The Models of Voids in the Friedman Universe

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ABSTRACT. The astronomical observations of the last years show that there are the regions in the Universe with much lower density of matter, than their surroundings. Theoretical studies of the regions (voids) in the model of the expanding Universe are carried on different directions. In this paper the voids have been built by means of matching Tolman and Friedman solutions. The Lichnerovich-Darmois matching conditions are used. It is shown that in expanding Universe with flat space the voids can not exist. So we have Friedman Universe with voids, with described by the Tolman solution. The models of voids in the Friedman Universe with negative spatial curvature have been built.

Key words: Universe, void, Friedman solution, space-time, matching conditions, density of energy.

1. Introduction

The astronomical observations of last years shows that there are the regions in the Universe with much lower density of matter than their surroundings (Thompson and Vishniac 1987; de Lapparent, Geller and Huchra 1986). Theoretical studies of these region (voids) in the models of the expanding Universe are carried on different directions (Redmouth 1988; Suto, Sato, and Sato 1984): small perturbations of homogeneous Universe; use of the Einstein-Straus model; use of the Tolman solution for the nonhomogeneous dust; consideration of the boundary of the void as the thin wall.

In this paper we use the Tolman spherically symmetric dust solution for the description of voids space-time, and Friedman solution for the description of the space-time of the surrounding Universe.

2. The Tolman solution

The Tolman solution for nonhomogeneous dust has the following form:

$$ds^2 = dt^2 - \frac{r'^2(R,t)}{f^2(R)}dR^2 - r^2(R,t)(d\Theta^2 + \sin^2\Theta d\varphi^2)\ , \qquad (1)$$

where

$$r(R,t) = \frac{m(R)}{1-f^2(R)}\left\{\begin{matrix} \sin^2(\alpha/2) \\ -\sinh^2(\alpha/2)\end{matrix}\right\} \text{for} \left\{\begin{matrix} f^2(R) < 1 \\ f^2(R) > 1\end{matrix}\right. ; \qquad (2)$$

$$t - t_0(R) = \frac{m(R)}{|1-f^2(R)|^{3/2}}\left\{\begin{matrix} \alpha - \sin\alpha \\ \sinh\alpha - \alpha\end{matrix}\right\} \text{for} \left\{\begin{matrix} f^2(R) < 1 \\ f^2(R) > 1\end{matrix}\right. ; \qquad (3)$$

$$r(R,t) = \left[\pm\frac{2}{3}m(R)^{1/2}(t - t_0(R))\right]^{2/3} \text{for } f^2(R) = 1. \qquad (4)$$

The velocity of light $c = 1$. The prime means ∂/∂_R. $m(R)$, $f(R)$ and $t_0(R)$ are the arbitrary functions of integration. $m(R)$ is the hole mass of the dust ball with radial coordinate R, $f(R)$ is the hole energy of the test particle, which is on the distance R from the centre. $t_0(R)$ determines the time of the collapse.

The density of the energy is given by

$$\varepsilon(R,t) = \frac{1}{8\pi\gamma}\frac{m'(R)}{r^2(R,t)r'(R,t)}, \qquad (5)$$

where γ is the Newton gravitational constant.

Friedman solution for homogeneous dust is the particular form of the Tolman one:

$$m(R) = a_0\left\{\begin{matrix} \sin^3(R) \\ \sinh^3(R) \\ R^3\end{matrix}\right\} \text{for} \left\{\begin{matrix} f^2(R) = \cos^2(R) \\ f^2(R) = \sinh^2(R) \\ f^2(R) = 1\end{matrix}\right. , \qquad (6)$$

$t_0(R) = 0$, $a_0 = const.$

3. The matching conditions

We use the Lichnerovicz-Darmous matching conditions, which consist in following: the first and the second differential forms of the matched metrics are the same on the matching hypersurface. We consider two different Tolman metrics and choose the hypersurface $R = R_b = const$ as the matching hypersurface. Then the matching conditions have the following form:

$$\begin{aligned} r_1(R_b, t_1) &= r_2(R_b, t_2), \\ f_1(R_b) &= f_2(R_b), \\ m_1(R_b) &= m_2(R_b), \end{aligned} \tag{7}$$

where index "1" and "2" mark the first and second matched metrics, respectively.

4. The voids, described by the flat space-time

Bonnor and Chamorro considered the voids as the Minkowski space-time (Bonnor, and Chamorro 1990; Bonnor, and Chamorro 1991). They have shown that such voids can not exist in the Friedman Universe. But it is possible to choose the definite Tolman Universe and such Universe can have the voids which are describe by the empty space-time. It was to be expected that this Tolman Universe must be sufficiently exotic.

Friedman Universe also can not have the voids which are described by the other Friedman space-time. Under this assumption the matching conditions are not fulfilled.

So we consider the Tolman space-time as the space-time of the "void", and Friedman space-time as the one of the surroundings.

5. Friedman model

Let us assume that the Universe have been described by the parabolic Friedman model. Under this condition the void is described by the parabolic Tolman model. The matching conditions demand the same spatial curvature of the void and of the surroundings space-time. The arbitrary function $f(R)$ determines the spatial curvature. So we choose the function $f(R)$ in the voids the the same, as it has been chosen in Friedman space-time $f^2(R) = 1$. Exactly this choice of $f^2(R)$ permit us to consider the space coordinate R as the same in the void and in the surrounding space-time. From the matching conditions we have obtained that on the matching hypersurface $t_T = t_F + t_0(R_b)$.

The average density of energy in the void is given by

$$\bar{\varepsilon} = \frac{M}{V} = \frac{\int\limits_0^{R_b} \varepsilon\sqrt{-g}\; dR\; d\Theta\; d\varphi}{\int\limits_0^{R_b} \sqrt{-g}\; dR\; d\Theta\; d\varphi} = \frac{\int\limits_0^{R_b} \frac{m'}{f(R)}\; dR}{\int\limits_0^{R_b} \frac{r^2 r'}{f(R)}\; dR}. \tag{8}$$

For the parabolic Tolman model with $f^2(R) = 1$ from (8) we obtain

$$\bar{\varepsilon} = \frac{\int\limits_0^{R_b} m'(R)\; dR}{\int\limits_0^{R_b} r^2 r'\; dR} = \frac{3m(R_b)}{r^3(R_b, t_T)}. \tag{9}$$

For the Friedman homogeneous model we can write the expression for the $\bar{\varepsilon}$ in the following form

$$\bar{\varepsilon} = \varepsilon(t) = \frac{m'(R)}{r^2 r'} = \frac{3m(R)}{r^3(R,t)}. \tag{10}$$

Because energy density (10) is independent from R, we can replace the value R in the expression (10) to the value R_b, then we obtain

$$\bar{\varepsilon} = \varepsilon(t) = \frac{3m(R_b)}{r^3(R_b, t_F)}. \tag{11}$$

¿From (7), (9) and (11) we can see that in parabolic Friedman model the voids can not exist, because the homogeneous energy density in the external space and the average density in the internal space are the same.

6. The voids in hyperbolic Friedman Universe

Let us consider the "voids" in the hyperbolic Friedman model. We take the mass function of the voids as

$$m_V = a_0 \frac{\sinh^{n+1} R}{\sinh^{n-2} R_b}, \tag{12}$$

where n is arbitrary whole number.

For the different n the observing mass M, the volumes V and the average density of the voids have been calculated. We have consider the possibility of the formation of the little and the big voids ($R_b \to 10^{-2}$, 10^{-1}, 1, 2). We have chosen $t_0(R) \to 0$, R_b, $\sinh R_b$. From calculation we have obtained that the Tolman time always is greater than Friedman one. There are not exist the voids when $t_0(R) = 0$. For $n \geq 3$ we have voids only in earlier Universe, now they can not exist.

The parameters of the models of the voids with $n = 1$ and $t_0(R_b) = R_b$ are given in the tables 1, 2, 3, and 4.

Table 1: Model of void for $R_b = 0.01$.

R_b	t_T	V_T/V_F	M_T/M_F	E_T/E_F
	0.01001	8.9		0.15
	0.0101	1.1		0.92
	0.011	1		1
	0.02	1		1
0.01	0.11	1.01	1	0.99
	0.26	1.13		0.88
	0.51	2.4		0.44
	0.76	7.26		0.13
	1.01	**22.14**		**0.05**
	1.51	156.8		0.0065

Table 2: Model of void for $R_b = 0.1$.

R_b	t_T	V_T/V_F	M_T/M_F	E_T/E_F
	0.10001	$8.5 \cdot 10^4$		$1.2 \cdot 10^{-5}$
	0.1001	$9.3 \cdot 10^2$		$1.1 \cdot 10^{-3}$
	0.101	11		0.09
	0.11	1.17		0.85
0.1	0.2	1.05	1	0.95
	0.35	1.12		0.89
	0.6	1.5		0.67
	0.85	2.86		0.35
	1.1	**5**		**0.2**
	1.6	24.5		0.041

Table 3: Model of void for $R_b = 1$.

R_b	t_T	V_T/V_F	M_T/M_F	E_T/E_F
	1.00001	$1.4 \cdot 10^9$		$7.4 \cdot 10^{-10}$
	1.0001	$1.5 \cdot 10^7$		$7 \cdot 10^{-8}$
	1.001	$1.5 \cdot 10^5$		$7 \cdot 10^{-6}$
	1.01	$1.6 \cdot 10^3$		$6.5 \cdot 10^{-4}$
1	1.1	29	1.04	0.038
	1.25	9.8		0.11
	1.5	4.2		0.11
	1.75	11.9		0.09
	2	**18.9**		**0.06**
	2.5	61		0.017

Table 4: Model of void for $R_b = 2$.

R_b	t_T	V_T/V_F	M_T/M_F	E_T/E_F
	2.00001	$4.5 \cdot 10^{1}0$		$2.4 \cdot 10^{-9}$
	2.0001	$4.9 \cdot 10^8$		$2.4 \cdot 10^{-7}$
	2.001	$4.9 \cdot 10^6$		$2.4 \cdot 10^{-5}$
	2.01	$5.1 \cdot 10^4$		$2.4 \cdot 10^{-3}$
2	2.1	653.3	1.15	0.0017
	2.25	108.2		0.011
	2.5	101.8		0.011
	2.75	111.6		0.01
	3	**152.9**		**0.007**
	3.5	400		0.003

Note, that the masses of the voids (M_T in the tables) are the same as the Friedman mass (M_F in the tables) in the region limited by R_b. But the volumes of the voids are greater than the corresponding Friedman volume.

E_T is the average energy density in the voids, E_F — the same in the Friedman space-time. Present time corresponds to the marked line.

¿From the tables we can see, that the ratio E_T/E_F is changed. This value is very little near the beginning of the Universe, then it increases some time, and decreases again. So at present time the voids can exist.

Conclusions show that these models describe the voids of different size. The voids are changing in time.

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