BRIEF DESCRIPTION OF THE RESTORATION METHOD OF A VECTOR VELOCITY FIELD IN GASEOUS DISKS OF SPIRAL GALAXIES

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ABSTRACT. This work is the brief description of the restoration method of a vector velocity field in gaseous disks of spiral galaxies from observed line-of-sight velocities (more detailed description see Lyakhovich et al., Astron.Zn. 1997).

Key words: Spiral galaxies, corotation radius, line-of-sight velocity field

From observations only line-of-sight velocities in gaseous disks of galaxies (only one component of velocities) can be obtain. Therefore for reconstruction of a complete vector velocity field it is necessary to use some model of the vector velocities. The most simple model is a model of pure circular motions. The line-of-sight velocities in a galactic gaseous disk in the framework of such model have the form

$$V_{obs}(R,\,\varphi) = V_s + V_{rot}(R)\cos\varphi\sin i,$$

where V_s is the systematic line-of-sight velocity of the galaxy, V_{rot} is the rotational velocity in the galactic plane, which depends only on, R, galactocentric radius, φ is the galactocentric azimuthal angle (equations of link between coordinates observed in the sky plane and galactocentric coordinates in the galactic plane see Lyakhovich et al., 1997), i is the inclination angle of the galaxy (angle of rotation from the sky plane to the galactic plane). However, there are the observational data (see, for example, Fridman and Polyachenko, 1984), which indicate to the existence of non-circular motions in gaseous disks of galaxies. It is natural to assume that these non-circular motions in spiral galaxies are connected to motions in spiral density waves.

The line-of-sight velocities accoding to the density wave theory have the form

$$\begin{aligned} V_{obs}(R,\,\varphi) &= V_s + V_r(R,\,\varphi)\sin\varphi\sin i + \\ &+ V_\varphi(R,\,\varphi)\cos\varphi\sin i. \end{aligned} \qquad (1)$$

where

$$\begin{aligned} V_r(R,\,\varphi) &= \tilde{V}_r(R,\,\varphi)\,, \\ V_\varphi(R,\,\varphi) &= V_{rot}(R) + \tilde{V}_\varphi(R,\,\varphi)\,. \end{aligned} \qquad (2)$$

Here V_r, V_φ are the radial and azimuthal galactocentric velocities in the gaseous disk, $\tilde{V}_r$ and $\tilde{V}_\varphi$ are the velocities caused by a density wave; for two-armed spiral galaxies they have the form:

$$\begin{aligned} \tilde{V}_r(R,\,\varphi) &= C_r(R)\,\cos(2\varphi - F_r(R))\,, \\ \tilde{V}_\varphi(R,\,\varphi) &= C_\varphi(R)\,\cos(2\varphi - F_\varphi(R))\,, \end{aligned} \qquad (3)$$

where C_r and C_φ are the radial and azimuthal amplitudes in spiral arms ($C_r \geq 0$, $C_\varphi \geq 0$), F_r and F_φ are the radial and azimuthal phases of velocity in spiral arms. Substituting (2) - (3) in (1), we obtain, that the observed velocities, in the framework of the model, should have the form

$$\begin{aligned} V_{obs}(R,\,\varphi) &= V_s + \sin i[a_1(R)\cos\varphi + b_1(R)\sin\varphi + \\ &+ a_3(R)\cos 3\varphi + b_3(R)\sin 3\varphi]\,, \end{aligned} \qquad (4)$$

where the coefficients a_1, b_1, a_3, b_3 are equal

$$\begin{aligned} a_1 &= +(C_r\sin F_r + C_\varphi\cos F_\varphi)/2 + V_{rot}, \\ b_1 &= -(C_r\cos F_r - C_\varphi\sin F_\varphi)/2, \\ a_3 &= -(C_r\sin F_r - C_\varphi\cos F_\varphi)/2, \\ b_3 &= +(C_r\cos F_r + C_\varphi\sin F_\varphi)/2\,. \end{aligned} \qquad (5)$$

Coefficients $a_1(R)$, $b_1(R)$, $a_3(R)$, $b_3(R)$ can be determined from observations of line-of-sight velocity field of the galaxy using the least-squares method (Lyakhovich et al., 1997).

System of the equations (5) consists of four equations with five unknown parameters $C_r(R)$, $C_\varphi(R)$, $F_r(R)$, $F_\varphi(R)$, $V_{rot}(R)$. Thus, one more equation is needed to make the system (5) complete. Such equation can be obtained from a density wave theory.

Variant 1. For galaxies with a small pitch angle of spiral arms, we have in WKB approximation (see Lyakhovich et al., 1997)

$$\begin{aligned} F_\varphi(R) &= F_r(R) - \pi/2, \quad \text{for} \quad R < R_c, \\ F_\varphi(R) &= F_r(R) + \pi/2, \quad \text{for} \quad R > R_c, \end{aligned} \qquad (6)$$

where R_c is the corotation radius of spiral arms. From (5) and (6) we obtain

$$C_r \cos F_r = b_3 - b_1, \qquad C_r \sin F_r = b_1 a_3/b_3 - a_3,$$
$$C_\varphi \cos F_\varphi = a_3 + b_1 a_3/b_3, \qquad C_\varphi \sin F_\varphi = b_1 + b_3,$$
$$V_{rot} = a_1 - a_3 b_1/b_3. \tag{7}$$

Thus, determining from observations $a_1(R)$, $a_3(R)$, $b_1(R)$, $b_3(R)$ and solving (7) we determine all required parameters.

Variant 2. For galaxies with a small pitch angle of spiral arms, we have in WKB approximation (see Lyakhovich et al., 1997)

$$\begin{aligned} F_r(R) &= F_\sigma(R) + \pi, \text{ for } R < R_c, \\ F_r(R) &= F_\sigma(R), \qquad \text{for } R > R_c. \end{aligned} \tag{8}$$

where F_σ is the density phase in spiral arms. From (5) and (8) we obtain

$$\begin{aligned} C_r \cos F_r &= b_3 - b_1 , \\ C_r \sin F_r &= (b_3 - b_1)\tan F_\sigma , \\ C_\varphi \cos F_\varphi &= 2a_3 + (b_3 - b_1)\tan F_\sigma, \\ C_\varphi \sin F_\varphi &= b_1 + b_3 , \\ V_{rot} &= a_1 - a_3 - (b_3 - b_1)\tan F_\sigma. \end{aligned} \tag{9}$$

The quantitative agreement between the first and second variants is the proof of correctness of the method.

Determination of a corotation radius R_c. Using (5), (6), (8) we can determine a corotation radius. Really, using (5), (6), and the fact, that $C_r > 0$ and $C_\varphi > 0$ we obtain

$$\begin{aligned} |b_3(R)| - |b_1(R)| \le 0, &\quad \text{for} \quad R < R_c, \\ |b_3(R)| - |b_1(R)| \ge 0, &\quad \text{for} \quad R > R_c. \end{aligned} \tag{10}$$

Thus, a corotation radius should be located in a region, where $|b_3(R)| - |b_1(R)|$ changes the sign.

Using (5), (8), and the fact, that $C_r > 0$ and $C_\varphi > 0$ we obtain

$$\begin{aligned} (b_3(R) - b_1(R)) \cos F_\sigma(R) &\le 0, \quad \text{for } R < R_c, \\ (b_3(R) - b_1(R)) \cos F_\sigma(R) &\ge 0, \quad \text{for } R > R_c. \end{aligned} \tag{11}$$

Thus, a corotation radius should be located in a region, where $(b_3(R) - b_1(R)) \cos F_\sigma(R)$ changes the sign.

The quantitative agreement between the first (10) and second (11) variants can be considered as a proof of correctness of this procedure.

Observed criterion for using of the method. From comparison between phases of the third harmonic of an observed velocity field and density phases in spiral arms we can determine region, where in galaxies equations (6) and (8) are fulfilled. Really, assuming, that the equations (6) and (8) are fulfilled, we obtain, that the third harmonic should have the form

$$H_3 = 1/2(C_\varphi(R) \mp C_r(R)) \cos(3\varphi - F_\sigma(R) - \pi/2). \tag{12}$$

Here "−" for $R < R_c$, "+" for $R > R_c$.

Knowing from observations a_3 and b_3 we can represent the third harmonic as $H_3 = C_3 \cos(3\varphi - F_3)$ ($C_3 \ge 0$). We obtain then

$$\begin{aligned} F_3 &= F_\sigma - \pi/2, \quad \text{for } R < R_c \text{ and } C_\varphi < C_r , \\ F_3 &= F_\sigma + \pi/2, \quad \text{for } R < R_c \text{ and } C_\varphi > C_r , \\ F_3 &= F_\sigma + \pi/2, \quad \text{for } R > R_c . \end{aligned} \tag{13}$$

This link between $F_3(R)$ and $F_\sigma(R)$ is the test for using of the method. This method was successfully used for restorations of the velociy fields of the galaxies NGC 157, NGC 3893, NGC 6181 (Fridman et al., 1997).

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References

Lyakhovich V.V., Fridman A.M., Khoruzhii O.V., Pavlov A.I.: 1997, *Soviet Astronomy Zh.*, **74**, 509.

Fridman, A.M., Polyachenko, V.L.: 1984, *Physics of gravitating systems.* N.Y. etc.: Springer–Verlag.

Fridman A.M., Khoruzhii O.V., Lyakhovich V.V., Avedisova V.S., Sil'chenko O.K., Zasov A.V., Afanasiev V.L., Rastorguev A.S., Dodonov S.N., Boulesteix J.: 1997, *Astroph. and Space Science* (in press).

Fridman A.M., Khoruzhii O.V., Lyakhovich V.V., Sil'chenko O.K., Zasov A.V., Afanasiev V.L., Dodonov S.N., Boulesteix J.: 1997, *Astrophys.J.* (in preparation).