BEST-FIT COSMOLOGICAL PARAMETERS FROM OBSERVATIONS OF THE LARGE SCALE STRUCTURE OF THE UNIVERSE

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ABSTRACT. The possibility to determine cosmological parameters on the basis of a wide set of observational data including the Abell-ACO cluster power spectrum and mass function, peculiar velocities of galaxies, the distribution of Ly-α clouds and CMB temperature fluctuations is analyzed. Assuming a flat universe, $\Omega_\Lambda + \Omega_m = 1$, with purely scalar perturbations, we show that a χ^2 minimization method applied on this data set determines quite precisely the values of the spectral index n of the primordial power spectrum, the baryon and massive neutrino density Ω_b and Ω_ν respectively, the Hubble constant $h \equiv H_0/(100\text{km/s/Mpc})$ and the value of the cosmological constant, Ω_Λ.

Varying all parameters we find that a tilted ΛMDM model with one species of massive neutrinos and the parameters $n = 1.13 \pm 0.10$, $\Omega_m = 0.41 \pm 0.11$, $\Omega_\nu = 0.06 \pm 0.028$, $\Omega_b = 0.039 \pm 0.014$ and $h = 0.70 \pm 0.12$ matches the observational data best.

Key words: Cosmology: theory: large scale structure of the Universe: dark matter; galaxies: clustering.

1. Introduction

The goal of this paper is to determine cosmological parameters on the basis of observational data on large scale structure (LSS) of the Universe obtained during last years. We consider a scale free power spectrum of purely scalar primordial perturbations which evolve in a multicomponent medium to form the large scale structure of the Universe. Today it is understood that models with a minimal number of free parameters, such as scale independent ($n = 1$) standard cold dark matter model (SCDM) or standard mixed (cold plus hot) dark matter model (SMDM), match observational data only marginally. Better agreement between predictions and observational data can be achieved in models with a larger number of parameters, e.g. cold dark matter (CDM) or mixed dark matter (MDM) with baryons, tilted primordial power spectra and cosmological constant (Valdarnini et al. 1998, Primack & Gross 1998).

The determination of cosmological parameters from some observational characteristics of the LSS of the Universe was carried out in many papers (e.g. Atrio-Barandela et al. 1997, Lineweaver & Barbosa 1998, Tegmark 1998, Bridle et al. 1999, Novosyadlyj 1999 and references therein). These papers differ by the number of parameters and the set of observational data included into the analysis. In this paper a total of 23 measurements from sub-galaxy scales (Ly-α clouds) over cluster scales up to the horizon scale (CMB quadrupole) is used to determine eight cosmological parameters, namely the tilt of the primordial spectrum n, the densities of cold dark matter Ω_{cdm}, hot dark matter Ω_ν, and baryons Ω_b, the density parameter of cosmological constant Ω_Λ, and the Hubble parameter h, the biasing parameter b_{cl}, the number of massive neutrino species N_ν.

Here we restrict ourselves to the analysis of spatially flat cosmological models with $\Omega_\Lambda + \Omega_m = 1$, where $\Omega_m = \Omega_{cdm} + \Omega_b + \Omega_\nu$, and to an inflationary scenario without tensor mode. It reduces the number of search parameters to seven. We also neglect the effect of a possible early reionization which could reduce the amplitude of the first acoustic peak in the CMB anisotropy spectrum.

2. Experimental data set and method

2.1. Observational data

We use the power spectrum of Abell-ACO clusters (Einasto et al. 1997, Retzlaff et al. 1998), measured in the range $0.03 \leq k \leq 0.2h$/Mpc, as observational input. Its amplitude and slope at lower and larger scales are quite sensitive to baryon content Ω_b, Hubble constant h, neutrino mass m_ν and number species of massive neutrinos N_ν (Novosyadlyj 1999). The total number of Abell-ACO data points with their errors used for minimization is 13, but not all of these points can be considered as independent measurements. Since we can accurately fit the power spectrum by an analytic expression depending on three parameters only (the amplitude at large scales, the slope at small ones and the scale of the bend); we assigned to the power spectrum 3 effective degrees of freedom.

The second observational data set which we use are the position and amplitude of first acoustic peak derived from data on the angular power spectrum of CMB temperature fluctuations. To determine the position and amplitude of the first acoustic peak we use a 6-th order polynomial fit to the data set on CMB temperature anisotropy, accumulated and compiled by Tegmark on his homepage with the new TOCO points added (Miller et al. 1999), 49 data points in total. The amplitude A_p and position ℓ_p of first acoustic peak determined from this fit are $80.81 \pm 18.0\mu$K and 256 ± 83 correspondingly. The statistical errors are estimated by edges of the χ^2-hyper-surface in the space of polynomial coefficients which corresponds to 68.3% (1σ) probability level under the assumption of Gaussian statistics. Also the mean weighted bandwidth of each experiment around l_p is added to obtain total $\Delta\ell_p$.

A constraint on the amplitude of the matter density fluctuation power spectrum at cluster scale can be derived from the cluster mass and X-ray temperature functions. It is usually formulated in terms of the density fluctuation in a top-hat sphere of $8h^{-1}$ Mpc radius, σ_8, which can be easily calculated for the given initial power spectrum. According to the recent optical determination of the mass function of nearby galaxy clusters (Girardi et al. 1998) and taking into account the results from other authors (see for references Borgani et al. 1999) we use the value $\tilde{\sigma}_8\tilde{\Omega}_m^{0.49-0.09\Omega_m} = 0.60 \pm 0.08$ Another constraint on the amplitude of the linear power spectrum of density fluctuations in our vicinity comes from the study of galaxy bulk flow, the mean peculiar velocity of galaxies in sphere of radius $50h^{-1}$ Mpc around our position. We use the data given by Kolatt & Dekel 1997, $\tilde{V}_{50} = (375 \pm 85)$ km/s.

An essential constraint on the linear power spectrum of matter clustering at galactic and sub-galactic scales $k \sim (2-40)h$/Mpc can be obtained from the Ly-α forest of absorption lines seen in quasar spectra (Gnedin 1998, Croft et al. 1998 and references therein). Assuming that the Ly-α forest is formed by discrete clouds of a physical extent near Jeans scale in the reionized inter-galactic medium at $z \sim 2-4$ Gnedin 1998 has obtained a constraint on the value of the r.m.s. linear density fluctuations $1.6 < \tilde{\sigma}_F(z=3) < 2.6$ (95% CL) at Jeans scale for $z = 3$ equal to $k_F \approx 38\Omega_m^{1/2}h$/Mpc (Gnedin 1999).

The procedure to recover the linear power spectrum from the Ly-α forest has been elaborated by Croft et al. 1998. Analyzing the absorption lines in a sample of 19 QSO spectra they have obtained the following constraint on the amplitude and slope of the linear power spectrum at $z = 2.5$ and $k_p = 1.5\Omega_m^{1/2}h$/Mpc, at (95% CL)

$$\tilde{\Delta}^2_\rho(k_p) \equiv k_p^3 P(k_p)/2\pi^2 = 0.57 \pm 0.26, \quad (1)$$

$$\tilde{n}_p \equiv \frac{\Delta log\ P(k)}{\Delta log\ k}\ |_{k_p} = -2.25 \pm 0.1, \quad (2)$$

.

In addition to the power spectrum measurements we use the constraints on the value of Hubble constant $\tilde{h} = 0.65 \pm 0.15$ which is a compromise between measurements made by two groups: Tammann & Federspiel 1997 and Madore et al. 1998. We also employ the nucleosynthesis constraints on the baryon density of $\tilde{\Omega_b h^2} = 0.019 \pm 0.0024$ (95% CL) given by Burles et al. 1999.

2.1. Method

In order to find the best fit model we must evaluate theoretical counterparts of above mentioned values.

For the solution of our searching problem we use the accurate analytic approximations of the MDM transfer function $T(k; z)$ depending on the parameters Ω_m, Ω_b, Ω_ν, N_ν, h by Eisenstein & Hu 1999.

The linear power spectrum of matter density fluctuations is

$$P(k; z) = Ak^n T^2(k; z) D_1^2(z)/D_1^2(0), \quad (3)$$

where A is the normalization constant and $D_1(z)$ is the growth factor, useful analytical approximation for which has been given by Carreta et al. 1999.

We normalize the spectra to the 4-year COBE data (Bennett et al. 1996) according to (Liddle et al. 1996, Bunn and White 1997).

The Abell-ACO power spectrum is connected with the matter power spectrum at $z = 0$ by a linear and scale independent cluster biasing parameter b_{cl}, which we include as a free parameter

$$P_{A+ACO}(k) = b_{cl}^2 P(k; 0). \quad (4)$$

For a given set of parameters n, Ω_m, Ω_b, h, Ω_ν, N_ν and b_{cl} the theoretical value of $P_{A+ACO}(k_j)$ can be calculated for each observed scale k_j. Let's denote these values by y_j $(j = 1, ..., 13)$.

The dependence of the position and amplitude of the first acoustic peak in the CMB power spectrum on cosmological parameters has been investigated using the public code CMBfast by Seljak & Zaldarriaga 1996. As expected, these characteristics are independent on the hot dark matter content.

We determine values ℓ_p and A_p for given parameters (n, h, Ω_b and Ω_Λ) on a 4-dimensional grid for parameter values inbetween the grid points we determine ℓ_p and A_p by linear interpolation. We denote ℓ_p and A_p by y_{14} and y_{15} respectively.

The theoretical values of the other experimental constraints are obtained as follows: the density fluctuation σ_8 is calculated according to

$$\sigma_8^2 = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k;0) W^2(8\mathrm{Mpc}\ k/h) dk, \qquad (5)$$

with $P(k; z)$ from Eq. (3). We set $y_{16} = \sigma_8 \Omega_m^{0.49-0.09\Omega_m}$ and $y_{17} = \sigma_8 \Omega^\alpha$, where $\alpha = 0.24$ for $\Omega_\Lambda = 0$ and $\alpha = 0.29$ for $\Omega_\Lambda > 0$, respectively.

The r.m.s. peculiar velocity of galaxies in a sphere of radius $R = 50h^{-1}$Mpc is equal

$$V_{50}^2 = \frac{1}{2\pi^2}\int_0^\infty k^2 P^{(v)}(k) e^{-k^2 R_f^2} W^2(50\mathrm{Mpc}\ k/h) dk, \qquad (6)$$

where $P^{(v)}(k)$ is the density-weighted power spectrum for the velocity field (Eisenstein & Hu 1999), $W(50\mathrm{Mpc}\ k/h)$ is a top-hat window function, and $R_f = 12h^{-1}$Mpc is a radius of Gaussian filter used for smoothing of the raw data. For the scales considered $P^{(v)}(k) \approx (\Omega^{0.6} H_0)^2 P(k; 0)/k^2$. We denote the r.m.s. peculiar velocity by y_{18}.

The value of the r.m.s. linear density perturbation from the formation of Ly-α clouds for corresponding z and k_F is given by

$$\sigma_F^2(z) = \frac{1}{2\pi^2}\int_0^\infty k^2 P(k; z) e^{(-k/k_F)^2} dk. \qquad (7)$$

It is denoted by y_{19}.

The corresponding values of the $\Delta_\rho^2(k_p, z)$ and slope $n(z)$ were obtained as defined in Eq. (1) and Eq. (2) at $z = 2.5$ and $k_p = 0.008H(z)/(1 + z)(\mathrm{km/s})^{-1}$, and are denoted by y_{20} and y_{21} accordingly.

For all tests excluding Gnedin's Ly-α test we used the density weighted transfer function $T_{cb\nu}(k, z)$ from Eisenstein & Hu 1999. For σ_F the function $T_{cb}(k, z)$ is used according to the prescription by Gnedin 1998. Note, however, that even in the model with maximal Ω_ν (~ 0.2) the difference between $T_{cb}(k, z)$ and $T_{cb\nu}(k, z)$ is $\leq 12\%$ for $k \leq k_p$.

Finally, the values of $\Omega_b h^2$ and h are denoted by y_{22} and y_{23} respectively.

Under the assumption that the errors on the data points are Gaussian, the deviations of the theoretical values from their observational counterparts can be characterized by χ^2:

$$\chi^2 = \sum_{j=1}^{23}\left(\frac{\tilde{y}_j - y_j}{\Delta\tilde{y}_j}\right)^2, \qquad (8)$$

where $\tilde{y}_j$ and $\Delta\tilde{y}_j$ are the experimental data and their dispersions, respectively. The set of parameters n, Ω_m, Ω_b, h, Ω_ν, N_ν and b_{cl} can be determined by minimizing χ^2 using the Levenberg-Marquardt method (Press et al. 1992). The derivatives of the predicted values w.r.t the search parameters required by this method are calculated numerically using a relative step size of 10^{-5}.

This method has been tested and has proven to be reliable, independent on the initial values of parameters and it has good convergence.

3. Results and conclusions

The determination of the parameters n, Ω_m, Ω_b, h, Ω_ν, N_ν and b_{cl} by the Levenberg-Marquardt χ^2 minimization method is realized in the following way: we vary the set of parameters n, Ω_m, Ω_b, h, Ω_ν and b_{cl} and find the minimum of χ^2, using all observational data described in previous section. Since the N_ν is discrete value, we repeat this procedure three times for N_ν=1, 2, and 3. The lowest of the three minima is the minimum of χ^2 for the complete set of free parameters.

The results are presented in the Table 1. The errors in the determined parameters are calculated as root square from diagonal elements of covariance matrix of the standard errors.

One can see the model with one sort of massive neutrinos provides the best fit to the data, $\chi^2_{min} \approx 4.6$. Note, however, that there are only marginal differences in χ^2_{min} for $N_\nu = 1, 2, 3$. Therefore, with the given accuracy of the data we cannot conclude whether – if massive neutrinos are present at all – their number of species is one, two, or three.

The number of degrees of freedom is $N_F = N_{\exp} - N_{\mathrm{par}} = 7$. The χ^2_{min} for all cases is within the expected range, $N_F - \sqrt{2N_F} \leq \chi^2_{\min} \leq N_F + \sqrt{2N_F}$ for the given number of degrees of freedom. This means that the cosmological paradigm which has been assumed is consistent with the data.

We summarize, that the observational data on LSS of the Universe considered here can be explained by a tilted ΛMDM inflationary model without tensor mode. The best fit parameters are: $n = 1.13 \pm 0.10$, $\Omega_m = 0.41 \pm 0.11$, $\Omega_\nu = 0.06 \pm 0.028$, $\Omega_b = 0.039 \pm 0.014$ and $h = 0.70 \pm 0.12$. All predictions of measurements are close to the real experimental values mentioned above and within the error bars of data. The CDM density

Table 1: Cosmological parameters determined for the tilted ΛMDM model with one, two and three species of massive neutrinos.

N_ν	χ^2_{min}	n	Ω_m	Ω_ν	Ω_b	h	b_{cl}
1	4.55	1.13±0.10	0.41±0.11	0.060±0.028	0.039±0.014	0.70±0.12	2.22±0.33
2	4.74	1.14±0.10	0.49±0.13	0.104±0.042	0.039±0.015	0.70±0.13	2.32±0.36
3	5.02	1.14±0.10	0.56±0.14	0.133±0.053	0.040±0.015	0.69±0.13	2.44±0.37

parameter is $\Omega_{cdm} = 0.31 \pm 0.12$ and Ω_Λ is moderate, $\Omega_\Lambda = 0.59 \pm 0.11$. The neutrino matter density corresponds to a neutrino mass $m_\nu = 94\Omega_\nu h^2 \approx 2.7\pm1.2$ eV. The value of Hubble constant is close to measurements made by Madore et al. 1998. The age of the Universe for this model equals 12.3 Gyrs which is in good agreement with the age of the oldest objects in our galaxy. The spectral index coincides with the COBE prediction. Relation between matter density Ω_m and cosmological constant Ω_Λ agrees well with measurements of cosmic deceleration and global curvature based on the SNIa observation (Schmidt et al. 1998).

The coincidence of the values of cosmological parameters obtained by different methods indicates that a wide set of cosmological measurements are correct and that their theoretical interpretation is consistent. However, we must also note that the accuracy of present observational data on the large scale structure of the Universe is still too modest to determine a set of cosmological parameters at high confidence level.

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