CALCULATE METHOD OF THE TWILIGHT SKY BRIGHTNESS IN THE SOLAR VERTICAL, STIPULATED BY A MULTIPLE SCATTERING OF A SUNLIGHT

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ABSTRACT. Use of a twilight sounding method of the Earth's atmosphere with the aim to reveal aerosols inflow to the upper and medial atmosphere needs a method of separation twilight sky brightness to the brightness of primary and secondary twilight. Method of secondary twilight brightness calculation on a declination of the logarithm of twilight sky brightness caused by a multiple scattering of sunlight is discussed. The calculation procedure of a declination of the logarithm of secondary twilight brightness on observations in solar vertical is given. The use equal declination of the logarithm of secondary twilight brightness for all viewing directions in solar vertical is proved.

Key words: remote sounding, atmospheric sounding, twilight sounding, multiple scattering

The aerosol content in the Earth's atmosphere essentially influences the processes of chemical reactions in an atmosphere and heat balance of the Earth. Present article discusses the twilight sounding method that is used for definition of the inflow and time variations of aerosol content in medial and upper Earth's atmosphere. In the twilight sounding method the brightness of the sky is measured during morning and evening twilight, and contents of an aerosol in an atmosphere is calculated on the base of quantity of a directional scattering factor. To define the directional scattering factor the discussed method uses only the brightness that is stipulated by a single scattering of sunlight (primary twilight brightness). Therefore it is necessary to split the brightness of the twilight sky into the primary twilight brightness, the brightness of the sky stipulated by a dispersion of sunlight of the higher orders (secondary twilight brightness) and the brightness of night sky. General brightness of the twilight sky is

$$B = B_1 + B_2 + B_b \tag{1}$$

where B_1 – primary twilight brightness; B_2 – secondary twilight brightness; B_b – the night sky brightness.

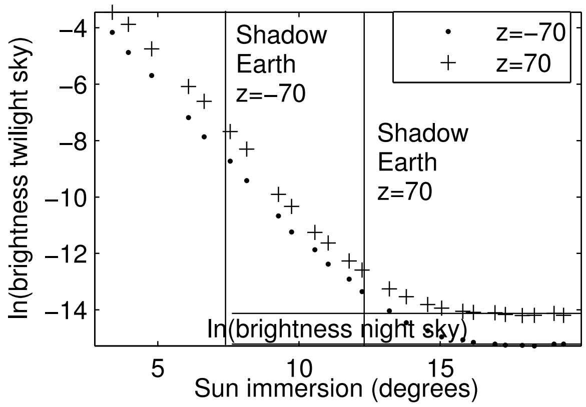

Figure 1: Observation in a solar vertical at a zenith distance 70 degrees in viewing directions on the Sun and from the Sun.

To split primary and secondary twilight brightness we used, so-called, gradient method (Zaginajlo Y.I. et al. 1999). According to this method in the zone of an Earth shadow where main part of twilight sky brightness is constituted by secondary twilight brightness, the dependence of the logarithmic twilight sky brightness on immersion of the Sun under horizon at a constant zenith distance of a viewing direction can be line fitted.

In a series of works (Smoktij O.I. 1967a, 1967b) the solution for sunlight multiple scattering in the spherically symmetric exponential atmosphere was offered. We showed (Shakun L.S. 2000), that it is possible to determine the relation between the declination of the approximating line in the zone of an Earth shadow and eigenvalues in the solution for multiple scattering of light in the exponential spherical atmosphere, the above mentioned solution. Then, using analytical formulas from the solution (Smoktij O.I. 1967b), it is possible to make an extrapolation of secondary twilight brightness in the zone of small immersions of the Sun where primary twilight brightness constitutes main part of all twilight sky brightness. The accuracy

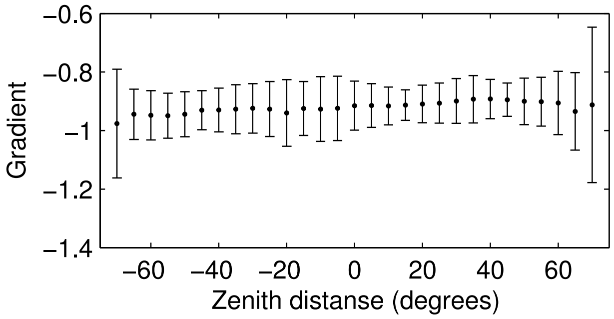

Figure 2: Declinations of fitting curves calculated separately in different viewing directions.

of calculation of secondary twilight brightness depends on accuracy of calculation of the Smoktij O.I. solution eigenvalues. Thus, it is necessary to calculate the declination of the line that approximates the secondary twilight brightness as accurately as possible. Let's call the declination of the line that approximates the secondary twilight brightness, the gradient of secondary twilight brightness.

Carrying out twilight observations in all the twilight vertical at one viewing direction it is possible to take no more than ten values of secondary twilight brightness in the Earth shadow zone. Obtained values are distributed nonuniformly on different viewing directions.

So, in the opposite of the Sun direction we can take about ten values of secondary twilight brightness, and in the direction to the Sun – one or two values. Thus, in all viewing directions the quantity of the values of secondary twilight brightness is not enough for accurate determination of declination of the approximating line (fig. 1). In the work (Shakun L.S. 2000) the gradient of secondary twilight brightness in the zone of an Earth shadow and the spectrum of Smoktij O.I. solution eigenvalues for the radiation transfer in spherically symmetric exponential atmosphere was connected with the height of an atmosphere in Smoktij's model. Most likely, this height of an atmosphere corresponds with the height of an atmosphere where an effective transfer of multiply scattered light takes place. It is logically to assume, that the height of an atmosphere where effective transfer of multiply scattered light takes place, does not depend on viewing direction. Therefore the gradient of secondary twilight brightness does not depend on viewing direction too. Really, the gradient of secondary twilight brightness differs insignificantly in different viewing directions, and in most cases this difference is less than the error of determination of the gradient of secondary twilight brightness (fig. 2). Thus, it is logically to determine the gradient of secondary twilight brightness using all the values of secondary twilight brightness. It is necessary to note, that all values of secondary twilight brightness will not lie on one line (fig. 1) because the zero-mark of the approximating line for each viewing direction varies. For simultaneous determination of zero-mark of the approximating lines and common declination of the lines it is necessary to put in special basis of the fitting functions.

The values of the secondary twilight brightness in the zone of an Earth shadow we shall fit with the following dependence:

$$\ln(B_2) = a_0 g + a_1 f_1(z) + \ldots + a_n f_n(z), \qquad (2)$$

where B_2 – the secondary twilight brightness; g – the immersion of the Sun under horizon; $f_i(z)$ – basis functions for determining the zero-mark of the approximating lines; a_0 - the gradient of secondary twilight brightness; a_i – zero-mark of the approximating lines.

At the selected viewing direction the fitting dependence should become:

$$\ln(B_2) = a_0 g + a_j, \qquad (3)$$

where j – the number of the selected viewing direction.

Thus, basis functions are defined as

$$f_j(z) = \begin{cases} 1, & where \quad z = z_j \\ 0, & where \quad z \neq z_j \end{cases} \qquad (4)$$

In order to make approximation it is necessary to allocate secondary twilight brightness from the measured twilight sky brightness. The most favorable zone to determinate the secondary twilight brightness is in the zone of an Earth shadow where primary twilight brightness can be neglected. In the zone of an Earth shadow measured brightness of the twilight sky may be represented as

$$B = B_2 + B_b + \varepsilon(B_2 + B_b) + \eta B_b, \qquad (5)$$

where B_2 – real secondary twilight brightness; B_b – real night sky brightness; ε – the relative accidental error of measurements of the twilight sky brightness; η – the relative accidental error of measurements of the night sky brightness.

In this case two types of accidental error of measurements can be distinguished. The first one is an accidental error of measurements of the twilight sky brightness as a results noise of the measuring devices and a flicker of natural atmosphere. The second accidental error, which we call an accidental measurement error of the night sky brightness, is a result of irregularity of the night sky brightness.

Except an accidental error of the night sky brightness the regular errors that depend on the way of determination the night sky brightness can be observed. The night sky brightness is being determined after the end of twilight by observations in a solar vertical. However the night sky brightness in the zone of secondary twilight domination differs slightly from the one measured on completion of twilight as a result of other orientation of solar vertical and rotation of the starry sky around of the celestial pole. The change of solar vertical position changes the position of viewing direction relatively to the objects on horizon. Illumination produced by objects on horizon, is non-uniform, therefore the real night sky brightness during the twilight will be different. Rotational displacement of the starry sky around the celestial pole results in bias of a viewing direction relatively to stars. Both factors can become apparent as trends that difficultly take into account in practice. Therefore measuring of night sky brightness after the end of twilight can be used only for estimation of the night sky brightness.

If we obtained the estimation of the night sky brightness, let subtract it from the total twilight sky brightness. Then in the zone of the Earth shadow the logarithm of the twilight sky brightness will be

$$\ln(B) = \ln(B_2 + B_b + \varepsilon(B_2 + B_b) + \eta B_b - \tilde{B}_b) = \\ = \ln(B_2) + \ln\left(1 + \varepsilon + \frac{(\varepsilon+\eta)B_b + \delta B}{B_2}\right), \quad (6)$$

where B – the observed secondary twilight brightness; $\tilde{B}_b$ – an estimation of the night sky brightness; $\delta B = B_b - \tilde{B}_b$ – an error of estimation of the night sky brightness.

In order to adequately determinate the gradient of secondary twilight by the least-squares method the next conditions must be fulfilled

$$1 + \varepsilon + \frac{(\varepsilon+\eta)B_b + \delta B}{B_2} \approx 1. \quad (7)$$

The measured night sky brightness after the end of twilight can be represented as:

$$B = B_b + (\varepsilon + \eta)B_b. \quad (8)$$

Assuming the night sky brightness as a constant value, it is easy to receive an estimation of the night sky brightness $\tilde{B}_b$ and an estimation of a standard deviation of the values of the night sky brightness $\sigma = \sqrt{<(\varepsilon+\eta),(\varepsilon+\eta)>}$. Let's find the immersions of the Sun at which the sky brightness meets the condition:

$$\frac{B - -\tilde{B}_b}{\sigma} > n. \quad (9)$$

It is correct for these immersions, that

$$\left|\varepsilon + \frac{(\varepsilon+\eta)B_b + \delta B}{B_2}\right| < \left|\varepsilon + \frac{1}{n} + \frac{\delta B}{B_2}\right|. \quad (10)$$

In the zone of immersions of the Sun under horizon where systematic error of an estimation of the night sky brightness exceeds the secondary twilight brightness we shall see the deviation from linearity of the logarithm of secondary twilight brightness. If the error of estimation of the night sky brightness is positive than we shall see the deviation from linear dependence to larger values and in the case of negative error – to smaller values (fig. 3).

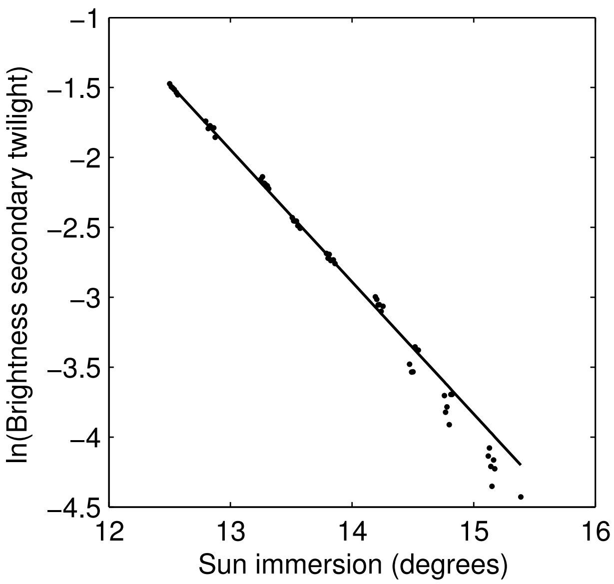

Figure 3: Approximation of secondary twilight brightness at a viewing direction on the Sun and a zenith distance 70 degrees.

As it is seen from above mentioned reasoning an error of each measuring depends on secondary twilight brightness. Therefore to adjust statistical significance of each observation and reduce the role of estimation error of the night sky brightness it is necessary to introduce the weight numbers.

Let measuring error of the device is ε_j and its standard deviation is ε; an estimation of the night sky brightness and its standard deviation are $\tilde{B}_b$ and σ accordingly. Then from the condition of equality of a standard deviation for all observations we have:

$$\left\langle w_j\left(\varepsilon_j(B_j - -\tilde{B}_b) + \sigma\right), w_i\left(\varepsilon_i(B_i - -\tilde{B}_b) + \sigma\right)\right\rangle = \\ = \varepsilon^2(B_j - -\tilde{B}_b)^2, \quad (11)$$

$$w_j^2 = \frac{\varepsilon^2}{\varepsilon^2 + \sigma^2/(B_j - -\tilde{B}_b)^2}, \quad (12)$$

where w_j – a weight number of the j twilight sky brightness.

It is evidently, that the less is the secondary twilight brightness and the higher nonuniformity of the night sky brightness the less are weight numbers.

Fig. 3 shows actual observations in a solar vertical at the constant zenith distance 70 degrees. It is seen, that the estimation of the night sky brightness is overstated and therefore secondary twilight brightness at great immersions noticeably declines downwards from the fitting line. If observations were obtained in the entire solar vertical, then each series of measuring turns into a point with higher statistical error. That is why in case observations were made in a solar vertical it is much more difficultly to reveal overstated estimations of the night sky brightness. In this case despite of overstated estimation of the night sky brightness, good estimation of secondary twilight brightness is obtained.

Let's compare now gradients of the secondary twilight brightness, that where obtained independently for each direction (fig. 2), and the gradient of secondary twilight brightness for the same observations, but on all directions of observations simultaneously. Average gradient of the secondary twilight brightness, obtained independently for each viewing direction, is equal 0.921 at the average standard deviation 0.0089, and the general gradient of secondary twilight brightness for all viewing directions is 0.933 at a standard deviation 0.0015. Thus, the method of the general gradient of secondary twilight brightness allows reaching considerably greater statistical significance of the factor of inclination of the logarithm of the secondary twilight brightness.

At last, let's compare the directional scattering factors obtained at approximation of secondary twilight brightness in each direction separately, and the one obtained as a result of determination of a general inclination of approximation of the secondary twilight brightness (fig. 4). The directional scattering factors obtained as a result of different approximations of secondary twilight brightness for different viewing directions, contain discontinuity of scattering angle dependence that is the result of discontinuity in approximation coefficients of the secondary twilight brightness. The directional scattering factors, obtained in case of general coefficient of an inclination of the secondary twilight brightness, have better smoothness and lower statistical error. Apparently, it is the evidence of better adequacy of such directional coefficients to physical reality.

Solid line marks the directional scattering factors that were determined at the general coefficient of inclination of approximation of the secondary twilight brightness. Points mark directional scattering factors that were obtained in the case of various coefficients of inclination of approximation lines of the secondary twilight brightness.

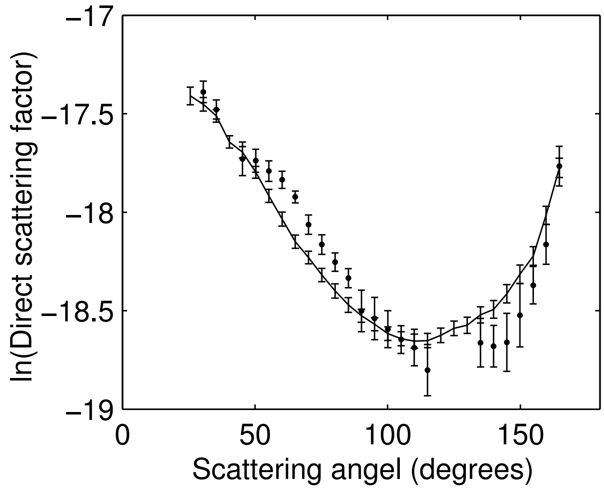

Figure 4: Directional scattering factors with a confidence interval 0.997 at height of 30 km in a wave length of 731 nm.

Thus the given procedure of determination of a general gradient of secondary twilight brightness guarantees better matching of directional scattering factors and lower statistical error. This allows preferring such method of determination of a gradient of secondary twilight brightness in the zone of an Earth shadow.

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