DETERMINATION OF POLE AND ROTATION PERIOD OF NOT STABILIZED ARTIFICIAL SATELLITE BY USE OF MODEL "DIFFUSE CYLINDER"

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ABSTRACT. The algorithm of determination of orientation of rotation axis (pole) and rotation period of satellite, simulated by a cylinder, which is precessing around of vector of angular moment of pulse with constant nutation angle is offered. The Lambert's law of light reflection is accepted. Simultaneously, dependence of light reflection coefficient versus phase angle is determined. The model's simulation confirm applicability of this method. Results of the calculations for artificial satellite No 28506 are carried out.

The determination of parameters of satellite movement around its centre from the photometric data is a typical reverse task of the satellite astronomy. The decision of this task allows to investigate influence of the environment factors on movement of satellite and to determine levels of these factors.

1. Choice of model

The model, accepted by us, first of all is applicable to rockets, to cylindrical satellite, which has lost stabilization after the ending of term of active work. We suggest the Lambert's law of light reflection as the elementary law which is taking into account characteristics of diffuse reflection of light. Most simple and effective model is the circular cylinder without the end faces. Such model is suitable, if the length of the cylinder in some times exceeds its diameter and the reflection of light from end faces can be neglected. The additional argument for a choice of this model is the fact, that the end faces of real rockets is not flat and does not essentially, contribute to the emission of a satellite. A more complex model having the form as the cylinder with hemispherical ends (having identical or different coefficients of light reflection) here was not considered.

2. The formulas

Further we shall consider, that the orbital elements of the satellite are known, and it is possible to calculate its place at the orbit at any time t; therefore are known the orts ε, k of "satellite-Sun" and "satellite-observer" directions; under the formula

$$\vec{b} = \frac{\vec{\varepsilon} + \vec{k}}{\mid \vec{\varepsilon} + \vec{k} \mid} \qquad (1)$$

it is possible to calculate the vector $\vec{b}$ which is the bisectrix of phase angle (angle "sun-satellite-observer" designate by a symbol α).

As a result of photometric observation and their correction through extinction of light in the atmosphere is received a series of magnitudes $\{m_j\}$ at the time $\{t_j\}$. We shall transform magnitudes to intensity of light I^o, which is reflected by the satellite to the observer, under the formula

$$I^o = 278000 \cdot r^2 \cdot \exp\left(-0.921 \cdot m\right), \qquad (2)$$

where r – distance up to the satellite in km. After processing, we shall receive a function of change of brightness as $I^o = I^o(t)$, where the index "o" designates, that the light curve is obtained from the observation. At a beginning of a time-scale we shall consider some moment t_0, which corresponds to the maximum of brightness and is located approximately in middle of interval of observations (is considered one passage of the satellite on the sky of the observer).

We shall consider the movement of the satellite concerning the centre of mass as precession of the its axis of symmetry (ort L) around of a moveless axis of rotation (ort Ω) with a nutation angle Θ (Fig.1).

Considering ort Ω as known, we shall construct the basis Ω, e_1, e_2 under the formulas (Fig.2):

$$\vec{e}_1 = \frac{\vec{\Omega} \times \vec{b}_0}{\mid \vec{\Omega} \times \vec{b}_0 \mid}; \quad \vec{e}_2 = \vec{\Omega} \times \vec{e}_1 \qquad (3)$$

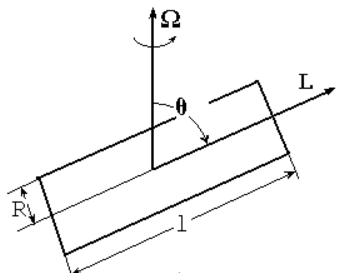

Fig.1

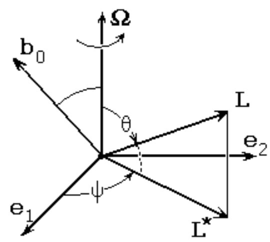

Fig.2

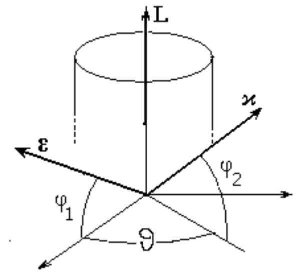

Fig.3

Here b_0 is ort b in the time t_0. Then the direction of ort L is described by the formula:

$$L = \Omega \cdot \cos\Theta + L \cdot \sin\Theta =$$

$$= \Omega \cdot \cos\Theta + [e_1 \cdot \cos\psi + e_2 \cdot \sin\psi] \cdot \sin\Theta \qquad (4)$$

where

$$\psi = \psi_0 + \omega(t - t_0); \qquad (5)$$

ω is angular velocity of the precession, ψ_0 – initial angle of the rotation. Knowing a mutual arrangement of orts ε, k, L, the sizes of model R and l, we shall calculate for a moment t under the given below formulas of light intensity I, reflected by the satellite to the observer:

$$I = I(R; l; \varepsilon; k; L(\Omega; \omega; \psi_0; \Theta; t)) \qquad (6)$$

The applied here technique of determination of a pole and rotation period of the satellite is updating of a method stated in work (Grigorevsky et al., 1979) at research of asteroids.

For various sets of parameters

$$\{\Omega, \omega, \psi_0, \Theta\} \qquad (7)$$

the theoretical curve of brightness variations $I^c = I^c(t)$ is calculated which is compared to an observable curve $I^o = I^o(t)$. For their comparison, the parameter

$$F = \sum_{j=1}^{N} \left((I^c)_j - (I^o_j) \right)^2 \qquad (8)$$

is used,which describes a degree of distinction of these curves. Let's find an optimum set of these parameters by minimizing F on parameters (7). For the calculations, the known formula for intensity of light (see, for example, McCue et al., 1971), reflected by the Lambert's cylinder was used:

$$I = \gamma l R \left\{ \frac{E}{2\pi} \cos\varphi_1 \cos\varphi_2 \left[(\pi - \vartheta)\cos\vartheta + \sin\vartheta \right] \right\} \qquad (9)$$

where E – normal illumination, created by the Sun in the vicinities of the Earth ($E = 135000$ lux), γ – factor of reflection in the Lambert's law, the angles $\varphi_1, \varphi_2, \vartheta$ are shown on Fig.3 and are functions of orts ε, k, L.

Let's designate as Φ_j expression in braces from (9) on the moment t_j; as it is impossible to determine meanings γ, l, R separately, included in (9) as product, we shall designate all this product by a symbol γ. Given factor is generally a function of a phase angle α. Considering that the given function changes rather slowly, we shall divide the interval $0° - 180°$ on 18 identical intervals, also we shall consider, what γ is constant at each of these intervals. Then the formula (8) start a kind

$$F = \sum_{k=1}^{18} \sum_{j(k)=1}^{j_k} (\gamma_k \Phi_{j(k)} - I^o_{j(k)})^2 =$$

$$= \sum_{k=1}^{18} \left(\gamma_k^2 \sum_{j(k)=1}^{j_k} \Phi^2_{j(k)} - 2\gamma_k \sum_{j(k)=1}^{j_k} I^o_j(k)\Phi_{j(k)} + \right.$$

$$\left. + \sum_{j(k)=1}^{j_k} (I^o_{j(k)})^2 \right) = \sum_{k=1}^{18} \left(a_k \gamma_k^2 - 2 d_k \gamma_k + c_k \right) \qquad (10)$$

where those points enter into the sum on $j(k)$, for which the angle α gets in k-interval (certainly, for some meanings k, there is $j_k = 0$, i.e. the appropriate intervals is empty and do not bring in the contribution to the sum F). We minimize F on parameters γ_k, considering these factors by independent, for this purpose we shall write down necessary conditions of extrema:

$$\frac{\partial F}{\partial \gamma_k} = 0 \Rightarrow 2a_k\gamma_k - 2d_k = 0 \Rightarrow \gamma_k = \frac{d_k}{a_k} \qquad (11)$$

For determined $\{\gamma_k\}$, the sufficient conditions of extrema (namely-minimum) are carried out also, it follows that the square-law form (10) is positively determined. Then

$$F_{min} = \sum_{k=1}^{18} \left(a_k \frac{d_k^2}{a_k^2} - 2\frac{d_k}{a_k} d_k + c_k \right) = \sum_{k=1}^{18} \left(\frac{a_k c_k - d_k^2}{a_k} \right) \qquad (12)$$

3. Technique of minimization

The calculations which are carried out on the formulas (9)–(12), are rather simple and occupy not a lot of computing time, therefore elementary method of a rectangular grid on three parameters with a decreasing step was applied to minimization on ω, ψ_0, Θ. The optimized meanings F_{min} depend only on equatorial coordinates $\alpha_\Omega, \delta_\Omega$ of ort Ω. For these parameters, a rectangular grid (table of dependence $F_{min} = F_{min}(\alpha_\Omega; \delta_\Omega)$) is also constructed, on which directly is determined the orientation of ort Ω; optimum meanings of a nutation angle Θ, siderical period of the rotation $P = \frac{2\pi}{\omega}$ and initial angle of rotation Ψ_0 simultaneously are determined also. At $\Theta \approx 90°$, the Lambert's cylinder has the maximal brightness, when orts L, b are approximately perpendicular. The maximum appropriate to the moment t_0 has a place, when $L \approx e_1$, it follow from (3) and (4). Therefore, the initial angle ψ_0 is close to zero. We chose "with a excess" an interval $-0.6 < \psi_0 < 0.6$; if to take into account exact dependence $\psi_0(\Theta)$, the interval can be reduced. Initial meaning of angular velocity ω_0 we shall determine from observable (synodic) period of change of shine $\omega_0 = 2\pi/P_{syn}$. Let's designate through t the maximal interval between t_0 and end points of a curve of shine. Then the maximal change of angle ψ from (5), caused by a variation of angular velocity $\Delta\omega$, is those:

$$\Delta\psi = \psi_0 + (\omega_0 + \Delta\omega)\Delta t - (\omega_0 + \omega_0\Delta t) = \Delta\omega \cdot \Delta t.$$

Let $\Delta\psi < \pi$ (that is the obviously overestimated estimation). Then $\Delta\omega < \pi/\Delta t$ is maximal (and overestimated) deviation of meaning ω from ω_0.

4. Results and discussion

For check of convergence of a method the numerical experiment was carried out, in which the orbit elements of the satellite 28506 (rocket "Sich-1M", passage 05.02.2005) were used. For the arbitrary chosen orientation of rotation axis ($\alpha_\Omega = 30°, \delta_\Omega = 30°$) the theoretical light curve with siderical period = 50 sec was constructed (at $\Theta = 90°, \psi_0 = 0$). Then for this light curve, as observable, with the above described technique, the reverse task was solved and the parameters of movement of the satellite around of the centre of mass are determined. The values of $\alpha_\Omega, \delta_\Omega, P, \Theta$, were restored with high accuracy. Let's notice, that in the area $\alpha_\Omega = 210°, \delta_\Omega = -30°$ (another end of an axis of rotation, i.e. the direction opposite to a pole of rotation) also exists a minimum, however rather dim and less deep. From this follows, that the given technique allows to allocate a true direction of rotation axis by considering a parallactic effect.

The calculations for real (received from observation 05.02.2005) light curve of the satellite 28506 are carried out also. Two diametrically opposite poles of rotation are revealed: a) $\alpha_\Omega = 125°, \delta_\Omega = -12°, P = 48.15$ sec; b) $\alpha_\Omega = 308°, \delta_\Omega = +12°, P = 52.73$ sec, and to both poles correspond approximately identical values of F_{min}. Thus, in this case, the parallactic shift of maxima has not rendered essential influence on meaning of F_{min}. It is explained, first of all, that the reflection of light by surface of the satellite strongly differs from the Lambertian and contains to an essential specular component. The presence of specular component is justified by a type of the light curve, at which there are rather sharp maxima of brightness. Indirectly the presence of specular component is confirmed by the grow for the determine values of g appreciable with increase of phase angle α. In future for applying the specified technique, will be chosen a satellite, for which the dependence $I^o(\alpha)$, averaged on the period (and, if is possible, on several passages), comes closer to the theoretical dependence $I^c(\alpha)$, calculated for sphere, which reflects light according to the Lambert's law.

References

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