INTEGRATION OF DIFFERENTIAL EQUATION FOR CELESTIAL BODIES' MOTION BY THE RUNGE-KUTT METHOD IN THE THIRD ORDER

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ABSTRACT. Equations of celestial bodies' motion in coordinates are solved by numerical Runge-Kutt method. The coefficients of Runge-Kutt method in the 10th order are obtained. The solutions are compared by methods of different order, for this purpose Runge-Kutt methods of the 4th, 5th and 10th orders have been realized in programs.

Key words: numerical integration; numerical simulation; methods of integration.

The Runge-Kutt method (hereafter R-K) is the highest order for the developed explicit R-K methods for solving differential equations. This 17-staged method was constructed by Hairer (1978). The method provides accuracy not worse than 10^{-15}. As is seen from the general scheme of R-K methods, it is necessary to know a_{ij}, b_i, c_i coefficients to construct an integration algorithm. In the general case these can be obtained from system solutions of linear equations with matrices of Wandermond type and free parameters. Hairer was thee first to obtain the above systems of linear equations. The analytical solution of these equation systems has been performed by us. At first coefficients ci and bi are determined from independent formulae with free parameters. Then nonzero coefficients a_{ij} are calculated. All in all the calculation of 119 coefficients a_{ij}, b_i, c_i for the R-K method of the 10th with the accuracy of 16 significant figures.

Table 1: The coefficients c and b

c	b
0.00000000000000000	0.03333333333333333
0.50000000000000000	-0.03333333333333333
0.52650910328800298	-0.12000000000000000
0.78976365493200447	0.00000000000000000
0.39392357321878806	0.00000000000000000
0.76665400000000000	-0.13000000000000000
0.28976365493200447	-0.18000000000000000
0.10847769751115627	0.00000000000000000
0.35738424175967745	0.27742918851774318
0.88252766196473235	0.18923747814892349
0.64261575824032255	0.27742918851774318
0.11747233803526765	0.18923747814892349
0.76665400000000000	0.13000000000000000
0.28976365493200447	0.18000000000000000
0.52650910328800298	0.12000000000000000
0.50000000000000000	0.03333333333333333
1.00000000000000000	0.03333333333333333

Table 2: The coefficients a_{ij}

	a
$a_{2,1}$	0.500000000000000000
$a_{3,1}$	0.249297267442865990
$a_{3,2}$	0.277211835845136990
$a_{4,1}$	0.197440913733001120
$a_{4,3}$	0.592322741199003350
$a_{5,1}$	0.197320549901158990
$a_{5,3}$	0.295083336687500560
$a_{5,4}$	-0.098480313369871490
$a_{6,1}$	0.131313418369284120
$a_{6,4}$	0.110154448085213840
$a_{6,5}$	0.525186133545502040
$a_{7,1}$	0.134200343335776690
$a_{7,4}$	0.696089003913604790
$a_{7,5}$	0.250497724903820550
$a_{7,6}$	-0.791023417221197550
$a_{8,1}$	0.072218277827166180
$a_{8,5}$	-0.058336331150054700
$a_{8,6}$	0.003047557905261750
$a_{8,7}$	0.091548192928783030
$a_{9,1}$	0.031255012451346830
$a_{9,6}$	0.000109123767407210
$a_{9,7}$	0.156725749578665400
$a_{9,8}$	0.169294355962258020

	a
$a_{10,1}$	0.011906555199089780
$a_{10,6}$	0.283437053152611850
$a_{10,7}$	-0.416312681148122000
$a_{10,8}$	0.264646513240441260
$a_{10,9}$	0.738850221520711440
$a_{11,1}$	0.023406547057290240
$a_{11,6}$	0.094493110802080540
$a_{11,7}$	-0.272872353991762250
$a_{11,8}$	0.224022150430302150
$a_{11,9}$	0.604381676387011710
$a_{11,10}$	-0.030815372444599860
$a_{12,1}$	0.045443773723211270
$a_{12,6}$	-0.001187993701161070
$a_{12,7}$	0.012035612762074010
$a_{12,8}$	0.075126916373159330
$a_{12,9}$	-0.018220889985728930
$a_{12,10}$	-0.000257153155717400
$a_{12,11}$	0.004532072019430440
$a_{13,1}$	0.178401191494440070
$a_{13,4}$	0.110154448085213840
$a_{13,5}$	0.525186133545502040
$a_{13,6}$	-0.489148527185302110
$a_{13,7}$	0.932444760083765200
$a_{13,8}$	-0.774475454248542890
$a_{13,9}$	-1.054903091927034580
$a_{13,10}$	0.131046704391395180
$a_{13,11}$	0.587049782903478850
$a_{13,12}$	0.620898052857084390
$a_{14,1}$	0.130220809295963570
$a_{14,4}$	0.696089003913604790
$a_{14,5}$	0.250497724903820550
$a_{14,6}$	-0.758949296998121780
$a_{14,7}$	-0.171517080561449580
$a_{14,8}$	-0.370217728103958150
$a_{14,9}$	0.124980912719016210
$a_{14,10}$	0.003353109831913580
$a_{14,11}$	-0.006632530935512160
$a_{14,12}$	0.429116584466715440
$a_{14,13}$	-0.037177853599988010
$a_{15,1}$	0.249297267442865990
$a_{15,2}$	0.277211835845136990
$a_{15,6}$	-0.145940581850349860
$a_{15,7}$	-0.799015888201494970
$a_{15,13}$	0.145940581850349860
$a_{15,14}$	0.799015888201494970
$a_{16,1}$	0.500000000000000000
$a_{16,3}$	-0.807097065368871710
$a_{16,15}$	0.807097065368871710
$a_{17,1}$	0.057320857210991220
$a_{17,2}$	-0.500000000000000000
$a_{17,3}$	-0.897470162794317560
$a_{17,6}$	-1.039909940192961010
$a_{17,7}$	-0.407356461949683440
$a_{17,8}$	-0.182830381505614480
$a_{17,9}$	-0.333659682493437140
$a_{17,10}$	0.395648533418683930
$a_{17,11}$	0.695056982748296490
$a_{17,12}$	0.271487312143254720
$a_{17,13}$	0.585423714778675740
$a_{17,14}$	0.958819065841793960
$a_{17,15}$	0.897470162794317560
$a_{17,16}$	0.500000000000000000

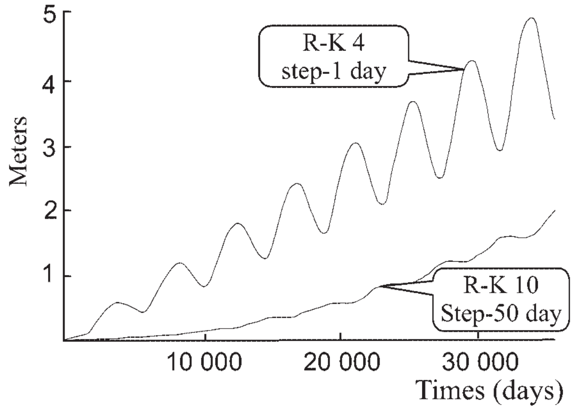

Figure 1: Accumultion of errors in integrating the motion: system of the Sun-Jupiter.

To illustrate the accuracy if R-K method of the 10th order (Kara, 2006), we have compared his solution with the accurate analytical one, and with a numerical solution by R-K method of the 4th order of a problem with 2 bodies in a pointwise approximation of the Sun-Jupiter and the Sun-Mercury system. The distance growth between analytical and numerical orbits is given in Fig.1 and Fig.2.

In case of the problem with N bodies there is not any analytical solution. Therefore the accuracy is estimated by Richardson method. The idea consists in our comparing simultaneosly two orbits in the node: orbits

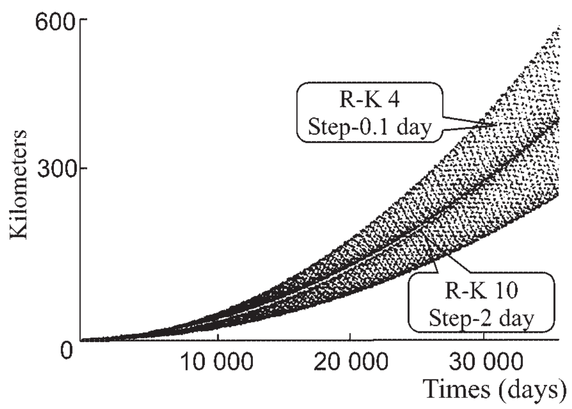

Figure 2: Accumultion of errors in integrating the motion: system of the Sun-Mercury.

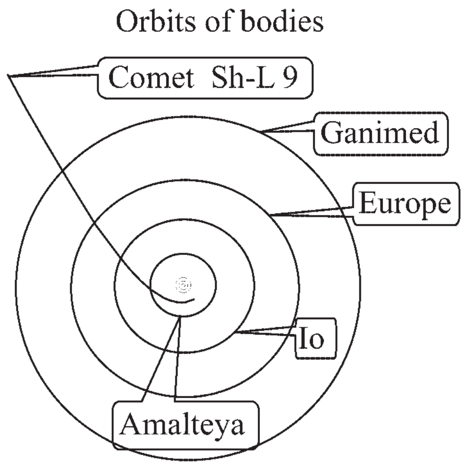

Figure 3: The orbit of Shoemaker - Levy 9 comet in joviocentric coordinate system (07.07.1990).

obtained with the whole and half-whole step of integration. The curve obtained like this is characterized by the growth of local itegration error. The comparison of error curves obtained by using the analytical method and a curve obtained by Richardson method demonstrates the growth of curves to be similar. The accuracy estimation by Richardson method can be obtained for the system with an arbitrary number of bodies. We have obtained the curves of integration error accumulation for the system the Sun - the Jupiter - the Saturn. These curves are similar to those in Fig.1 and Fig.2.

We can illustrate the use of R-K method of the 10th order for modeling motion in the solar system, as an example, consider the motion of comet Shoemacer-Levy 9. This well-known comet was captured by Jupiter in july 1992, and in july 1994 it collided with the planet. To determine initial conditions we took advantage of orbital element for the comet borrowing them from the paper by G. Sitarsky.

Two problems were set. The first problem by 29th of January the initial parameters for major planets of the solar system well have been obtained and equation of motions integrated. The second problem. Using the initial conditions obtained it is necessary to integrate the motion system approximately 200 years back for investigations the comet orbit evolution.

To solve the first problem the coordinates and velocities of major planets for 29th of January 1990 were borrowed from Astronomical Year - book 1990. In calculations the mass planets were used from the theory of DE405/LE405. The motion equations were integrated up to the moment of the 8th of July 1992. As a result of integration it was found that the moment of the first approximation was on the 7th of July 1992 at 18:36 of Universal Time, minimum distance between the comet and Jupiter amounted to 11000km (Fig.3).

This date differs from that shown in the paper by G. Sitarsky in 3.5 hours. Further integration within the frame of point model is impossible. At small reciprocal distances the tidal forces and perturbations arising because of nonsphericity of mass distribution in Jupiter are getting essential. Besides, we have not taken into account Jupiter's satellites influence.

To solve the second problem we took advantage of the same initial conditions. As a result of integration we have got a set of trajectories indicating the comet orbit evolution in the past. The comet orbit has not practically changed for the last 50 years of its existence.

As is seen from calculations, the comet orbit was more distant from the Sun before close approach Jupiter in January 1945. As a result of calculation the orbits lying between Jupiter and Saturn have been obtained. The outcome of this work is calculation of coefficients for the algorithm by R-K method 10 as accurate as 16 signs. This method can be successfully used for integration of celestial bodies motion in coordinates.

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