEV and VSOLJ databases. Thus "running parabolae" are better fit because of variations of the shape of the individual cycles.

The accuracy estimate of the fit $\sigma[x_C]$ reaches its local minimum of $0.^{\rm m}083$ at $\Delta t = 60.^{\rm d}5$, despite the global one is at $\Delta t \to \infty$, i.e., according to this criterion, it is "better" to assume the signal to be constant and to neglect variations. The signal-to-noise ratio is at maximum of 9.3 at $\Delta t = 44.^{\rm d}8$. This value should be used for further fits.

2.4. "O-C" analysis

This approach allows to study period variations with much smaller level of detectability than the methods of determination of local period. An extensive description and examples one may find e.g. in Kopal and Kurth

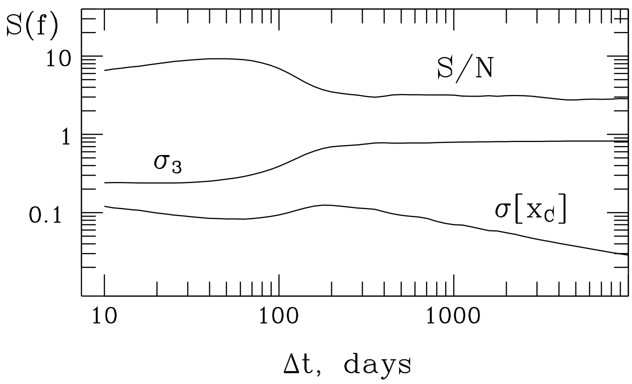

Figure 4: Scalegram for S Aql using "running parabolae".

(1957), Tsessevich (1970) and Kalimeris et al. (1994). The methods of "best accuracy" determination of the moments of "characteristic events" are currently proposed by Marsakova and Andronov (1996) and Chinarova and Andronov (2000). Some methods of study "O-C" diagrams are available using the program *OO* described in this volume (Andronov, 2001a).

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