THE METHOD FOR FAST DETERMINATION OF GEOSTATIONARY EARTH SATELLITE ORBIT FROM ANGULAR COORDINATES MEASUREMENTS

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ABSTRACT. Classical Laplas method for determination of Keplerian orbital elements is described. The values obtained are improved by the method of differential corrections. The software has been provided and realizes such an algorithm to calculate orbital elements for high artificial Earth satellites. The approach suggested is used in processing artificial Earth satellites observations at Nikolayev Astronomical Observatory (NAO). Examples are given of orbit calculations for some geostationary satellites.

Key words: Satellites: optical observations, satellite orbits.

Introduction

Regular optical observations of geostationary artificial Earth satellites (GEO) have been carried out at Nikolaev Astronomical Observatory since 2002. The observations are made with two telescopes by using a "combined method" which was suggested by Nikolayev astronomers in 2000 Kovalchuk at al.(2000): with a multichannel telescope (MKT) (D=120 mm, F=2040 mm) and a speedy automatic complex (SAC) (D=300 mm, F=1500 mm). When using MKT telescopes the active telecommunication GEO are observed as a continious row within 2 or 5 howrs, when observing with SAC passive GEO (cosmic debris) are seen as short series only 20 minutes long in every two hours. We have already obtained and stored considerable bulk of observations. In order to estimate the accuracy of satellite observation, the knowledge of trajectory motion is needed. As is known, for the calculation of a satellite orbit it is necessary to take into account a lot of perturbing forces – the Earth's nonsphericity, the Moon's, the Sun's gravity, as well as atmospere resistance, the light pressure and so on. However, if it is necessary to obtain osculatory keplerian orbital elements for the epoch of observation, wilhout taking interest in a prolonged prediction of the satellite motion, this problem can be solved comparatively in a non-complicated way. Created and tested at the obtained NAO observations GEO algorithm for calculating osculatory elements has shown a good stability and convergence in calculating process. On the basis of the suggested software it is possible to rapidly and efficiently evaluate the error of observations due to comparing the individual observation with the satellite position on the osculating orbit.

1. The structure of calculations

In calculating, the system the Earth-satellite is examined within the frames of the problem with 2 bodies and is considered isolated. A twoaxial ellipsoid of rotation is taken for the Earth's form. At some moments of time ti the angular observations in the 2rd equatorial system $\alpha_i, \delta_i, i = 1, 2, ..., N$ are obtained from the Earth's surface.

The geocentric observer's position at the moment t_i, a radius-vector of the observing site $\vec{R}_i$ can be found if the geographical latitude of observing site φ and local sidereal time s_i are known Abalakin at al. (1971), Aksenov (1977):

$$R_{xi} = -G_1\, cos\varphi\, coss_i,$$

$$R_{yi} = -G_1\, cos\varphi\, sins_i,$$

$$R_{zi} = -G_2\, sin\varphi,$$

where

$$G_1 = \frac{1}{\sqrt{1-(2f-f^2\sin^2\varphi)}} + h,$$

$$G_2 = \frac{(1-f)^2}{\sqrt{1-(2f-f^2\sin^2\varphi)}} + h,$$

f – the Earth's polar compression,
h – altitude of the observing site above the sea level.
The process of orbit determination is divided into two stages: initial orbit determination – an approximate construction of a wholly unknown orbit initially and improvement of the orbit.

1.1. An initial orbit determination

An initial orbit determination is carried out by using Laplas method Escobal (1965). Theoretically, for the initial orbit determination only three observations of the satellite positions are needed. This method is based upon the investigation of differential equation of celestial body motion

$$\frac{d^2\vec{r}}{dt^2} = -k_E^2\, M_E \frac{\vec{r}}{r^3},$$

where
$\vec{r}$ is a geocentric satellite radius-vector, $k_E = 0.07436574 \frac{r.e.^{3/2}}{min\, m.e.^{1/2}}$ is a geocentric gravitational constant, M_E – the Earth's mass.
Passing in these differential equations to the modified time we obtain

$$\frac{d^2\vec{r}}{dt^2} = \ddot{\vec{r}} = -\frac{\vec{r}}{r^3},$$

where

$$\tau \equiv k_E(t-t_0),$$

t_0 is the time referred to the beginning of the year, t is nowdays.
The unit vector $\vec{L}_i$ directed along the vector of inclined distance $\rho_i\vec{L}_i = \vec{\rho}_i$ of the satellite:

$$\vec{L}_i = \begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} cos\delta_i\, cos\alpha_i \\ cos\delta_i\, sin\alpha_i \\ sin\delta_i \end{pmatrix}_i, i = 1,2,3.$$

Differentiating the previous equation twice we substitute the second derivative $\ddot{\vec{r}}$ and taking into account $\vec{r} = \rho\vec{L} - \vec{R}$, we obtain

$$\ddot{\rho}\,\vec{L} + 2\,\dot{\vec{L}}\dot{\rho} + \rho\Big(\ddot{\vec{L}} + \frac{\vec{L}}{r^3}\Big) = \ddot{\vec{R}} + \frac{\vec{R}}{r^3}.$$

The vector $\vec{L}$ is known from observations, the radius-vector $\vec{R}$ has been found earlier; the derivatives $\dot{\vec{L}},\ddot{\vec{L}},\dot{\vec{R}},\ddot{\vec{R}}$ can be determined numerically by using the Lagrangian polynomial for three nodes.
For the second observation we have

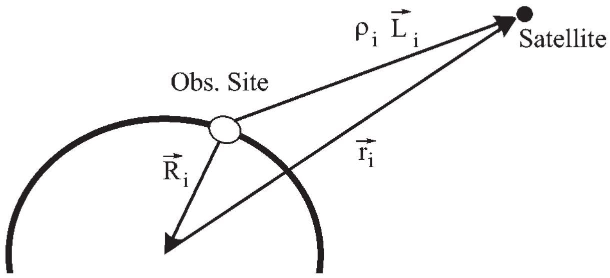

Figure 1: Basic vectors of the model

$$\dot{\vec{L}}_2 = \frac{\tau_3}{\tau_1(\tau_1-\tau_3)}\vec{L}_1 - \frac{\tau_1+\tau_3}{\tau_1\tau_3}\vec{L}_2 - \frac{\tau_1}{\tau_3(\tau_3-\tau_1)}\vec{L}_3,$$

$$\ddot{\vec{L}}_2 = \frac{2}{\tau_1(\tau_1-\tau_3)}\vec{L}_1 + \frac{2}{\tau_1\tau_3}\vec{L}_2 + \frac{2}{\tau_3(\tau_3-\tau_1)}\vec{L}_3,$$

$$\dot{\vec{R}}_2 = \frac{\tau_3}{\tau_1(\tau_1-\tau_3)}\vec{R}_1 - \frac{\tau_1+\tau_3}{\tau_1\tau_3}\vec{R}_2 - \frac{\tau_1}{\tau_3(\tau_3-\tau_1)}\vec{R}_3,$$

$$\ddot{\vec{R}}_2 = \frac{2}{\tau_1(\tau_1-\tau_3)}\vec{R}_1 + \frac{2}{\tau_1\tau_3}\vec{R}_2 + \frac{2}{\tau_3(\tau_3-\tau_1)}\vec{R}_3,$$

where $\tau_1 = k_E(t_1-t_2)$, $\tau_3 = k_E(t_3-t_2)$.
Taking into account that r_2 is known, we write the system in three scalar equations

$$\ddot{\rho}_2\, L_{2x} + 2\,\dot{L}_{2x}\dot{\rho}_2 + \rho_2\Big(\ddot{L}_{2x} + \frac{L_{2x}}{r_2^3}\Big) = \ddot{R}_{2x} + \frac{R_{2x}}{r_2^3},$$

$$\ddot{\rho}_2\, L_{2y} + 2\,\dot{L}_{2y}\dot{\rho}_2 + \rho_2\Big(\ddot{L}_{2y} + \frac{L_{2y}}{r_2^3}\Big) = \ddot{R}_{2y} + \frac{R_{2y}}{r_2^3},$$

$$\ddot{\rho}_2\, L_{2z} + 2\,\dot{L}_{2z}\dot{\rho}_2 + \rho_2\Big(\ddot{L}_{2z} + \frac{L_{2z}}{r_2^3}\Big) = \ddot{R}_{2z} + \frac{R_{2z}}{r_2^3},$$

The determinant of this system is easily brought into the form

$$D_\Delta = 2\begin{vmatrix} L_{2x} & \dot{L}_{2x} & \ddot{L}_{2x} \\ L_{2y} & \dot{L}_{2y} & \ddot{L}_{2y} \\ L_{2z} & \dot{L}_{2z} & \ddot{L}_{2z} \end{vmatrix}.$$

By the Kramer rule, considering $D_\Delta \neq 0$,

$$\rho_2 = 2\frac{\begin{vmatrix} L_{2x} & \dot{L}_{2x} & \ddot{R}_{2x} \\ L_{2y} & \dot{L}_{2y} & \ddot{R}_{2y} \\ L_{2z} & \dot{L}_{2z} & \ddot{R}_{2z} \end{vmatrix}}{D_\Delta} + 2\frac{\begin{vmatrix} L_{2x} & \dot{L}_{2x} & R_{2x} \\ L_{2y} & \dot{L}_{2y} & R_{2y} \\ L_{2z} & \dot{L}_{2z} & R_{2z} \end{vmatrix}}{r_2^3 D_\Delta},$$

and similarly

$$\dot{\rho}_2 = \frac{\begin{vmatrix} L_{2x} & \ddot{R}_{2x} & \ddot{L}_{2x} \\ L_{2y} & \ddot{R}_{2y} & \ddot{L}_{2y} \\ L_{2z} & \ddot{R}_{2z} & \ddot{L}_{2z} \end{vmatrix}}{D_\Delta} + \frac{\begin{vmatrix} L_{2x} & R_{2x} & \ddot{L}_{2x} \\ L_{2y} & R_{2y} & \ddot{L}_{2y} \\ L_{2z} & R_{2z} & \ddot{L}_{2z} \end{vmatrix}}{r_2^3 D_\Delta}.$$

Introducing notations

$$D_a = \begin{vmatrix} L_{2x} & \dot{L}_{2x} & \ddot{R}_{2x} \\ L_{2y} & \dot{L}_{2y} & \ddot{R}_{2y} \\ L_{2z} & \dot{L}_{2z} & \ddot{R}_{2z} \end{vmatrix}, \; D_b = \begin{vmatrix} L_{2x} & \dot{L}_{2x} & R_{2x} \\ L_{2y} & \dot{L}_{2y} & R_{2y} \\ L_{2z} & \dot{L}_{2z} & R_{2z} \end{vmatrix},$$

$$D_c = \begin{vmatrix} L_{2x} & \ddot{R}_{2x} & \ddot{L}_{2x} \\ L_{2y} & \ddot{R}_{2y} & \ddot{L}_{2y} \\ L_{2z} & \ddot{R}_{2z} & \ddot{L}_{2z} \end{vmatrix}, \; D_d = \begin{vmatrix} L_{2x} & R_{2x} & \ddot{L}_{2x} \\ L_{2y} & R_{2y} & \ddot{L}_{2y} \\ L_{2z} & R_{2z} & \ddot{L}_{2z} \end{vmatrix},$$

we have

$$\rho_2 = 2\frac{D_a}{D_\Delta} + \frac{2}{r_2^3}\frac{D_b}{D_\Delta},$$

$$\dot{\rho}_2 = \frac{D_c}{D_\Delta} + \frac{2}{r_2^3}\frac{D_d}{D_\Delta},$$

where all the determinants D, D_a, D_b, D_c, D_d are known. If we introduce the equation symbols

$$A = 2\frac{D_a}{D_\Delta},\; B = 2\frac{D_b}{D_\Delta},\; C = \frac{D_c}{D_\Delta},\; D = \frac{D_d}{D_\Delta},$$

and add here an evident geometric bond (theorem of cosines)

$$r_2^2 = \rho_2^2 - 2\rho_2 \vec{L}_2 \vec{R}_2 + R_2^2,$$

taking into account that the scalar product of two known vectors in the coordinate form is

$$-2\vec{L}_2\vec{R}_2 = -2(L_{2x}R_{2x} + L_{2y}R_{2y} + L_{2z}R_{2z}) \equiv C_S$$

then the problem is reduced to solving the system of three nonlinear equations with three unknowns:

$$\rho_2 = A + \frac{B}{r_2^3},\; \dot{\rho}_2 = C + \frac{D}{r_2^3},\; r_2^2 = \rho_2^2 + C_S\,\rho_2 + R_2^2.$$

Solving it by numerical methods we can find all the unknown scalars $\rho_2, \dot{\rho}_2, r_2$. After this we find vectors as well

$$\vec{r}_2 = \rho_2\vec{L}_2 + \vec{R}_2,\; \dot{\vec{r}}_2 = \dot{\rho}_2\,\vec{L}_2 + \rho_2\,\dot{\vec{L}}_2 + \dot{\vec{R}}_2,$$

from which it is easy to pass over to Keplerian orbital elements of the satellite.

1.2. Orbital elements improvement

At the next stage of calculations the improvement of orbital elements obtained by differential correction takes place Escobal (1965).

Let for the given satellite the measurements of angular coordinates be carried out which we conventionally designate by letter A_0. Using the first obtained approximation of orbital elements for the corresponding moments we calculate angular coordinate values. Let A_c denote the results obtained. If the orbital elements have been determined correct, the residual $\Delta = A_0 - A_c$ must be equal to zero. However, commonly this residual does not turn into zero. This testifies to the inaccuracy of initial orbit determination, to the errors of observing site position determination, to the presence of perturbations influencing the satellite when it moves from the p osition towards the epoch to any other position.

Let us we residuals Δ_i obtained which represent the differences between observational results and calculated positions for the moments of observation t_i.

Thus, the vector of state

$$\Big[\vec{r}_2, \dot{\vec{r}}_2\Big]_{t=t_2} = \Big[x_2, y_2, z_2, \dot{x}_2, \dot{y}_2, \dot{z}_2,\Big]_{t=t_2}$$

is determined in the first approximation. Suppose further, that the satellite's positions and velocities are precomputed for two moments of time

$$\Big[x_i, y_i, z_i, \dot{x}_i, \dot{y}_i, \dot{z}_i,\Big]_{t=t_i}, i = 1, 3.$$

Then for all the moments of time t_i ($i = 1, 2, 3$) we can find topocentric satellite's coordinates α_{tc}, δ_{tc} – these are calculated values of measured magnitudes. Moreover, there are measured values of topocentric coordinates corresponding to moments t_i, so

$$\Delta\alpha_i = (\alpha_{t0})_i - (\alpha_{tc})_i,$$

$$\Delta\delta_i = (\delta_{t0})_i - (\delta_{tc})_i$$

represent 6 residuals. These residuals show the degree of perturbations affecting the orbit and the magnitude of the above inaccuracies.

As the measured angles are functions of a state vector at some arbitrary epoch, they can be presented in the form of functions.

$$\alpha_t = \alpha_t\Big(x_2, y_2, z_2, \dot{x}_2, \dot{y}_2, \dot{z}_2\Big),$$

$$\delta_t = \delta_t\Big(x_2, y_2, z_2, \dot{x}_2, \dot{y}_2, \dot{z}_2\Big).$$

Substituting differentials of these functions for finite increments:

calculated. Because of errors in observations, inaccuracies of observational site coordinates and deviation of the satellite motion from the Keplerian orbit the discrepancies between the observed and estimated $(O - C)$ positions increase faster and faster in the course of time. Much better agreement with a number of observation is attained if a series points for the orbit determination is located at the beginning, in the middle and at the end of the series. However, it is evident that the orbit determination from three points will inevitably result in observational information losses.

1.3 The orbit improvement by using all the observations

The described method of differential corrections or differential orbit correlation has been extended for N-observations. In this case we don't obtain 6 but 2N conventional equation of the form

$$\Delta\alpha_i = \frac{\partial\alpha_i}{\partial x_k}\Delta x_k + \frac{\partial\alpha_i}{\partial y_k}\Delta y_k + \frac{\partial\alpha_i}{\partial z_k}\Delta z_k + \frac{\partial\alpha_i}{\partial \dot{x}_k}\Delta \dot{x}_k + \frac{\partial\alpha_i}{\partial \dot{y}_k}\Delta \dot{y}_k + \frac{\partial\alpha_i}{\partial \dot{z}_k}\Delta \dot{z}_k,$$

$$\Delta\delta_i = \frac{\partial\delta_i}{\partial x_k}\Delta x_k + \frac{\partial\delta_i}{\partial y_k}\Delta y_k + \frac{\partial\delta_i}{\partial z_k}\Delta z_k + \frac{\partial\delta_i}{\partial \dot{x}_k}\Delta \dot{x}_k + \frac{\partial\delta_i}{\partial \dot{y}_k}\Delta \dot{y}_k + \frac{\partial\delta_i}{\partial \dot{z}_k}\Delta \dot{z}_k,\ i = 1, 2, ..., N.$$

with six unknown corrections to the vector of state at the moment t_k - $\Delta x_k, \Delta y_k, \Delta z_k, \Delta \dot{x}_k, \Delta \dot{y}_k, \Delta \dot{z}_k$.

This system is solved by the least square method, as a result we have 6 normal equations of the form

$\lambda_j = a_j\Delta x_k + b_j\Delta y_k + c_j\Delta z_k + d_j\Delta \dot{x}_k + e_j\Delta \dot{y}_k + f_j\Delta \dot{z}_k$, where j=1,2,...6.

Or in the matrix form

$$\begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \lambda_4 \\ \lambda_5 \\ \lambda_6 \end{bmatrix} = M_N \begin{bmatrix} \Delta x_N \\ \Delta y_N \\ \Delta z_N \\ \Delta \dot{x}_N \\ \Delta \dot{y}_N \\ \Delta \dot{z}_N \end{bmatrix}, \text{ where}$$

$$M_N = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 & e_1 & f_1 \\ a_2 & b_2 & c_2 & d_2 & e_2 & f_2 \\ a_3 & b_3 & c_3 & d_3 & e_3 & f_1 \\ a_4 & b_4 & c_4 & d_4 & e_4 & f_4 \\ a_5 & b_5 & c_5 & d_5 & e_5 & f_5 \\ a_6 & b_6 & c_6 & d_6 & e_6 & f_6 \end{bmatrix}.$$

The further decision course is not different from the method described above for three observations.

2. The obtained result

The described method of orbital elements determination was algorithmized and the software was created on its basis.The following examples illustrate the processing of GEO observations carried out at Scientific Reseach Institute NAO.

The use of this software enables to solve various problems starting from finding out the malfunction moments of the observational apparatus complex to high-accurate determination for telecommunication GEO orbits observed in long uniform series and "cosmic refuse".

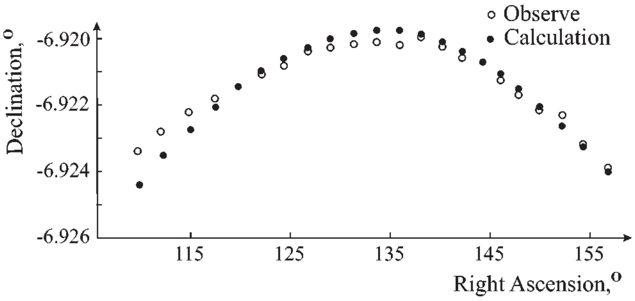

Figure 2: The positions of satellite 25515 on the 17th-18th, January, 2003.

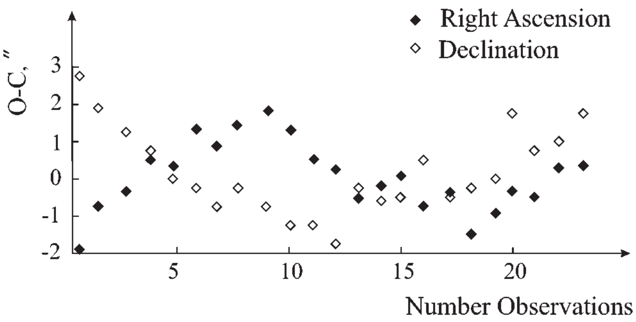

Figure 3: Residuals $O-C$ of satellite 25515 within 17-18th, January, 2003.

The absolute values $O-C$ proved to be rather significant (Fig.3). Observational data analysis shows that in the middle of observations after point 13 a malfunction in the apparatus part has taken place. Therefore, the first and the second part of the evening should be processed independently. The values of residuals $O-C$ do not exceed $1'' - 1.5''$.

The procedure suggested was tested on a short series of observations for passive geostationars (cosmic refuse) and it showed a good convergence (Fig.4,5).

In case within an observational period the perturbed accelerations can be neglected (for the high satellites it is justified) and the observational site is known with

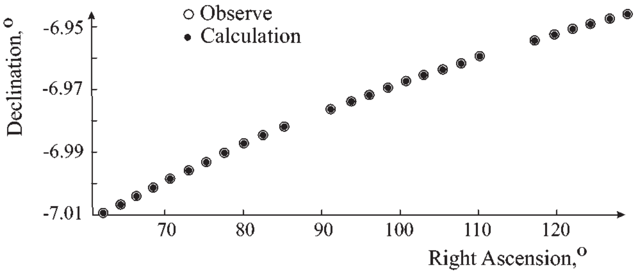

Figure 4: The positions of satellite 27441 during the 7-8th of November 2003

orbit determination is located at the beginning, in the middle and at the end of the series. However, it is evident that the orbit determination from three points will inevitably result in observational information losses.

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$$\Delta\delta_i = \frac{\partial\delta_i}{\partial x_k}\Delta x_k + \frac{\partial\delta_i}{\partial y_k}\Delta y_k + \frac{\partial\delta_i}{\partial z_k}\Delta z_k + \frac{\partial\delta_i}{\partial \dot{x}_k}\Delta\ \dot{x}_k + \frac{\partial\delta_i}{\partial \dot{y}_k}\Delta\ \dot{y}_k + \frac{\partial\delta_i}{\partial \dot{z}_k}\Delta\ \dot{z}_k,\ i = 1, 2, ..., N.$$

with six unknown corrections to the vector of state at the moment t_k – $\Delta x_k, \Delta y_k, \Delta z_k, \Delta\ \dot{x}_k, \Delta\ \dot{y}_k, \Delta\ \dot{z}_k$.

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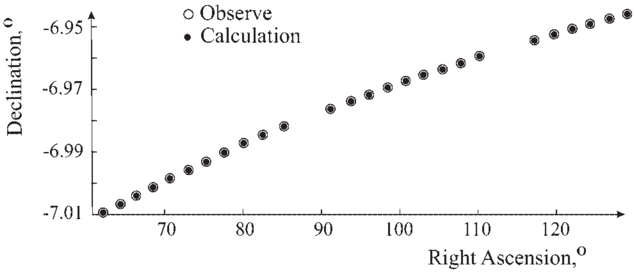

Figure 4: The positions of satellite 27441 during the 7-8th of November 2003.

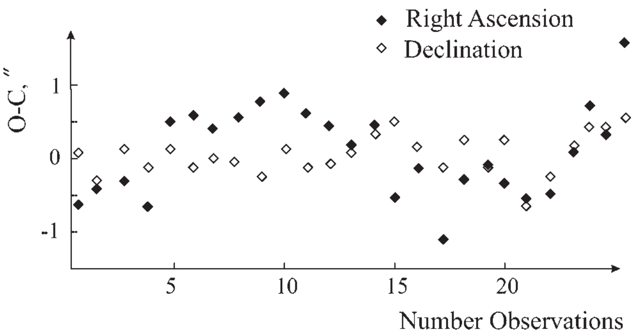

Figure 5: Residuals $O - C$ of satellite 27441 within 7-8th of November 2003.

2. The obtained result

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The procedure suggested was tested on a short series of observations for passive geostationars (cosmic refuse) and it showed a good convergence (Fig. 4, 5).

In case within an observational period the perturbed accelerations can be neglected (for the high satellites it is justified) and the observational site is known with a sufficient accuracy (the error does not exceed several metres), then the value of residuals $O - C$ can serve as an accuracy criterion for observations. On this basis the software enables to evaluate the accuracy of the

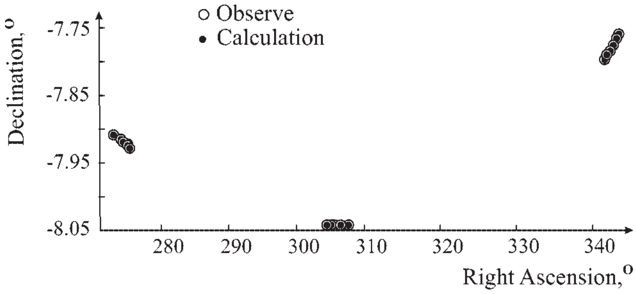

Figure 6: The positions of satellite 95035 within 21st-22nd of July 2004.

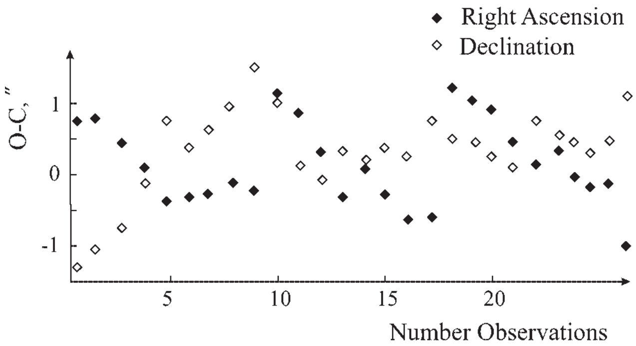

Figure 7: Residuals $O - C$ of satellite 95035 during 21st and 22nd of July 2004.

calculated orbital elements.

For this purpose, in the numerical model each observed coordinate pair is incremented within the limits of O to $O - C$. The value of increment satisfies a uniform distribution. Based upon modified observed coordinates the modified orbital elements are calculated. This process is repeated 150 times, thereafter from the obtained data files of modified orbital elements, their average values and root-mean-square deviations are computed. The value of a root-mean-square orbital element deviation from the mean value of data files specifies the accuracy of this element determination derived from angular observations.

An example of a good continious series of observations can be given. It was the evening of the 7-8th of November, 2003, satellite 27441 was observed. The root-mean-square deviations of orbital elements are as follows:

$p = 42160 \pm 0.6$ km,
$e = 0.00026 \pm 0.00002$,
$M_0 = 224 \pm 4^o$,
$i = 0.05021 \pm 0.00004^o$,
$\omega = 179.9 \pm 0.8^o$,
$\Omega = 84.02 \pm 0.05^o$.

The example of a good series of observations is the evening on the 21st-22nd July, 2004, satellite 95035. The root-mean-square deviations of orbital elements are as follows:

$p = 38307.1 \pm 1.3$ km,
$e = 0.001497 \pm 0.000005$,
$M_0 = 310 \pm 3^o$,
$i = 0.96779 \pm 0.00005^o$,
$\omega = 350.6 \pm 0.7^o$,
$\Omega = 32.01 \pm 0.002^o$.

Summary

On the Laplas method basis with further improving the method of differential corrections, the software has been created which permits to carry out the primary processing of terrestrial optical observations of geostationary satellites. Though the process of calculations is of an iteration character, it converges rather fast, and in practice it has shown good stability. As a result, both the quality of observations and the expected accuracy of the satellite's orbital elements can be estimated. The possibilities of modern computer facilities enable to perform all the computations within several seconds. The procedure described has been tested and is successfully used at Scientific Reseach Institute "Nikolayev Astronomical observatory".

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