TIME-DEPENDENT DISK ACCRETION IN BINARY SYSTEMS

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ABSTRACT. The analytic investigation of time-dependent accretion in disk is carried out. We consider a disk in a binary system at outburst which has fixed tidally truncated outer radius. The standard model (Shakura–Sunyaev 1973) of the disk is considered. The fully analytic solutions in two different opacity regimes are characterized by power–law variations of accretion rate with time. The solutions supply asymptotic description of the disk evolution after the peak of outburst while the disk is fully ionized. The X-ray flux of multicolor (black-body) α-disk is obtained to vary quasi-exponentially. The application to X-ray novae is briefly discussed concerning observed faster-than-power decays of X-ray light curves. The case of time-dependent advective disk is mentioned.

Key words: Stars: binary: novae, cataclysmic variables; X-rays: bursts; stars: individual: A 0620-00

1. Introduction

The problem of time-dependent accretion is closely related to the phenomena of flares widely observed in binary systems. We consider emission of the flaring source to be generated by the accretion disk and the light curve to be regulated by the accretion rate variations. Such sources are typified by the low massive X-ray binaries and cataclysmic variables.

After Weizsäcker (1948) who considered the evolution of a protoplanetary cloud, the analytic investigations of non-stationary accretion were carried out by Lüst (1952), Lynden-Bell & Pringle (1974), Lyubarskii & Shakura (1987, hereafter LS87) as applied to accretion disks.

LS87 suggested three stages of evolution of a time-dependent accretion disk. Initially a finite torus of the increased density is formed around a gravitational centre. Viscosity causes the torus to spread and develop into the disk (1st stage). After disk approaching the centre the accretion rate reaches the maximum value (2nd stage) and begins to descend (3rd stage). During this stage the total angular momentum of the disk is conserved.

In a binary system variations of accretion rate can be due to the non-stationary exchange of mass between the components of the binary (mass–overflow instability model) or due to the disk instability processes (see Kato et al. 1998 and references therein). At some instant the accretion rate onto the centre begins to augment. We assume that the maximum accretion rate through the inner boundary of the disk corresponds to a peak of outburst and accretion rate decreases afterwards. In a binary system the third stage of LS87 cannot be realized, because the accretion disk around a primary would be confined by the gravitational influence of a secondary. Such disks do not preserve their angular momentum, transferring it to the orbital motion.

In this paper, which is an extension of our previous work (Lipunova & Shakura 1999), we outline the main features of time-dependent accretion in a binary system and make the first step at applying our model to X-ray nova A0620-00 flare of 1975.

2. Basic non–stationary accretion disks equation

In the approximation of Newtonian potential the velocity of a free particle orbiting at r is assumed to be a Kepler one. This is a good approximation to the law of motion for particles in the standard under–Eddington disk, whereas in the advection–dominated accretion flow (ADAF) the particles are substantially subjected to the radial gradient of pressure. The time–independent angular velocity is assumed, although there can possibly be certain variations of ω in the non-Keplerian advective disks, when time–dependent pressure gradient is involved.

Then the basic equation of time–dependent accretion is given by:

$$\frac{\partial \Sigma_{\rm o}}{\partial t} = \frac{1}{2}\,\frac{(GM)^2}{h^3}\,\frac{\partial}{\partial h}\left(\left[\frac{\partial h_*}{\partial h}\right]^{-1}\frac{\partial F}{\partial h}\right) , \qquad (1)$$

where Σ_o is the surface density, $F = W_{r\varphi} r^2$, $W_{r\varphi}$ being the height–integrated viscous shear stresses between adjacent layers, h and h_* are the Keplerian and the real specific angular momenta, t is the time, M is the central mass. In the case of the Keplerian disk $\partial h_* / \partial h = 1$.

3. Viscous evolution of Keplerian disk

To solve Eq. (**??**) one needs to know the relation between F and Σ_o. The special case when $F \propto \Sigma_o h^l$ was investigated by Lynden-Bell & Pringle (1974). We use the needed relation in a form

$$\Sigma_o = \frac{(GM)^2\, F^{1-m}(h,t)}{2\,(1-m)\,D\,h^{3-n}} \tag{2}$$

(see also Filipov 1984), suggested by LS87 for α-disks. Then, seeking the solution in the form $F(h,t) = F(t) f(\xi)$, the time–dependent part of the solution is turned to be

$$F(t) = \left(\frac{h_o^{n+2}}{\lambda\, m\, D\,(t+t_0)} \right)^{1/m} \tag{3}$$

and accretion rate is:

$$\dot M(h,t) = 2\pi f'(h/h_o) F(t)/h_o\ , \tag{4}$$

where D is the constant defined by the vertical structure of the disk, $\xi = h/h_o$, $h_o = (GMr_{\rm out})^{1/2}$, t_0 and λ are to be defined from the initial and boundary conditions, respectively; m and n are the dimensionless constant depending of the specific opacity (see Table 1). We calculated D (Lipunova & Shakura 1999) adopting the results of Ketsaris and Shakura (1998).

In a binary system the accretion picture has particular features. The main feature is the limitation of the outer radius due to tidal interactions. As Ichikawa and Osaki (1994) showed the tidal effects are generally small in the accretion disk, except near to the tidal truncation radius, which is given by the last non-intersecting periodic particle orbit in the disk (Paczyński 1977). We consider the size of the disk to be maximum and invariant over the period of outburst. As the drain of angular momentum occurs in a narrow region near this truncation radius (Ichikawa, Osaki 1994), we treat the region near this radius as the δ-type channel, not considering the details of the process. The derived $f(\xi)$ is shown in Fig. **??**. Let us notice that (**??**) implies a considerably *steeper* time dependence than the solution by LS87 does.

Table 1: Short list of parameters in solutions for two opacity regimes for the Keplerian disk.

	m	n	λ
Thomson scattering: ${}_{\rm T} \gg {}_{\rm ff}$	2/5	6/5	−3.482
Free-free transitions: ${}_{\rm ff} \gg {}_{\rm T}$	3/10	4/5	−3.137

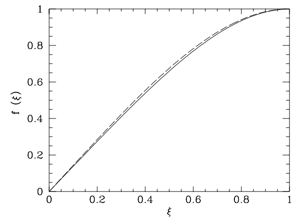

Figure 1: The solution $f(\xi)$ in two cases: when ${}_{\rm T} \gg {}_{\rm ff}$ (solid line) and ${}_{\rm ff} \gg {}_{\rm T}$ (dashed line).

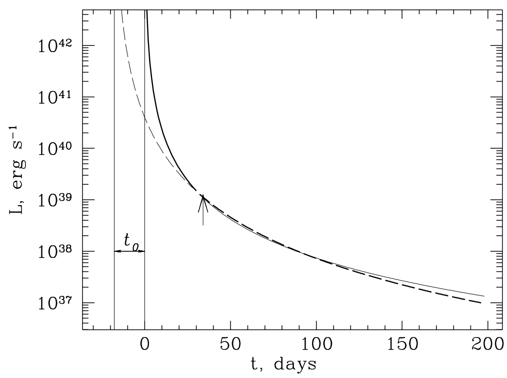

Figure 2: The bolometric luminosity in Thomson opacity regime (solid line) and in the free–free opacity regime (dashed line). Their bold parts represent the resulting light curve of the disk. The arrow marks one of two intersections when $F_1(h,t) = F_2(h, t+t_0)$.

4. Bolometric light curve of time-dependent Keplerian disk

For the most luminous, inner, parts of the disk we take $\dot M(t) = \dot M(0,t)$ given by (**??**). The overall emission of the disk is defined by the gravitational energy release $L = \eta\, \dot M(t)\, c^2$, where η is the efficiency of the process. At early t, when the Thomson scattering is dominant, the bolometric luminosity of the disk varies as follows:

$$L_{\rm T}(t) \propto t^{-5/2}\,. \tag{5}$$

As the temperature decreases, the law of decline switches to:

$$L_{\rm ff}(t) \propto (t+t_0)^{-10/3}\,. \tag{6}$$

These dependencies give asymptotic laws for *bolometric* luminosity variations of the disk. Transfer between the opacity regimes occurs when the solutions in two regimes sew at the half disk radius. This moment corresponds to:

$$t = t_{\rm tr} \approx 0.52\, t_{\rm E}\, (m_{\rm x} (R_\odot / r_{\rm out})^3 (0.1/\eta)^2 (0.5/\mu))^{1/5}\ ,$$

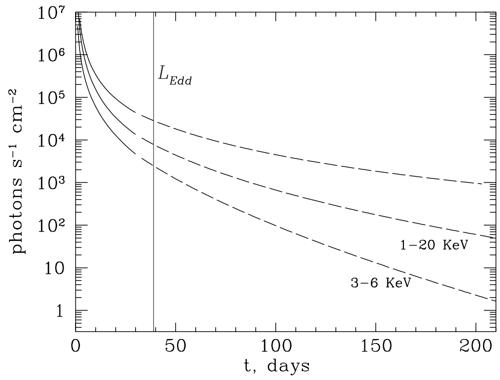

Figure 3: The flux from one side of accretion disk at 1 kpc. The curves show the bolometric flux (upper curve), the 1–20 KeV flux (middle curve) and the 3–6 KeV flux (lower curve) during the Thomson opacity regime (solid parts) and the free–free opacity regime (dashed parts).

where $m_x = M/M_\odot$, t_E is the time, when $L = L_{Edd} \approx 1.3 \times 10^{38}\, m_x$ erg s^{-1}. Generally speaking, the solution before t_E appears to have no application. Yet recall that the Eddington limit is uncertain since a disk has non-spherical geometry.

Fig. **??** represents the bolometric light curve of the disk for $m_x = 3$, $\alpha = 0.3$, $\mu = 0.5$, $r_{out} = R_\odot$. For these parameters $t_{tr} \approx 34^d$ (the vertical arrow) and $t_0 \approx 17^d$. The second intersection of the curves in Fig. **??** at $t \approx 95^d$ corresponds to the other intersection of functions $F_1(h,t) = F_2(h, t + t_0)$, meanwhile the physical parameters of the disk are different. Thus the disk is at the same (free-free) opacity regime as before.

When T_c decreases to the value $\sim 10^4$ K, the convection (which is presumably appeared in the zones of partial ionization) starts to influence greatly on the disk's structure and the diffusive type of radiation transfer (which we use) is no longer valid. For $m_x = 3$ and $\alpha = 0.3$ this happens at $t \approx 190$ days. For investigation of the disk evolution on larger time-scales see e.g. Cannizzo et al. (1995), Cannizzo (1998), Kim et al. (1999).

We remark that the observed X-ray light curves can have *different* (most probably, steeper) law of decay. Indeed, energy band of an X-ray detector usually covers the region harder 1 KeV where the multi–color photon spectrum of the disk (having appropriate temperature) can have specific distribution. The narrower the observed band the more different observed curve could look like in comparison with the expected bolometric flux light curve.

5. Observed light curves

Explaining the observed faster-than-power decay of outbursts in soft X-ray transients one must take into account that the observed slope of the curve depends on width and location of the observing interval. Of course, in each particular case this difference also reflects the spectral distribution of energy coming from the source.

We show here how the slope of the curve changes in the simplest case of multi-color black body disk spectrum according to which spectral range is observed. We calculate I_ν and integrate it over three energy ranges: 3-6 KeV, 1-20 KeV and that one in which practically all energy is emitted. Fig. **??** shows the photon flux variations in two X-ray energy ranges (those of *Ariel 5* and *EXOSAT* or *Ginga* observatories) and the bolometric flux variation, for the face-on disk at an arbitrary distance of 1 kpc. The vertical line marks the time after which bolometric luminosity of the disk's one side is less than L_{Edd}.

One can see almost linear trend of the X-ray flux when bolometric luminosity is under the Eddington limit (to the right from the vertical line in Fig. **??**), especially in intervals of ~ 50 days. The decline becomes closer to the exponential one with time. The slope of the curve depends on α, m, r_{out} and other parameters. For the same parameters as in Fig. **??** the e-folding time falls in the range $20 - 30$ days for the lower curve (3–6 KeV). For instance, smaller α will result in less steep decline.

The natural explanation of such result is the following: because the spectral shape of the disk emission has Wien-form (exponential fall-off) at the considered X-ray ranges the law of variation of X-ray flux is roughly proportional to $\exp\left(-h_P\, \nu / k\, T^{eff}(t)\right)$. In the free-free regime of opacity we have $T^{eff}(t) \propto L_{ff}^{1/4}(t) \propto \dot{M}(t)^{1/4} \propto t^{-10/12}$. Consequently, the observed X-ray flux varies like $\exp\left(-t^{5/6}\right)$ which is quite close to exponential behavior.

6. Discussion and conclusion

In this work, we presented the analytic solutions to time-dependent accretion in binary systems. For two opacity regimes the full analytic time-dependent solutions for Keplerian disk are obtained and *asymptotic* light curve is calculated with smooth transition between opacity regimes. During the decline phase accretion disks around black holes appear to be dominated by the free-free and free-bound opacity in order to comply with the Eddington limit on luminosity. This phase is characterized by the power-law decay of accretion rate $\propto t^{-10/3}$. It is shown that observed decay time scale depends on the real energetic band of detector (Fig. **??**).

The results obtained in this work can be applied to the accreting systems having variable emission of flare type if emission is essentially due to the fully ionized

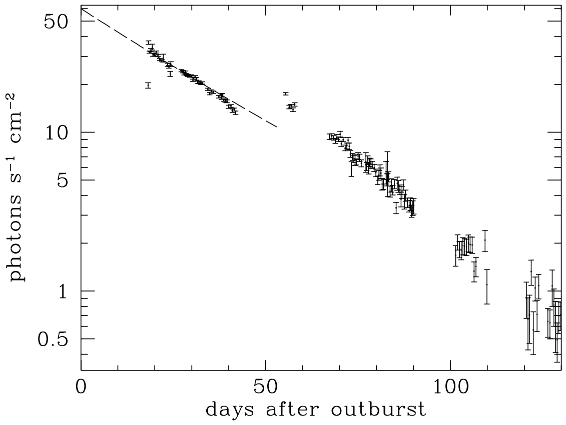

Figure 4: Comparison of the model (dashed line) with the *Ariel-5* A0620-00 (1975) 3–6 KeV light curve.

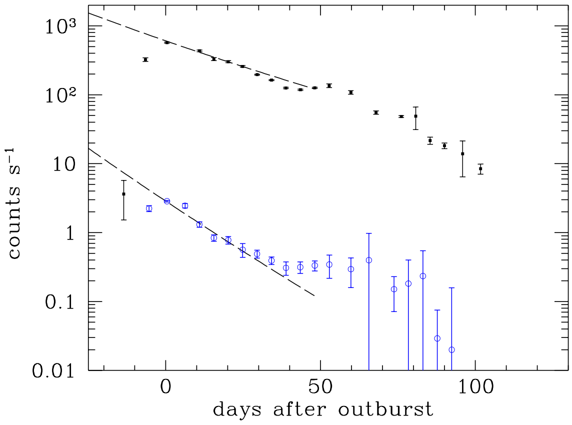

Figure 5: Comparison of the model (dashed lines) with the *Vela 5B* A0620-00 (1975) 3–12 KeV (dots) and 6–12 KeV light curves (open circles).

accretion disk around a black hole, or a neutron star, or a white dwarf.

Typical X-ray novae (XN) outburst light curves (see Tanaka & Shibazaki (1996), Chen at al. (1997) for the review) show quasi-exponential decay. Up to date several approaches have been used to account for XN features (see e.g. Mineshige et al., 1993; Cannizzo et al., 1995; King & Ritter, 1998) Using the results of this work we can explain the general features of XN light curves in the early phase. We show that nearly exponential X-ray decays $\propto \exp\left(-t^{5/6}\right)$ appear taking into account the fact that the X-ray light curves are observed in the energetic range where the spectrum of the disk has Wien-form.

Figures **??** and **??** represent the comparison of the model for parameters $m_{\mathrm{x}} = 10$, $\alpha = 0.5$, $r_{\mathrm{out}} = 2.17 R_\odot$ with data[1] for XN A0620-00 observed in 1975 by *Ariel-5* and *Vela 5B*. The orbital inclination angle of the disk is taken 66°, the distance is 0.87 kpc (e.g. Tanaka & Shibazaki, 1996). The model parameters were fitted firstly to the *Ariel-5* light curve and then the *Vela 5B* data was applied. The presented model can explain the observations before ~ 50 day, and apparently fails after. This implies that some effects, yet ignored in presented scenario, becomes important. Some possibilities can be suggested: the disk becomes partly ionized (and convection presumably begins), an additional mass supply to the disk occurs, or the irradiation effects of the outer parts of the disk become significant.

If one adopts for the structure of advection-dominated accretion flow (ADAF) the self-similar solution (e.g. Narayan & Yi, 1995), it can be inferred (Lipunova & Shakura 1999) that such disks possibly exhibit the exponential with time behaviour in a binary system and are quickly depleted if α is not small. If this is the case the abrupt steep falls observed in several XN (Tanaka & Shibazaki 1996) in the last phase of the decay, at luminosity levels 10^{36} erg s^{-1}, can be interpreted in terms of quickly depleting ADAF with relevant values of $\alpha \sim 10^{-1}$.

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