STRUCTURE AND EVOLUTION OF THE PROTOTYPE MAGNETIC CATACLYSMIC VARIABLE AM HER: CLUES FROM X-RAY (CHANDRA) AND OPTICAL POLARIMETRY AND PHOTOMETRY. FOUR-COMPONENT MODEL OF THE AUTO-CORRELATION FUNCTION

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ABSTRACT. In this paper, we present the results of a study of the variability of the prototype polar AM Her using one of the complementary mathematical methods. Results of modeling of the auto-correlation analysis of a 24117 second long Chandra observation are discussed. The data have been binned to 1–second intervals. The corresponding auto-correlation function has been modeled to take into account the non-sinusoidal orbital variability of the object, onto which a strong flickering is added. The model of a second order trigonometric polynomial (orbital) + a first–order Auto-Regressive process ("shot–noise" flickering) does not seem to be not sufficient. A much better approximation can be achieved with a a four-parameter fit with exponential decay times $\tau_1 = 174$s and $\tau_2 = 9.8$s. A possible physical mechanism may be attributed to the longer time of the flare of the plasma blob ("spaghetti") falling onto the white dwarf as compared to flares corresponding to smaller blobs resulting from magneto-hydrodynamical instabilities. The results are compared with numerical models.

Key words: Stars: binary: cataclysmic; stars: individual: AM Her; data reduction.

1. Introduction

AM Herculis is the prototype of cataclysmic variables, in which the magnetic field of the white dwarf is strong enough (10-300 MGs) to prevent creation of the accretion disk and to keep the white dwarf in synchronism with the orbital motion. Since the discovery of circular and linear polarization of thes object in 1976 (Tapia 1977) and creation of the "standard" model (Chanmugam and Wagner, 1977), the system has attracted attention, and was a subject of multi-wavelength studies from ground-based and orbital observatories. Recent reviews on these exciting objects have been published by Warner (1995) Hellier (2002) and Andronov (2001).

To study long-term variations of the light curve, the photometric monitoring has started initially in the Astronomical Observatory of the Odessa State (now National) University, and, from 1989, it has been continued in the Crimean Astrophysical Observatory polarimetrically and photometrically in the UBVRI bands (1.25m telescope AZT-11) and in the wide R band (2.6m Shain Telescope). The highlights of this monitoring have been recently summarized by Andronov et al. (2002).

In 2000, we have organized an international campaign of simultaneous UBVRI ground-based optical and space (Chandra) X-Ray observations of AM Her. Detailed results of this campaign obtained will be presented elsewhere.

In this paper, we present result of one of the method of analysis, i.e. the modeling of the auto-correlation function (ACF) of the Chandra data taking into account non-sinusoidal shape of the orbital variability and the "shot noise".

2. Auto-Correlation Analysis

Previous studies of the behaviour of Auto-Correlation functions (ACF) have been usually made using relatively short data segments of optical data interrupted by measurements of background and comparison star. In this case, the subtraction of the sample mean value causes significant bias of the ACF (cf. Sutherland et al. 1978). Andronov (1994) presented a complete set of equations describing a bias in a general case of trend described by linear combinations of arbitrary basic functions and of arbitrary length. Assuming a "shot noise" model for rapid variability of this object, one may approximate the signal in terms of the first-order auto-regressive model (AR-1):

$$x_k = \psi x_{k-1} + \varepsilon_k, \qquad (1)$$

where x_k and ε_k are the signal and "exciting noise" at the k^{th} time, respectively, and ψ is the coefficient related to the time of exponential decay τ :

$$\psi = \exp(-T/\tau), \qquad \tau = -T/\ln\psi. \qquad (2)$$

Here T is the time step between subsequent data (cf. Jenkins and Watts, 1968).

For the signal consisting of a linear combination of m independent processes:

$$x_k = \sum_{\alpha=1}^{m} x_{\alpha k} \qquad (3)$$

one may obtain an autocovariation function

$$R_u = \sum_{k=1}^{N-u} x_k x_{k+u} \approx \sum_{\alpha=1}^{m}\sum_{k=1}^{N-u} x_{\alpha k} x_{\alpha,k+u} = \sum_{\alpha=1}^{m} R_{\alpha u}. \qquad (4)$$

The connection between the auto-correlation and auto-covariation functions $r_{\alpha u} = R_{\alpha u}/R_{\alpha 0}$, so

$$r_u = \sum_{\alpha=1}^{m} z_\alpha r_{\alpha u}, \qquad z_\alpha = R_{\alpha 0}/\sum_{\beta=1}^{m} R_{\beta 0}. \qquad (5)$$

is a partial contribution of variance of the α^{th} process to the total variance. The "equal to" sign may be written for mathematical expectation for stationary infinite data runs. For real finite-length data, it is only an approximation. We may suggest that 24117 points is a number, which is large enough for modeling.

In Fig. 1 and 2, the autocorrelation functions are shown for 3 realizations of AR-1 process with the parameter $\psi = 0.99$ and 0.999, respectively. The theoretical ACF $\rho_u = \psi^u$ is asymptotically decaying to zero, whereas the ACFs for realizations show apparent waves, including zero crossings.

The coincidence of theoretical and "observational" ("sample") curves is better at small shifts u than at large ones. Thus, for modeling, one should apply additional weights decaying with u.

One may note a good coincidence of sample ACFs with theoretic curves for $\psi = 0.99$ (and less, not shown). However, for $\psi = 0.999$, one may note systematical difference between theoretical and sample curves. Such a distorsion is caused by high correlations of data which increases the statistical error of the mean value, as was discussed by Sutherland et al. (1987). This effect had been studied by Andronov (1994) for arbitrary unbiased auto-correlation functions and basic functions (e.g. polynomials) used for the LS trend removal. The characteristic times obtained using this method for different stars have been published by Andronov (1999) and Halevin et al. (2002).

2. Modeling ACF for the Chandra Data

The phase light curve of AM Her exhibits two-hump structure, onto which the flickering is superimposed (cf. Szkody and Brownlee 1977). Previous studies of flickering assumed detrended segments of data, for which the values of $e-$folding time τ_e varied from 30 to 200 s from run to run (Bailey 1977). Panek (1980) estimated τ_e in a range of 70-96 sec, noting that the change of the adopted trend (e.g. from linear to cubic) may cause difference of estimates by a factor of 2-3 for the same run.

Shakhovskoy et al. (1992) had modelled the ACF by an expression

$$\rho_u = a\exp(-uT/\tau_d)\cos(u\pi\ T/2\tau_0)(1-u/N), \qquad (6)$$

where τ_d is characteristic time for an exponential decay, and τ_0 is the zero-crossing time.

In this paper, we use another approach, modeling not the residuals of the data from some fit, but the complete multi-component curve. To distinguish between contributions with different time scales, we use logarithmic axis for the time shift (Fig. 3). The final formula adopted for the smoothed "calculated" ACF is

$$\begin{aligned} r_{Cu} \;=\; & (z_1\cdot\cos(u\cdot 2\pi T/P_{orb}) + \\ & z_2\cdot\cos(u\cdot 4\pi T/P_{orb}) + \\ & z_3\cdot\exp(-uT/\tau_1) + \\ & z_4\cdot\exp(-uT/\tau_2))\cdot(1-u/N) \end{aligned} \qquad (7)$$

Here first two contributions correspond to the orbital variability and two-hump structure, respectively. It was natural to add the third term corresponding to the "shot noise" with time τ_1. However, as one may see from Fig.3, at short time scales, there is some additional component, which may also be approximated by a decaying exponent, but with much shorter time scale τ_2.

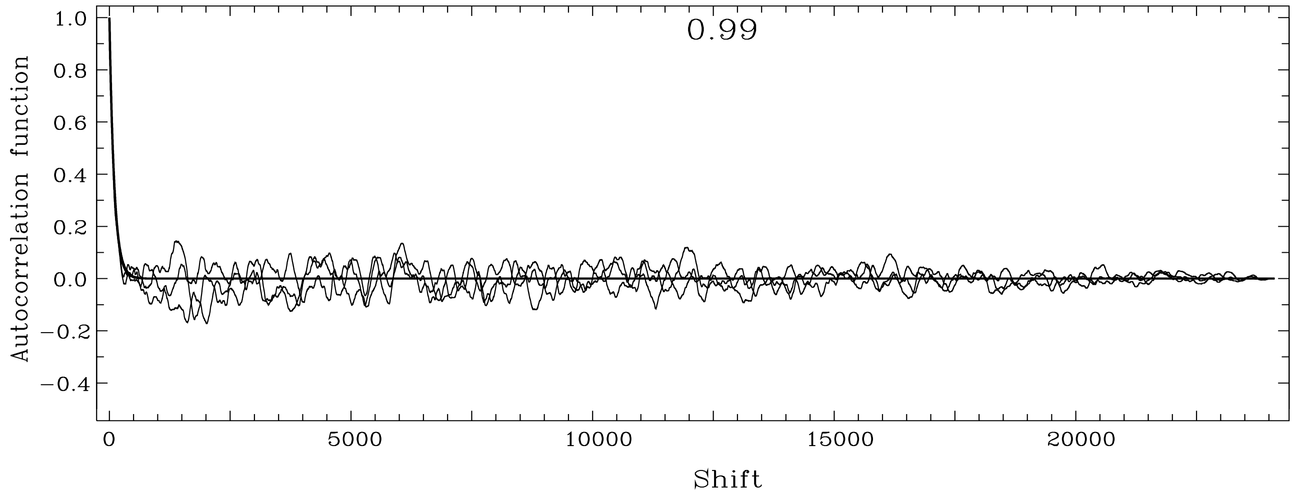

Figure 1. Auto-correlation function (ACF) for 3 realizations of the first-order Auto-regressive (AR-1) process and theoretical decaying exponent for $\psi = 0.99$.

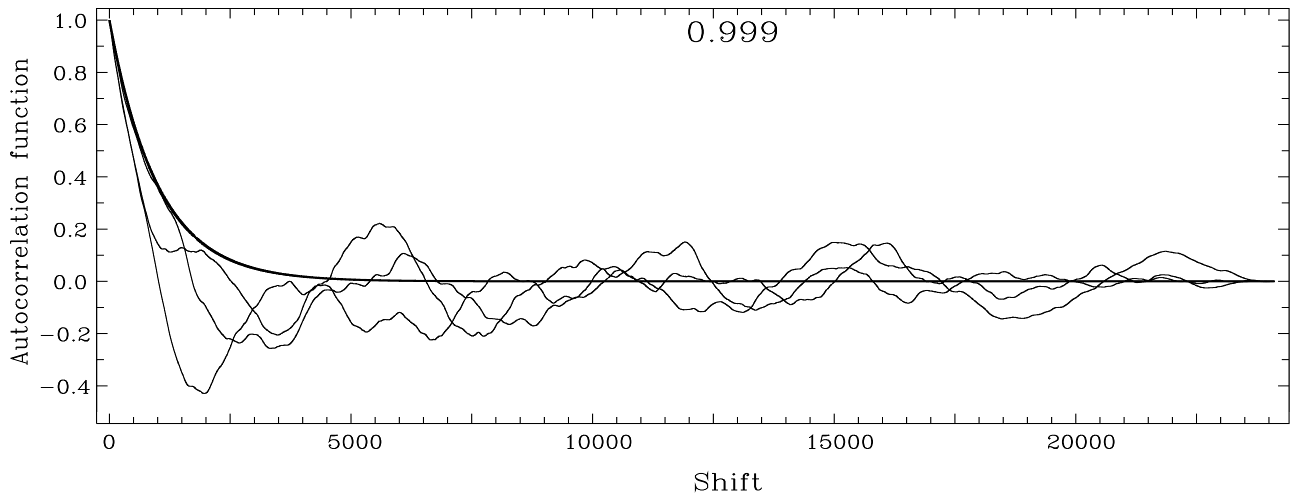

Figure 2. Auto-correlation function (ACF) for 3 realizations of the first-order Auto-regressive (AR-1) process and theoretical decaying exponent for $\psi = 0.999$.

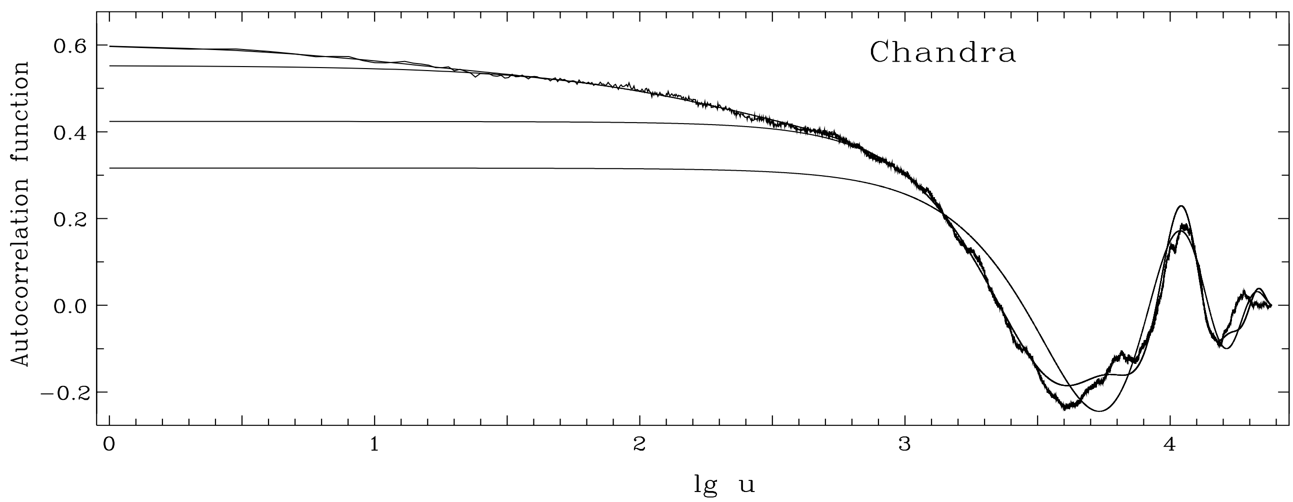

Figure 3. Auto-correlation function (ACF) in logarithmic scale for the Chandra observations and 4 particular sums of the model of ACF. The 4-parameter model (upper curve) practically coincides with the observational ACF.

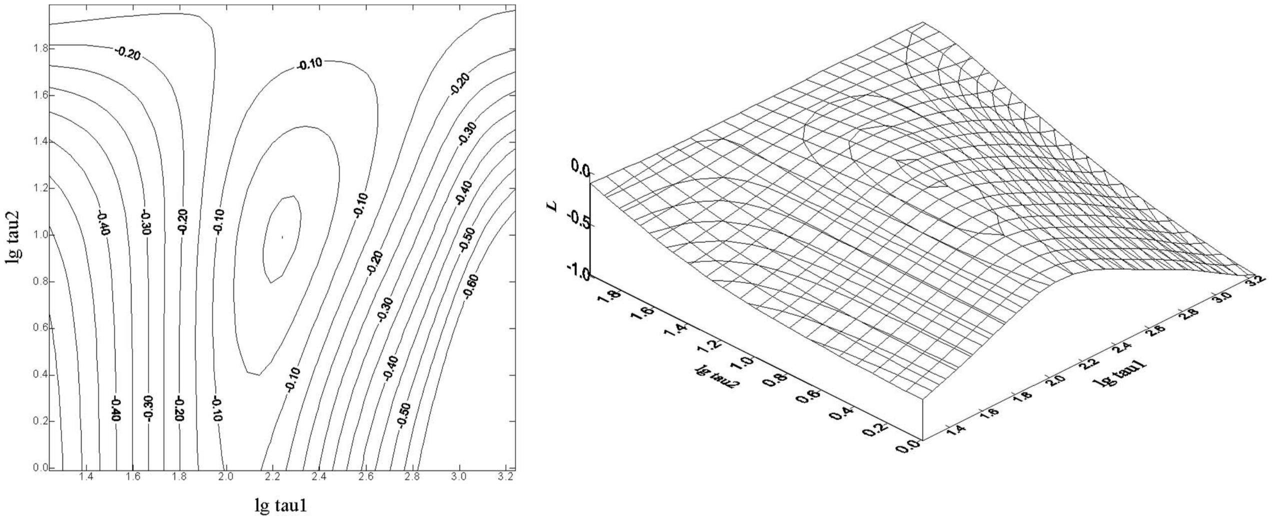

Figure 1: Dependence of the test function $L(\tau_1, \tau_2)$ on exponential decay times τ_1 and τ_2.

To determine both parameters τ_1 and τ_2, we have computed the test function

$$\Phi(\tau_1, \tau_2) = \sum_{k=1}^{N-1} w_u \cdot (r_u - r_{Cu})^2 \qquad (8)$$

with determination of parameters $z_1..z_4$ using linear LS method for each pair of (τ_1, τ_2). Here $w_u = (N-u)/Nu$ are weight coefficients which take into account the decreasing number of points used for determination of the sample ACF $(N-u)$ with increasing u. Additional division by u is unusual. We have chosen it, because, in logarithmic scale, $d \lg u/du = \lg e \cdot (du/u)$. So this divizor allows to approximate minimization of the sum by minimization of the corresponding integral, i.e. to make the approximation better over a whole range of shifts.

For illustrative purposes, we have transformed the function Φ to the logarithm of the likelyhood function $L(\tau_1, \tau_2) = \lg \exp(-\Phi(\tau_1, \tau_2)/2\Phi_0) = -\Phi(\tau_1, \tau_2)/(2\Phi_0) \cdot \lg e$. Here Φ_0 is the minimal value corresponding to statistically optimized values of τ_1 and τ_2. This function is shown in Fig 4.

The best fit values correspond to $\tau_1 = 174$s and $\tau_2 = 10$s. For complicated weights, the accuracy estimates may be computed using the generalized formulae published by Andronov (1997) instead of simplified expressions. The coefficients z_α are equal to $z_1 = 0.316 \pm 0.003$, $z_2 = 0.107 \pm 0.004$, $z_3 = 0.129 \pm 0.024$, $z_4 = 0.050 \pm 0.032$.

Despite the fourth coefficient exceeds it's error estimate by a factor of only 1.6, the four-component model corresponds to Φ_0 which is by a factor of 1.4 smaller than that for the third-component model with one AR process. Thus we may conclude that the four-component model is statistically significant. The relatively large error estimate just corresponds to nearly parallel behaviour of ACFs with both exponentially decaying shapes. The contrubution of non-correlated noise is $z_5 = 1 - z_1 - z_2 - z_3 - z_4 = 0.398$ is rather large because of of small photon count rate.

The faster decay of the ACF for small shifts argues that the flickering is the sum of least two AR processes with different characteristic times rather than one "shot-noise" process with non-exponential flares.

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