INTERACTING GRAVITATIONAL EXCITONS AND OBSERVABLE EFFECTS FROM EXTRA DIMENSIONS

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ABSTRACT. It is supposed that a multidimensional manifold undergoes a spontaneous compactification $M \longrightarrow M^4 \times \prod_{i=1}^n M_i$, where M^4 is the 4−dimensional external space-time and M_i are compact internal spaces. Conformal excitations of the internal space metric can be observed as massive and massless scalar fields (gravitational excitons) in the external space-time. Specific interaction features of gravitational excitons with gravitons are considered.

Key words: Gravitation theory; cosmology: cosmological models; field theory.

1. Introduction

The large scale dynamics of the observable part of our present time universe is well described by the Friedmann model with 4-dimensional Friedmann-Robertson-Walker (FRW) metric. However, it is possible that space-time at short (Planck) distances might have a dimensionality of more than four and possess a rather complex topology. String theory (Green, Schwarz, Witten 1987) and its recent generalizations — p-brane, M- and F-theory (Strominger, Vafa 1996; Duff 1996) widely use this concept and give it a new foundation. From this viewpoint, it is natural to generalize the Friedmann model to multidimensional cosmological models (MCM) with topology (Ivashchuk, Melnikov, Zhuk 1989)

$$M = \mathbf{R} \times M_0 \times M_1 \times \ldots \times M_n \ , \qquad (1.1)$$

where for simplicity the $M_i \quad (i = 0, \ldots, n)$ can be assumed to be d_i-dimensional Einstein spaces. M_0 usually denotes the $d_0 = 3$ - dimensional external space.

One of the main problems in multidimensional models consists in the dynamical process leading from a stage with all dimensions developing on the same scale to the actual stage of the universe, where we have only four external dimensions and all internal spaces have to be compactified and contracted to sufficiently small scales, so that they are apparently unobservable. To make the internal dimensions unobservable at the actual stage of the universe we have to demand their contraction to scales $10^{-17}cm$ — $10^{-33}cm$ (between the Fermi and Planck lengths). This leads to an effectively four-dimensional universe.

In the present paper we briefly review some recent results connected with different aspects of multidimensional gravitational models (Günther, Zhuk 1997a,b, 1998; Günther, Starobinsky, Zhuk 1999).

1. We show that inhomogeneous fluctuations of the scale factors of internal factor spaces in MCMs can be interpreted as scalar particles (gravitational excitons) in our observable $(D_0 = 4)-$dimensional external space-time.

2. We point out some specific features of interaction and propagation of gravitational excitons and gravitons in the presence of inhomogeneous scale factor backgrounds.

3. A consideration of the interaction of gravitational excitons with abelian gauge fields allows us to indicate some specific astrophysical implications related to gravitational excitons as well as some possible observable consequences connected with their existence.

4. Starting from the fact that according to present day observations the dynamical behaviour of the universe after inflation is well described by the standard Friedmann model in the presence of a perfect fluid we show that this approach can be generalized to the description of the postinflationary stage in multidimensional cosmological models. We derive a class of models where, from one side, the internal spaces are stable compactified near Planck scales and, from the other side, the external universe behaves in accordance with

the standard Friedmann model.

2. Gravitational excitons

In this section we describe the basic features of our model and show how gravitational excitons necessarily occure in higher dimensional gravitational theories (Günther, Zhuk 1997a,b).

Let us consider a multidimensional space-time manifold

$$M = \bar{M}_0 \times M_1 \times \ldots \times M_n \tag{2.1}$$

with decomposed metric on M

$$g = g_{MN}(X)dX^M \otimes dX^N = g^{(0)} + \sum_{i=1}^{n} e^{2\beta^i(x)} g^{(i)}, \tag{2.2}$$

where x are some coordinates of the $D_0 = d_0 + 1$ - dimensional manifold $\bar{M}_0$ and

$$g^{(0)} = g^{(0)}_{\mu\nu}(x) dx^\mu \otimes dx^\nu. \tag{2.3}$$

Let manifolds M_i be d_i-dimensional Einstein spaces with metric $g^{(i)} = g^{(i)}_{m_i n_i}(y_i) dy_i^{m_i} \otimes dy_i^{n_i}$, i.e.,

$$R_{m_i n_i}\left[g^{(i)}\right] = \lambda^i g^{(i)}_{m_i n_i}, \qquad m_i, n_i = 1, \ldots, d_i \tag{2.4}$$

and

$$R\left[g^{(i)}\right] = \lambda^i d_i \equiv R_i. \tag{2.5}$$

In the case of constant curvature spaces parameters λ^i are normalized as $\lambda^i = k_i(d_i - 1)$ with $k_i = \pm 1, 0$. Internal spaces M_i $(i = 1, \ldots, n)$ may have nontrivial global topology, being compact (i.e. closed and bounded) for any sign of spatial topology (Wolf 1967, Fagundes 1992, 1993).

With total dimension $D = 1 + \sum_{i=0}^{n} d_i$, κ^2 a D-dimensional gravitational constant, Λ - a D-dimensional bare cosmological constant and S_{YGH} the standard York-Gibbons-Hawking boundary term (York 1972, Gibbons, Hawking 1977), we consider an action of the form

$$S = \frac{1}{2\kappa^2} \int\limits_M d^D X \sqrt{|g|} \left\{ R[g] - 2\Lambda \right\} + S_{add} + S_{YGH}. \tag{2.6}$$

The additional potential term

$$S_{add} = -\int\limits_M d^D X \sqrt{|g|} \rho(x) \tag{2.7}$$

is not specified and left in its general form, taking into account the Casimir effect (Candelas, Weinberg 1984), the Freund - Rubin monopole ansatz (Freund, Rubin 1980), a perfect fluid (Gavrilov, Ivashchuk, Melnikov 1995; Kasper, Zhuk 1996) or other hypothetical potentials (Bleyer, Zhuk 1995; Günther, Kriskiv, Zhuk 1998). In all these cases ρ depends on the external coordinates through the scale factors $a_i(x) = e^{\beta^i(x)}$ $(i = 1, \ldots, n)$ of the internal spaces.

After dimensional reduction the action reads

$$\begin{aligned} S &= \frac{1}{2\kappa_0^2} \int\limits_{\bar{M}_0} d^{D_0} x \sqrt{|g^{(0)}|} \prod_{i=1}^{n} e^{d_i \beta^i} \times \\ &\times \left\{ R\left[g^{(0)}\right] - G_{ij} g^{(0)\mu\nu} \partial_\mu \beta^i \, \partial_\nu \beta^j + \right. \\ &+ \left. \sum_{i=1}^{n} R\left[g^{(i)}\right] e^{-2\beta^i} - 2\Lambda - 2\kappa^2 \rho \right\}, \end{aligned} \tag{2.8}$$

where $\kappa_0^2 = \kappa^2 / V_I$ is the D_0-dimensional gravitational constant and

$$V_I = \prod_{i=1}^{n} v_i = \prod_{i=1}^{n} \int\limits_{M_i} d^{d_i} y \sqrt{|g^{(i)}|} \tag{2.9}$$

defines the internal space volume corresponding to the scale factors $a_i \equiv 1, i = 1, \ldots, n$. $G_{ij} = d_i \delta_{ij} - d_i d_j$ $(i, j = 1, \ldots, n)$ is the midisuperspace metric (Ivashchuk, Melnikov, Zhuk 1989; Rainer, Zhuk 1996). The action functional (2.8) is written in the Brans-Dicke frame. A conformal transformation to the Einstein frame

$$g^{(0)}_{\mu\nu} = \Omega^2 \tilde{g}^{(0)}_{\mu\nu}, \tag{2.10}$$

with

$$\Omega = exp\left(-\frac{1}{D_0 - 2} \sum_{i=1}^{n} d_i \beta^i \right) \tag{2.11}$$

yields

$$\begin{aligned} S &= \frac{1}{2\kappa_0^2} \int\limits_{M_0} d^{D_0} x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - \right. \\ &- \left. \bar{G}_{ij} \tilde{g}^{(0)\mu\nu} \partial_\mu \beta^i \, \partial_\nu \beta^j - 2U_{eff} \right\}. \end{aligned} \tag{2.12}$$

The tensor components of the midisuperspace metric (target space metric on $\mathbf{R}_T^n$) $\bar{G}_{ij}$ $(i, j = 1, \ldots, n)$, its inverse metric $\bar{G}^{ij}$ and the effective potential are respectively

$$\bar{G}_{ij} = d_i \delta_{ij} + \frac{1}{D_0 - 2} d_i d_j, \tag{2.13}$$

$$\bar{G}^{ij} = \frac{\delta^{ij}}{d_i} + \frac{1}{2 - D} \tag{2.14}$$

and

$$U_{eff} = \left(\prod_{i=1}^{n} e^{d_i \beta^i} \right)^{-\frac{2}{D_0 - 2}} \left[-\frac{1}{2} \sum_{i=1}^{n} R_i e^{-2\beta^i} + \Lambda + \kappa^2 \rho \right]. \tag{2.15}$$

We recall that ρ depends on the scale factors of the internal spaces: $\rho = \rho\left(\beta^1, \ldots, \beta^n\right)$. Thus, we are led to the action of a self-gravitating $\sigma-$model with flat target space $(\mathbf{R}_T^n, \bar{G})$ (2.13) and self-interaction described by the potential (2.15). It can be easily seen that the problem of the internal spaces stable compactification reduces now to a search of models that provide minima of the effective potential (2.15). It is important to note that the conformal transformation (2.10) is performed with respect to the external metric $g^{(0)}$. Thus, stable configurations of the internal spaces do not depend on the choice of the frame. However, in the present paper the Einstein frame is considered as the physical one (Cho 1992; Litterio et al 1996).

As next step we bring midisuperspace metric (target space metric) (2.13) by a regular coordinate transformation

$$\varphi = Q\beta, \quad \beta = Q^{-1}\varphi \tag{2.16}$$

to a pure Euclidean form

$$\bar{G}_{ij} d\beta^i \otimes d\beta^j = \sigma_{ij} d\varphi^i \otimes d\varphi^j = \textstyle\sum_{i=1}^n d\varphi^i \otimes d\varphi^i,$$

$$\bar{G} = Q^T Q, \quad \sigma = \mathrm{diag}(+1, +1, \ldots, +1). \tag{2.17}$$

(The superscript ^{T} denotes the transposition.) An appropriate transformation $Q: \ \beta^i \mapsto \varphi^j = Q^j_i \beta^i$ is given e.g. by

$$\begin{aligned} \varphi^1 &= -A \textstyle\sum_{i=1}^n d_i \beta^i, \\ \varphi^i &= [d_{i-1}/\Sigma_{i-1}\Sigma_i]^{1/2} \textstyle\sum_{j=i}^n d_j(\beta^j - \beta^{i-1}), \end{aligned} \tag{2.18}$$

$i = 2, \ldots, n$, where $\Sigma_i = \sum_{j=i}^n d_j$,

$$A = \pm \left[\frac{1}{D'} \frac{D-2}{D_0 - 2} \right]^{1/2} \tag{2.19}$$

and $D' = \sum_{i=1}^n d_i$. So we can write action (2.12) as

$$\begin{aligned} S &= \frac{1}{2\kappa_0^2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - \right. \\ &- \left. \sigma_{ik} \tilde{g}^{(0)\mu\nu} \partial_\mu \varphi^i \, \partial_\nu \varphi^k - 2U_{eff} \right\} \end{aligned} \tag{2.20}$$

with effective potential

$$U_{eff} = e^{\frac{2}{A(D_0-2)}\varphi^1} \left(-\frac{1}{2} \sum_{i=1}^n R_i e^{-2(Q^{-1})^i{}_k \varphi^k} + \Lambda + \kappa^2 \rho \right). \tag{2.21}$$

In this section let us for simplicity consider models with a constant scale factor background localized in one of the minima $\vec{\varphi}_c, c = 1, ..., m$ of the effective potential $\left.\frac{\partial U_{eff}}{\partial \varphi^i}\right|_{\vec{\varphi}_c} = 0$. Then, for small field fluctuations $\xi^i \equiv \varphi^i - \varphi^i_{(c)}$ around the minima the potential (2.21) reads

$$U_{eff} = U_{eff}(\vec{\varphi}_c) + \frac{1}{2} \sum_{i,k=1}^n \bar{A}_{(c)ik} \xi^i \xi^k + O(\xi^i \xi^k \xi^l), \tag{2.22}$$

where the Hessians

$$\bar{A}_{(c)ik} := \left.\frac{\partial^2 U_{eff}}{\partial \xi^i \, \partial \xi^k}\right|_{\vec{\varphi}_c} \tag{2.23}$$

are not vanishing identically. The action functional (2.20) reduces to a family of action functionals for the fluctuation fields ξ^i

$$\begin{aligned} S &= \frac{1}{2\kappa_0^2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - 2U_{eff}(\vec{\varphi}_c) - \right. \\ &- \left. \sigma_{ik} \tilde{g}^{(0)\mu\nu} \partial_\mu \xi^i \, \partial_\nu \xi^k - \bar{A}_{(c)ik} \xi^i \xi^k \right\}, \\ & \quad c = 1, ..., m. \end{aligned} \tag{2.24}$$

It remains to diagonalize the Hessians $\bar{A}_{(c)ik}$ by appropriate $SO(n)-$rotations S_c :
$\xi \mapsto \psi = S_c \xi, \quad S_c^T = S_c^{-1}$

$$\bar{A}_c = S_c^T M_c^2 S_c, \quad M_c^2 = \mathrm{diag}(m^2_{(c)1}, m^2_{(c)2}, \ldots, m^2_{(c)n}), \tag{2.25}$$

leaving the kinetic term $\sigma_{ik} \tilde{g}^{(0)\mu\nu} \partial_\mu \xi^i \, \partial_\nu \xi^k$ invariant

$$\sigma_{ik} \tilde{g}^{(0)\mu\nu} \partial_\mu \xi^i \, \partial_\nu \xi^k = \sigma_{ik} \tilde{g}^{(0)\mu\nu} \partial_\mu \psi^i \, \partial_\nu \psi^k, \tag{2.26}$$

and we arrive at action functionals for decoupled normal modes of linear $\sigma-$models in the background metric $\tilde{g}^{(0)}$ of the external space-time:

$$\begin{aligned} S &= \frac{1}{2\kappa_0^2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - 2\Lambda_{(c)eff} \right\} + \\ &+ \sum_{i=1}^n \frac{1}{2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \times \\ &\times \left\{ -\tilde{g}^{(0)\mu\nu} \psi^i_{,\mu} \psi^i_{,\nu} - m^2_{(c)i} \psi^i \psi^i \right\} \\ & \quad c = 1, \ldots, m, \end{aligned} \tag{2.27}$$

where $\Lambda_{(c)eff} \equiv U_{eff}(\vec{\varphi}_c)$ plays the role of a D_0-dimensional effective cosmological constant and the factor $\sqrt{V_I/\kappa^2}$ has been included into ψ for convenience: $\sqrt{V_I/\kappa^2}\psi \to \psi$.

Thus, conformal excitations of the metric of the internal spaces behave as massive scalar fields developing on the background of the external space-time. By analogy with excitons in solid state physics where they are excitations of the electronic subsystem of a crystal, the excitations of the internal spaces were called gravitational excitons (Günther, Zhuk 1997a).

Different models which ensure minima of the effective potential were described in (Günther, Zhuk 1997a,b; Günther, Kriskiv, Zhuk 1998). Some of them (including the pure geometrical case with $\rho \equiv 0$) satisfy following physical conditions

$$\begin{array}{lccl} (i) & a_{(c)i} & = & e^{\beta^i_c} \gtrsim L_{Pl}, \\ (ii) & m_{(c)i} & \leq & M_{Pl}, \\ (iii) & \Lambda_{(c)eff} & \to & 0. \end{array} \tag{2.28}$$

The first condition expresses the fact that the internal spaces should be unobservable at the present time and stable against quantum gravitational fluctuations. This condition ensures the applicability of the classical gravitational equations near positions of minima of the effective potential. The second condition means that the curvature of the effective potential at minimum points should be less than the Planckian one. Of course, gravitational excitons can be excited at the present time if $m_i \ll M_{Pl}$. The third condition reflects the fact that the cosmological constant at the present time is very small: $|\Lambda| \le 10^{-56}\mathrm{cm}^{-2} \approx 10^{-121}\Lambda_{Pl}$, where $\Lambda_{Pl} = L_{Pl}^{-2}$. Strictly speaking, in the multi-minimum case ($c > 1$) we can demand $a_{(c)i} \sim L_{Pl}$ and $\Lambda_{(c)eff} \to 0$ only for one of the minima to which corresponds the present universe state. For all other minima it may be $a_{(c)i} \gg L_{Pl}$ and $|\Lambda_{(c)eff}| \gg 0$.

3. Interaction of gravitational excitons and gravitons

In this section we give a brief sketch of some of the basic features of the interaction between gravitational excitons and gravitons corresponding to fluctuations of the external metric $\tilde{g}^{(0)}$ (Günther, Starobinsky, Zhuk 1999). To simplify notations we shall drop the index (0) for the external metric: $\tilde{g}^{(0)}_{\mu\nu} \equiv \tilde{g}_{\mu\nu}$ and use the abbreviations

$$A_{ij} := \frac{\partial^2 U_{eff}}{\partial\beta^i\partial\beta^j}, \quad b_i := \frac{\partial U_{eff}}{\partial\beta^i} \; . \tag{3.1}$$

As starting point of our consideration we choose the Euler-Lagrange equations for the scale factors and the external metric derived by variation of the action functional (2.12)

$$\tilde{R}_{\mu\nu} - \frac{1}{2}\tilde{g}_{\mu\nu}\tilde{R} - T_{\mu\nu}\left[\beta, \tilde{g}\right] = 0 \tag{3.2}$$

and

$$\bar{G}_{ij}\Box\beta^j \equiv \bar{G}_{ij}\frac{1}{\sqrt{|\tilde{g}^{(0)}|}}\partial_\mu\left(\sqrt{|\tilde{g}^{(0)}|}\tilde{g}^{(0)\mu\nu}\partial_\nu\beta^j\right) = b_i(\beta)\,, \tag{3.3}$$

where

$$\begin{aligned} T_{\mu\nu}\left[\beta,\tilde{g}\right] &= \bar{G}_{ij}\partial_\mu\beta^i\partial_\nu\beta^j - \\ &\quad -\frac{1}{2}\tilde{g}_{\mu\nu}\left(\bar{G}_{ij}\tilde{g}^{\alpha\beta}\partial_\alpha\beta^i\partial_\beta\beta^j + 2U_{eff}\right) \\ &\equiv \bar{G}_{ij}\partial_\mu\beta^i\partial_\nu\beta^j + \tilde{g}_{\mu\nu}\kappa_0^2 L_\beta[\beta] \; . \end{aligned} \tag{3.4}$$

Assuming that there exists a well defined splitting of the physical fields $(\tilde{g}, \beta)$ into not necessarily constant background components $(\bar{g}, \bar{\beta})$ and small perturbational (fluctuation) components (h, η)

$$\begin{aligned} \tilde{g}_{\mu\nu} &= \bar{g}_{\mu\nu} + h_{\mu\nu}\,, \\ \beta^i &= \bar{\beta}^i + \eta^i \end{aligned} \tag{3.5}$$

we can perform a perturbational analysis of the interaction dynamics of our model splitting the field equations (3.2) and (3.3) into an equation set defining the dynamics of the background fields (zeroth order contribution of fluctuations)

$$\bar{R}_{\mu\nu} - \frac{1}{2}\bar{g}_{\mu\nu}\bar{R} - T_{\mu\nu}\left[\bar{\beta}, \bar{g}\right] = 0\,, \tag{3.6}$$

$$\Box\bar{\beta}^i = \left[\bar{G}^{-1}\right]^{ij} b_j(\bar{\beta}) \tag{3.7}$$

and a set of background depending linearized field equations for the fluctuational components (first order contribution)

$$\begin{aligned} \tfrac{1}{2}h_{\mu\nu;\lambda}{}^{;\lambda} &- h^\lambda{}_{(\mu;\nu);\lambda} + \frac{1}{2}h_{;\mu;\nu} + \\ + &\quad \frac{1}{2}\bar{g}_{\mu\nu}\left(h^{\alpha\lambda}{}_{;\alpha;\lambda} - h_{;\lambda}{}^{;\lambda}\right) + \frac{1}{2}h_{\mu\nu}\bar{R} - \\ - &\quad \frac{1}{2}\bar{g}_{\mu\nu}h^{\kappa\lambda}\bar{R}_{\kappa\lambda} + \kappa_0^2 L_{\bar{\beta}}h_{\mu\nu} + \\ + &\quad \frac{1}{2}\bar{g}_{\mu\nu}\bar{G}_{ij}\partial_\kappa\bar{\beta}^i\partial_\lambda\bar{\beta}^j h^{\kappa\lambda} + \\ + &\quad \bar{G}_{ij}\left(\partial_\mu\bar{\beta}^i\partial_\nu\eta^j + \partial_\mu\eta^i\partial_\nu\bar{\beta}^j\right) - \\ - &\quad \bar{G}_{ij}\bar{g}_{\mu\nu}\bar{g}^{\kappa\lambda}\partial_\kappa\bar{\beta}^i\partial_\lambda\eta^j - \bar{g}_{\mu\nu}b_j(\bar{\beta})\eta^j = 0 \end{aligned} \tag{3.8}$$

and

$$\begin{aligned} \Box\eta^i - \left[\bar{G}^{-1}\right]^{ij}A_{jk}(\bar{\beta})\eta^k &= \frac{1}{\sqrt{|\bar{g}|}}\partial_\nu\left(\sqrt{|\bar{g}|}h^{\mu\nu}\partial_\mu\bar{\beta}^i\right) - \\ &\quad -\frac{1}{2}\bar{g}^{\mu\nu}\partial_\mu\bar{\beta}^i\partial_\nu h \; . \end{aligned} \tag{3.9}$$

Here $\bar{R}_{\mu\nu}$ and the semicolon denote the Ricci-tensor and the covariant derivative with respect to the background metric $\bar{g}_{\mu\nu}$. Additionally we have used the formula

$$\tilde{g}^{\mu\nu} = \bar{g}^{\mu\nu} - h^{\mu\nu} \tag{3.10}$$

which is valid up to linear terms in h. Indices in $h_{\mu\nu}$ are raised and lowered by the background metric $\bar{g}_{\mu\nu}$, e.g. $h = h_{\mu\nu}\bar{g}^{\mu\nu}$.

Let us now generalize the normal mode formalism applied in section 2 for the derivation of gravitational excitons over constant scale factor backgrounds to models with non-constant scale factor backgrounds (Günther, Zhuk 1998). For this purpose we diagonalize matrix $\left[\bar{G}^{-1}A\right]^i_k \equiv \left[\bar{G}^{-1}\right]^{ij}A_{jk}(\bar{\beta})$ by an appropriate background depending $SO(n)-$rotation $S = S(\bar{\beta})$

$$S^{-1}\bar{G}^{-1}AS \overset{def}{=} M^2 = \mathrm{diag}\left[m_1^2(\bar{\beta}), \ldots, m_n^2(\bar{\beta})\right] \tag{3.11}$$

and rewrite Eq. (3.9) in terms of generalized normal modes (gravitational excitons) $\psi = S^{-1}\eta$:

$$\begin{aligned} \bar{g}^{\mu\nu}D_\mu D_\nu\psi - M^2(\bar{\beta})\psi &= \left(h^{\mu\nu} - \frac{1}{2}\bar{g}^{\mu\nu}h\right)_{;\nu} D_\mu\bar{\varphi} + \\ &\quad + h^{\mu\nu}D_\mu D_\nu\bar{\varphi}\,, \end{aligned} \tag{3.12}$$

where $\bar{\varphi}$ are $SO(n)-$rotated background scale factors $\bar{\varphi} = S^{-1}\bar{\beta}$ and M^2 can be interpreted as background depending diagonal mass matrix for the gravitational excitons.

D_μ denotes a covariant derivative

$$D_\mu := \partial_\mu + \Gamma_\mu + \omega_\mu \ , \quad \omega_\mu := S^{-1}\partial_\mu S \tag{3.13}$$

with $\Gamma_\mu + \omega_\mu$ as connection on the fibre bundle $E(\bar{M}_0, \mathbf{R}^{D_0} \oplus \mathbf{R}^n_T) \to \bar{M}_0$ consisting of the base manifold $\bar{M}_0$ and vector spaces $\mathbf{R}^{D_0}_x \oplus \mathbf{R}^n_{Tx} = T_x\bar{M}_0 \oplus \left\{\left(\eta^1(x), \ldots, \eta^n(x)\right)\right\}$ as fibres. So, the background components $\bar{\beta}^i(x)$ via the effective potential U_{eff} and its Hessian $A_{ij}(\bar{\beta})$ play the role of a medium for the gravitational excitons $\psi^i(x)$. Propagating in $\bar{M}_0$ filled with this medium they change their masses as well as the direction of their "polarization" defined by the unit vector in the fibre space

$$\xi(x) := \frac{\psi(x)}{|\psi(x)|} \in S^{n-1} \subset \mathbf{R}^n, \tag{3.14}$$

where S^{n-1} denotes the $(n-1)-$dimensional sphere.

From (3.8), (3.9) and (3.12) we see that in the lowest order (linear) approximation of the used perturbation theory a non-constant scale factor background is needed for an interaction between gravitational excitons and gravitons. For constant scale factor backgrounds $\bar{\beta} = const$ the system is necessarily located in one of the minima $\bar{\beta} = \beta_{(c)}$ of the effective potential U_{eff} so that $b_i(\beta_{(c)}) = 0, \quad \kappa_0^2 L_{\beta_{(c)}} = U_{eff}(\beta_{(c)}) = \Lambda_{(c)eff}$ and gravitational excitons and gravitons can only interact via nonlinear (higher order) terms. In the linear approximation they decouple over constant scale factor backgrounds due to vanishing terms in (3.8), (3.9) and (3.12).

4. Interaction of gravitational excitons with abelian gauge fields and possible astrophysical implications

In this section we consider the zero mode case, i.e., the case of an abelian vector potential that depends only on the external coordinates: $A_M = A_M(x) \quad (M = 1, \ldots, D)$. Thus, for non-zero components of the field strength tensor we have: $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \quad (\mu, \nu = 1, \ldots, D_0)$ and $F_{\mu m_i} = \partial_\mu A_{m_i} - \partial_{m_i} A_\mu = \partial_\mu A_{m_i} \quad (m_i = 1, \ldots, d_i; i = 1, \ldots, n)$.

Dimensional reduction of the action for the gauge field yields

$$\begin{aligned} S_{em} &= -\frac{1}{2}\int_M d^D X \sqrt{|g|} F_{MN}F^{MN} \qquad (4.1) \\ &= -\frac{1}{2}\int_{\bar{M}_0} d^{D_0}x\sqrt{|g^{(0)}|}\prod_{i=1}^n e^{d_i\beta^i}\, \{F_{\mu\nu}F^{\mu\nu} + \\ &\quad +2g^{(0)\mu\nu}\sum_{i=1}^n e^{-2\beta^i(x)}\bar{g}^{(i)m_in_i}\partial_\mu A_{m_i}\partial_\nu A_{n_i}\Bigg\}, \end{aligned}$$

where we introduced the constant metric

$$\bar{g}^{(i)m_in_i} \equiv \frac{1}{v_i}\int_{M_i} d^{d_i}y\sqrt{|g^{(i)}|}g^{(i)m_in_i}(y^i) \tag{4.2}$$

and included the factor $\sqrt{V_I}$ into A_M for convenience: $\sqrt{V_I}A_M \to A_M$. In Eq. (4.1) we assumed $F^{\mu\nu} = g^{(0)\mu\kappa}g^{(0)\nu\delta}F_{\kappa\delta}$.

The exact field strength 2-form $F = dA, \ A = A_\mu dx^\mu$ with components $F_{\mu\nu}$ is invariant under gauge transformations $A \to A^f = A + df$, i.e., $A_\mu \to A^f_\mu = A_\mu + \partial_\mu f$, $F^f = dA + d^2 f = dA = F$, with $f(x)$ any smooth function. Gauge invariance of F implies a gauge invariance of the action functional (4.1).

Action functional (4.1) is written in the Brans-Dicke frame. After conformal transformation (2.10) and with an ansatz

$$A_\mu = \Omega^k \tilde{A}_\mu \tag{4.3}$$

for the vector potential, where $\Omega(x)$ is given by (2.11), the effective action in the Einstein frame reads

$$\begin{aligned} S_{em} = -\frac{1}{2}\int_{\bar{M}_0} & d^{D_0}x\sqrt{|\tilde{g}^{(0)}|}\left\{\Omega^{2(k-1)}\bar{F}_{\mu\nu}\bar{F}^{\mu\nu} + \right. \\ & \left. + \ 2\tilde{g}^{(0)\mu\nu}\sum\nolimits_{i=1}^n e^{-2\beta^i(x)}\bar{g}^{(i)m_in_i}\partial_\mu A_{m_i}\partial_\nu A_{n_i}\right\}, \end{aligned} \tag{4.4}$$

where

$$\bar{F}_{\mu\nu} := D_\mu\tilde{A}_\nu - D_\nu\tilde{A}_\mu \tag{4.5}$$

and

$$D_\mu := \partial_\mu + \partial_\mu(\ln\Omega^k)\,. \tag{4.6}$$

The external space indices are raised and lowered by the metric $\tilde{g}^{(0)}$. Explicitly we have in (4.4)

$$\begin{aligned} \bar{F}_{\mu\nu}\bar{F}^{\mu\nu} &= \tilde{F}_{\mu\nu}\tilde{F}^{\mu\nu} - \\ & -2\tilde{F}^{\mu\nu}\left[\tilde{A}_\mu\partial_\nu(\ln\Omega^k) - \tilde{A}_\nu\partial_\mu(\ln\Omega^k)\right] \\ & +2\tilde{g}^{(0)\mu\kappa}\partial_\mu(\ln\Omega^k)\partial_\kappa(\ln\Omega^k)\tilde{A}^\nu\tilde{A}_\nu - \\ & -2\left(\tilde{A}^\mu\partial_\mu(\ln\Omega^k)\right)^2 , \end{aligned} \tag{4.7}$$

where $\tilde{F} = d\tilde{A}$.

In order to preserve the gauge invariance of the action functional when passing from the Brans-Dicke frame to the Einstein frame we have to keep the vector potential unchanged, i.e. we have to fix the conformal weight at $k = 0$ (Günther, Starobinsky, Zhuk 1999). As result we arrive at an action functional

$$\begin{aligned} S_{em} = -\frac{1}{2}\int_{\bar{M}_0} & d^{D_0}x\sqrt{|\tilde{g}^{(0)}|} \times \\ & \times \left\{e^{\frac{2}{D_0-2}\sum_{i=1}^n d_i\beta^i(x)}F_{\mu\nu}F^{\mu\nu} + \right. \\ & \left. + \ 2\tilde{g}^{(0)\mu\nu}\sum\nolimits_{i=1}^n e^{-2\beta^i(x)}\bar{g}^{(i)m_in_i}\partial_\mu A_{m_i}\partial_\nu A_{n_i}\right\}, \end{aligned} \tag{4.8}$$

with a dilatonic coupling of the abelian gauge potential to the gravitational excitons. The components A_{m_i} play the role of additional scalar fields.

Similar to string cosmology models describing the dynamics of electromagnetic fields with dilatonic coupling (Gasperini, Giovannini, Veneziano 1995a,b) we can expect in theory (4.8) an amplification of electromagnetic vacuum fluctuations (due to the presence of a dynamical gravitational exciton background) which can result in the observable cosmic microwave background anisotropy.

Let us now discuss some astrophysical implications of the interaction between gravitational excitons and photons. For simplicity we consider the one internal space case ($i = 1$) and suppose that the scale factor background is localized in a minimum $a_{(c)} = \exp \beta^1_{(c)}$ of the effective potential (2.15). Then, for small scale factor fluctuations $\eta = \beta^1 - \beta^1_{(c)}$ action (2.6) with $S_{add} \equiv S_{em}$ (where S_{em} is described by Eqs. (4.1) and (4.8)) reads

$$\begin{aligned} S = \frac{1}{2\kappa_0^2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - 2\Lambda_{(c)eff} \right\} + \\ + \frac{1}{2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ -\tilde{g}^{(0)\mu\nu} \psi_{,\mu}\psi_{,\nu} - m^2_{(c)}\psi\psi \right\} - \\ - \frac{1}{2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ F_{\mu\nu}F^{\mu\nu} - \right. \\ \left. - 2\sqrt{\frac{d_1}{(D_0-2)(D-2)}} \kappa_0 \psi F_{\mu\nu}F^{\mu\nu} \right\} + \ldots , \end{aligned} \tag{4.9}$$

where we used the notations of Eq. (2.27) and fluctuations η and ψ are connected with each other as follows:

$$\eta = \kappa_0 \sqrt{\frac{D_0 - 2}{d_1(D-2)}} \psi \ . \tag{4.10}$$

As mentioned above, $\kappa_0^2 = 8\pi/M^2_{Pl}$ is the D_0-dimensional (usually $D_0 = 4$) gravitational constant. In Eq. (4.9) we normalize the electromagnetic field as: $(a_{(c)})^{d_1/(D_0-2)} F_{\mu\nu} \longrightarrow F_{\mu\nu}$ and the last term there describes the interaction between gravitational excitons and photons. In a tree-level approximation this term corresponds to the diagram at Fig.1 and describes a

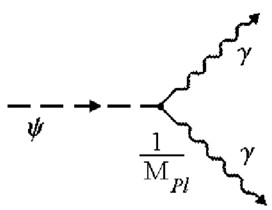

Figure 1: Decay of a gravitational exciton into two photons.

gravitational exciton decaying into two photons. For the probability of this decay we can easily get the estimate

$$\Gamma \sim \left(\frac{1}{M_{Pl}}\right)^2 m^3_{(c)} = \left(\frac{m_{(c)}}{M_{Pl}}\right)^3 \frac{1}{T_{Pl}} \ , \tag{4.11}$$

which results in a life-time of the gravitational excitons with respect to this decay

$$\tau = \frac{1}{\Gamma} \sim \left(\frac{M_{Pl}}{m_{(c)}}\right)^3 T_{Pl} \ . \tag{4.12}$$

This equation shows that for excitons with masses $m_{(c)} \leq 10^{-21} M_{Pl} \sim 10^{-2} GeV \sim 20 m_e$ (where m_e is the electron mass) the life-time $\tau \geq 10^{19} sec > t_{univ} \sim 10^{18} sec$ is greater than the age of the universe. Thus, gravitational excitons are very weakly interacting particles and can be considered as Dark Matter (DM). The type of the DM depends on the DM particle masses. It is hot for $m_{DM} \leq 50 - 100 eV$, warm for $100 eV \leq m_{DM} \leq 10 KeV$ and cold for $m_{DM} \geq 10 - 50 KeV$. The gravitational exciton masses are defined by the scales of compactification: $m_{(c)} \sim \left(a_{(c)}\right)^{-(D-2)/(D_0-2)}$ (Günther, Zhuk 1997a,b). It is clear that it is hardly possible to use the diagram at Fig.1 to estimate from experiments the gravitational exciton masses and respectively the scale of the internal spaces compactification. The reason consists in the term $1/M_{Pl}$ in the vertex of the diagram. However, by analogy with axions (Gnedin 1997a,b, 1999) it is possible that in strong magnetic field there can occure oscillations between gravitational excitons and photons which are described by the diagram at Fig.2, which corresponds to an in-

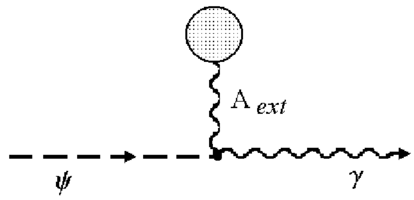

Figure 2: Conversion of a gravitational exciton into a photon in the presence of a strong magnetic background field.

teraction term $L_{eff} \sim \kappa_0 \psi F^{\mu\nu}_{ext} f_{\mu\nu}$. $F^{\mu\nu}_{ext}$ is an external magnetic field and $f_{\mu\nu}$ describes photons. The probability of magnetic conversion of gravitational excitons into photons (and vice versa) can be much greater then (4.11) and will result in observable lines in spectra of astrophysical objects.

5. Multidimensional perfect-fluid cosmology with stable compactified internal dimensions

Here we present a toy example of a multidimensional cosmological model (MCM) which shows the principal possibility to achieve a postinflational Friedmann-Robertson-Walker dynamics for the external ($D_0 = 4$)−dimensional space-time keeping the internal factor spaces stable compactified. This model is out of the scope of MCM with stable compactification found in (Günther, Zhuk 1997a,b). The main difference consists in an additional time-dependent term in the effective potential that provides the needed dynamical behaviour of the external space-time. This term is induced by a special type of fine-tuning of the parameters of a multicomponent perfect fluid. Although such a fine-tuning is a strong restriction on the matter content of the model, many important cases of physically interest are described by this class of perfect fluid. We note that a similar class of perfect fluids was considered in (Kasper, Zhuk 1996), where MCMs were integrated in the case of an absent cosmological constant and Ricci-flat internal spaces. As result particular solutions with static internal spaces had been obtained. In this section we show that these solutions are not stable and a bare cosmological constant and internal spaces with non-vanishing curvature are necessary conditions for their stabilization. With the help of suitably chosen parameters the model can be further improved to solve two problems simultaneously. First, the internal spaces undergo stable compactification. Second, the external space behaves in accordance with the standard Friedmann model.

To reach this goal let us start for simplicity from a homogeneous metric ansatz for the multidimensional cosmological model

$$\begin{aligned} g &= g_{MN} dX^M \otimes dX^N \qquad (5.1)\\ &= -\exp\left[2\gamma(\tau)\right] d\tau \otimes d\tau + \sum_{i=0}^{n} \exp\left[2\beta^i(\tau)\right] g_{(i)}, \end{aligned}$$

where we assumed the product manifold given by (1.1) with all factor spaces M_i, $i = 0, \ldots, n$ as Einstein manifolds and a foliated external space-time $\bar{M}_0 = \mathbf{R} \times M_0$. The scalar curvature corresponding to the metric (5.1) reads

$$\begin{aligned} R &= \sum_{i=0}^{n} R_i \exp\left(-2\beta^i\right) + \exp\left(-2\gamma\right) \times \qquad (5.2)\\ &\times \sum_{i=0}^{n} d_i \left[2\ddot{\beta}^i - 2\dot{\gamma}\dot{\beta}^i + \left(\dot{\beta}^i\right)^2 + \dot{\beta}^i \sum_{j=0}^{n} d_j \dot{\beta}^j\right]. \end{aligned}$$

Matter fields we take into account in a phenomenological way as a m−component perfect fluid with energy-momentum tensor

$$T_N^M = \sum_{a=1}^{m} T^{(a)}{}_N^M , \qquad (5.3)$$

$$\begin{aligned} T^{(a)}{}_N^M = \operatorname{diag}\Big(& -\rho^{(a)}(\tau), \underbrace{P_0^{(a)}(\tau), \ldots, P_0^{(a)}(\tau)}_{d_0 \text{ times}}, \ldots, \\ & \ldots, \underbrace{P_n^{(a)}(\tau), \ldots, P_n^{(a)}(\tau)}_{d_n \text{ times}} \Big) \qquad (5.4) \end{aligned}$$

and equations of state

$$P_i^{(a)} = \left(\alpha_i^{(a)} - 1\right) \rho^{(a)}, \quad i = 0, \ldots, n, \quad a = 1, \ldots, m. \qquad (5.5)$$

It is easy to see that physical values of $\alpha_i^{(a)}$ according to $-\rho^{(a)} \le P_i^{(a)} \le \rho^{(a)}$ run the region $0 \le \alpha_i^{(a)} \le 2$. The conservation equations we impose on each component separately

$$T^{(a)}{}_{N;M}^M = 0. \qquad (5.6)$$

Denoting by an overdot differentiation with respect to time τ, these equations read for the tensors (5.4)

$$\dot{\rho}^{(a)} + \sum_{i=0}^{n} d_i \dot{\beta}^i \left(\rho^{(a)} + P_i^{(a)}\right) = 0 \qquad (5.7)$$

and have according to (5.5) the simple integrals

$$\rho^{(a)}(\tau) = A^{(a)} \prod_{i=0}^{n} a_i^{-d_i \alpha_i^{(a)}}, \qquad (5.8)$$

where $a_i \equiv e^{\beta^i}$ are scale factors of M_i and $A^{(a)}$ are constants of integration. It is not difficult to verify that the Einstein equations with the energy-momentum tensor (5.3)-(5.8) are equivalent to the Euler-Lagrange equations for the Lagrangian (Ivashchuk, Melnikov 1995; Zhuk 1996)

$$\begin{aligned} L &= \frac{1}{2} e^{-\gamma+\gamma_0} G_{ij} \dot{\beta}^i \dot{\beta}^j - \qquad (5.9)\\ &- e^{\gamma+\gamma_0} \left(-\frac{1}{2} \sum_{i=0}^{n} R_i e^{-2\beta^i} + \kappa^2 \sum_{a=1}^{m} \rho^{(a)} + \Lambda\right). \end{aligned}$$

Here we use the notation $\gamma_0 = \sum_0^n d_i \beta^i$.

The Lagrangian (5.9) can be obtained by dimensional reduction of the action functional (2.6).

Via conformal transformation of the external space-time metric from the Brans-Dicke to the Einstein frame:

$$g = g_{MN} dX^M \otimes dX^N$$

$$
\begin{aligned}
&= \bar{g}^{(0)} + \sum_{i=1}^{n} \exp{[2\beta^i(x)]} g^{(i)} \\
&= \Omega^2 \tilde{g}^{(0)} + \sum_{i=1}^{n} \exp{[2\beta^i(x)]} g^{(i)}, \qquad (5.10)
\end{aligned}
$$

with Ω^2 given by (2.11) we find that the external scale factors in the Brans-Dicke frame $a_0 = e^{\beta^0} \equiv a$ and in the Einstein frame $\tilde{a}_0 = e^{\tilde{\beta}^0} \equiv \tilde{a}$ are connected with each other by the relation

$$
a = \left(\prod_{i=1}^{n} e^{d_i \beta^i} \right)^{-\frac{1}{D_0 - 2}} \tilde{a}. \qquad (5.11)
$$

The energy densities $\rho^{(a)}$ of the perfect fluid components are given by (5.8) and with the help of relation (5.11) can be rewritten as

$$
\rho^{(a)} = \rho_0^{(a)} \prod_{i=1}^{n} a_i^{-\xi_i^{(a)}}, \qquad (5.12)
$$

where

$$
\rho_0^{(a)} = A^{(a)} \frac{1}{\tilde{a}^{\alpha_0^{(a)} d_0}} \qquad (5.13)
$$

and

$$
\xi_i^{(a)} = d_i \left(\alpha_i^{(a)} - \frac{\alpha_0^{(a)} d_0}{d_0 - 1} \right). \qquad (5.14)
$$

For the effective potential (2.15) we get accordingly

$$
\begin{aligned}
U_{eff} &= \left(\prod_{i=1}^{n} e^{d_i \beta^i} \right)^{-\frac{2}{D_0 - 2}} \left[-\frac{1}{2} \sum_{i=1}^{n} R_i e^{-2\beta^i} + \right. \\
&\left. + \Lambda + \kappa^2 \sum_{a=1}^{m} \rho^{(a)} \right]. \qquad (5.15)
\end{aligned}
$$

We now investigate MCMs containing a special subclass of this effective potential (5.15): potentials with separating scale factor contributions from internal and external factor spaces

$$
\begin{aligned}
U_{eff} &= \underbrace{\left(\prod_{i=1}^{n} e^{d_i \beta^i} \right)^{-\frac{2}{D_0 - 2}} \left[-\frac{1}{2} \sum_{i=1}^{n} R_i e^{-2\beta^i} + \Lambda \right]}_{U_{int}} + \\
&+ \underbrace{\kappa^2 \sum_{a=1}^{m} \rho_0^{(a)}}_{U_{ext}}. \qquad (5.16)
\end{aligned}
$$

We will show below, that such a separation on the one hand provides a stable compactification of the internal factor spaces due to a minimum of the first term $U_{int} = U_{int}(\beta^1, \ldots, \beta^n)$ as well as a dynamical behaviour of the external factor space due to $U_{ext} = U_{ext}(\tilde{\beta}^0)$. On the other hand this separation crucially simplifies the calculations and allows an exact analysis. The price that we have to pay for the separation is a fine-tuning of the parameters of the multicomponent perfect fluid

$$
\begin{aligned}
\alpha_0^{(a)} &= \tfrac{2}{d_0} + \tfrac{d_0 - 1}{d_0} \alpha^{(a)} \\
\alpha_i^{(a)} &= \alpha^{(a)}, \quad i = 1, \ldots, n, \quad a = 1, \ldots, m.
\end{aligned} \qquad (5.17)
$$

Only in this case we have

$$
\xi_i^{(a)} = -\frac{2 d_i}{d_0 - 1} \qquad (5.18)
$$

yielding the compensation of the exponential prefactor for the perfect fluid term in the effective potential (5.15). The corresponding components $\rho_0^{(a)}$ read, respectively,

$$
\rho_0^{(a)} = A^{(a)} \frac{1}{\tilde{a}^{2 + (d_0 - 1)\alpha^{(a)}}}. \qquad (5.19)
$$

Although the fine-tuning (5.17) is a strong restriction, there exist some important particular models that belong to this class of multicomponent perfect fluids. For example, if $\alpha^{(a)} = 1$ the a-th component of the perfect fluid describes radiation in the space M_0 and dust in the spaces $M_1, \ldots, M_n$. This kind of perfect fluid satisfies the condition $\sum_{i=0}^{n} d_i \alpha_i^{(a)} = D$ and is called superradiation (Liebscher, Bleyer 1985). If $\alpha^{(a)} = 2$ we obtain the ultra-stiff matter in all M_i $(i = 0 \ldots, n)$ which is equivalent, e.g., to a massless minimally coupled free scalar field. In the case $\alpha^{(a)} = 0$ we get the equation of state $P_0^{(a)} = [(2 - d_0)/d_0] \rho^{(a)}$ in the external space M_0 which describes a gas of cosmic strings if $d_0 = 3 : P^{(a)} = -\frac{1}{3} \rho^{(a)}$ (Spergel, Pen 1997) and vacuum in the internal spaces $M_1, \ldots, M_n$. If $\alpha^{(a)} = 1/2$ and $d_0 = 3$ we obtain dust in the external space M_0 and a matter with equation of state $P_i^{(a)} = -\frac{1}{2} \rho^{(a)}$ in the internal spaces $M_i, \quad i = 1, \ldots, n$.

Let us first consider the conditions for the existence of a minimum of the potential $U_{int}(\beta^1, \ldots, \beta^n)$. According to (Günther, Zhuk 1997b) potentials U_{int} of type (5.16) have a single minimum if the bare cosmological constant and the curvature scalars of the internal spaces are negative $R_i, \Lambda < 0$. The scale factors $\{\beta_c^i\}_{i=1}^{n}$ at the minimum position of the effective potential are connected by a fine-tuning condition

$$
\frac{R_i}{d_i} e^{-2\beta_c^i} = \frac{2\Lambda}{D - 2} \equiv \widetilde{C}, \qquad i = 1, \ldots, n \qquad (5.20)
$$

and the masses squared of the corresponding gravitational excitons are degenerate and given as

$$
\begin{aligned}
m_1^2 &= \ldots = m_n^2 = m_{exci}^2 \\
&= -\frac{4\Lambda}{D - 2} \exp{\left[-\frac{2}{d_0 - 1} \sum_{i=1}^{n} d_i \beta_c^i \right]} \\
&= 2 \left| \widetilde{C} \right|^{\frac{D - 2}{d_0 - 1}} \prod_{i=1}^{n} \left| \frac{d_i}{R_i} \right|^{\frac{d_i}{d_0 - 1}}. \qquad (5.21)
\end{aligned}
$$

Further it was shown in (Günther, Zhuk 1997b) that the value of the potential U_{int} at the minimum is connected with the exciton mass by the relation

$$\Lambda_{int} := U_{int}(\beta_c^1, \ldots, \beta_c^n) = -\frac{d_0 - 1}{4} m_{exci}^2 . \quad (5.22)$$

From equations (5.20), (5.21) we see that exciton masses and minimum position $a_{(c)i} = \exp \bar{\beta}_c^i$ are constants that solely depend on the value of the bare cosmological constant Λ, the (constant) curvature scalars R_i and dimensions d_i of the internal factor spaces. This means that we have automatically $\Omega_c = \bar{M}_0$ from the very onset of the model. Hence the exciton approach in the present linear form breaks down only when the excitations become too strong so that higher order terms must be included in the consideration or the phenomenological perfect fluid approximation itself becomes inapplicable.

Let us now turn to the dynamical behaviour of the external factor space. For simplicity we consider the zero order approximation, when all excitations are freezed, in the homogeneous case: $\tilde{\gamma} = \tilde{\gamma}(\tilde{\tau})$ and $\tilde{\beta} = \tilde{\beta}(\tilde{\tau})$. Then the action functional (2.27) with

$$\begin{aligned} U_{(c)eff} &\equiv U_{eff}\left[\vec{\beta}_c, \tilde{\beta}(\tilde{\tau})\right] \\ &= U_{int}(\beta_c^1, \ldots, \beta_c^n) + U_{ext}\left[\tilde{\beta}(\tilde{\tau})\right] \\ &\equiv \Lambda_{int} + \bar{\rho}_0(\tilde{\tau}) \end{aligned} \quad (5.23)$$

after dimensional reduction reads:

$$\begin{aligned} S &= \frac{1}{2\kappa_0^2} \int\limits_{\bar{M}_0} d^{D_0}x \sqrt{|\tilde{g}^{(0)}|} \left\{ \tilde{R}\left[\tilde{g}^{(0)}\right] - 2U_{(c)eff} \right\} \\ &= \frac{V_0}{2\kappa_0^2} \int d\tilde{\tau} \left\{ e^{\tilde{\gamma} + d_0\tilde{\beta}} e^{-2\tilde{\beta}} R[g^{(0)}] + \right. \\ &\quad + d_0(1-d_0) e^{-\tilde{\gamma} + d_0\tilde{\beta}} \left(\frac{d\tilde{\beta}}{d\tilde{\tau}} \right)^2 - \\ &\quad \left. - 2e^{\tilde{\gamma} + d_0\tilde{\beta}} \left(\Lambda_{int} + \bar{\rho}_0 \right) \right\} + \\ &\quad + \frac{V_0}{2\kappa_0^2} d_0 \int d\tilde{\tau} \frac{d}{d\tilde{\tau}} \left(e^{-\tilde{\gamma} + d_0\tilde{\beta}} \frac{d\tilde{\beta}}{d\tilde{\tau}} \right), \end{aligned} \quad (5.24)$$

where usually $R[g^{(0)}] = kd_0(d_0 - 1), \quad k = \pm 1, 0$. The constraint equation $\partial L / \partial \tilde{\gamma} = 0$ in the synchronous time gauge $\tilde{\gamma} = 0$ yields

$$\left(\frac{1}{\tilde{a}} \frac{d\tilde{a}}{d\tilde{t}} \right)^2 = -\frac{k}{\tilde{a}^2} + \frac{2}{d_0(d_0 - 1)} \left(\Lambda_{int} + \bar{\rho}_0(\tilde{a}) \right), \quad (5.25)$$

which results in

$$\begin{aligned} &\tilde{t} + const \\ &= \int \frac{d\tilde{a}}{\left[-k + \frac{2\Lambda_{int}}{d_0(d_0-1)} \tilde{a}^2 + \frac{2\kappa^2}{d_0(d_0-1)} \sum_{a=1}^m \frac{A^{(a)}}{\tilde{a}^{(d_0-1)\alpha^{(a)}}} \right]^{1/2}}, \\ &= \int \frac{d\tilde{a}}{\left[-k + \frac{\Lambda_{int}}{3} \tilde{a}^2 + \frac{\kappa^2}{3} \sum_{a=1}^m \frac{A^{(a)}}{\tilde{a}^{2\alpha^{(a)}}} \right]^{1/2}}, \end{aligned} \quad (5.26)$$

where in the last line we put $d_0 = 3$.

Thus in the zero order approximation we arrived at a Friedmann model in the presence of negative cosmological constant Λ_{int} and a multicomponent perfect fluid. The perfect fluid has the form of a gas of cosmic strings for $\alpha^{(a)} = 0$, dust for $\alpha^{(a)} = 1/2$ and radiation for $\alpha^{(a)} = 1$. As $0 \le \alpha^{(a)} \le 2$, the cosmological constant plays a role only for large $\tilde{a}$ and because of the negative sign of Λ_{int} the universe has a turning point at the maximum of $\tilde{a}$. To be consistent with present time observation we should take

$$|\Lambda_{int}| \le 10^{-121} \Lambda_{Pl} . \quad (5.27)$$

We note that due to (5.26) and in contrast with (2.27) the minimum value $U_{(c)eff}$ of the effective potential in (5.23) cannot be interpreted as a cosmological constant, even as a time dependent one. Coming back to the gravitational excitons we see that according to (5.22) the upper bound (5.27) on the effective cosmological constant leads to ultra-light particles with mass $m_{exci} \le 10^{-60} M_{Pl} \sim 10^{-32} eV$. This is much less than the cosmic background radiation temperature at the present time $T_0 \sim 10^{-4} eV$. It is clear that such light particles up to present time behave as radiation and can be taken into account as an additional term $\rho_r = \frac{\kappa_0^2 A_r/3}{\tilde{a}^2}$ in (5.26). It can be easily seen that we reconstruct the standard scenario if we consider the one-component ($m = 1$) case with $\alpha^{(1)} = 1/2$, $\kappa^2 A^{(1)} \sim 10^{61}$ and $\kappa_0^2 A_r \sim 10^{117}$. Here we have at early stages a radiation dominated universe and a dust dominated universe at later stages of its evolution.

For completeness we note that via equations (5.21) and (5.22) the value of the effective cosmological constant has a crucial influence on the relation between the compactification scales of the internal factor spaces and their dimensions. In the case of only one internal negative curvature space $M_1 = H^{d_1}/\Gamma$ with $R_1 = -d_1(d_1 - 1)$ and compactification scale $a_{(c)1} = 10 L_{Pl}$ we have e.g. the relation $\Lambda_{int} = -(d_1 - 1) 10^{-2(d_1+2)} L_{Pl}$, so that the bound (5.27) implies a dimension of this space of at least $d_1 = 59$. Taking instead of one internal space a set of 2-dimensional hyperbolic $g-$tori $\left\{ M_i = H^2/\Gamma \right\}_{i=1}^n$ (Lachieze-Rey, Luminet 1995) with compactification scale $a_{(c)i} = 10^2 L_{Pl}$ it is easy to check that we need at least $n = 29$ such spaces to satisfy (5.27).

Of course, other values of the cosmological constant lead to other exciton masses and compactification - dimensionality relations. So, it is also possible to get models with much more heavier gravitational excitons. For $\Lambda_{int} = -10^{-8} \Lambda_{Pl}$ we have e.g. $m = 10^{-4} M_{Pl}$ and the excitons are very heavy particles that should be considered as a cold dark matter. If we take the one-component case $\alpha^{(1)} = 1$ we get at early times a radiation dominated universe with smooth transition to a cold dark matter dominated universe at later stages. But for this example it is necessary to introduce a mechanism that provides a reduction of the huge cosmological constant to the observable value $10^{-121}\Lambda_{Pl}$.

6. Conclusion

In the present paper we reviewed some recent results on inhomogeneous scale factor fluctuations as they necessarily occure in higher dimensional gravitational models after stable compactification of internal factor spaces. We showed that such scale factor fluctuations should be interpreted as massive or massless scalar particles propagating in the external space-time and interacting with other particles.

As simplest examples we considered lowest order approximations of the interaction of gravitational excitons with gravitons and photons.

Due to the specific gradient-like coupling terms of gravitational excitons and gravitons, in the used lowest order approximation of the theory an interaction between them occures only in the presence of a non-constant scale factor background. For constant scale factor backgrounds the system is necessarily located in one of the minima of the effective potential so that an interaction is only possible via nonlinear (higher order) coupling terms.

The analysis of the interaction between gravitational excitons and abelian gauge field showed that due to the high life-time of gravitational excitons with respect to the decay channel into photons the excitons should be interpreted as Dark Matter. Nevertheless, by analogy with axions it is possible that in strong magnetic fields there can occure oscillations between gravitational excitons and photons which will result in observable lines in spectra of astrophysical objects.

The last section of this review was devoted to the question of compatibility of the considered multidimensional gravitational models with the postinflational Friedman-Robertson-Walker dynamics of the observable part of our universe. For this purpose we considered a MCM with bare cosmological constant and a perfect fluid as matter source. It can be easily seen that there are only two classes of perfect fluids with stably compactified internal spaces. These kind of solutions are of utmost interest because an absent time variation of the fundamental constants in experiments (Marciano 1984; Kolb, Perry, Walker 1986) shows that at the present time the extra dimensions, if they exist, should be static or nearly static.

The first class (Günther, Zhuk 1997a,b) consists of models with $\alpha_0^{(a)} = 0$. It leads to the vacuum equation of state in the external space M_0. All other $\alpha_i^{(a)} (i = 1, \ldots, n)$ can take arbitrary values. This model can be used for a phemenological description of a muitidimensional inflationary universe with smooth transition to a matter dominated stage.

For models of the second class the stability is induced by a fine-tuning of the equation of state of the perfect fluid in the external and internal spaces (5.17). This class includes many important particular models and allows considerations of perfect fluids with different equations of state in the external space, among them also such that result in a Friedmann-like dynamics. Thus, this class of models can be applied for the description of the postinflationary stage in multidimensional cosmology. For the considered models we found necessary restrictions on the parameters which, from the one hand, ensure stable compactification of the internal spaces near Planck length and, from the other hand, guarantee dynamical behaviour of the external (our) universe in accordance with the standard scenario for the Friedmann model.

This toy model gives a promising example of a multidimensional cosmological model which is not in contradiction to observations. Although, a fine-tuning is necessary to get an effective cosmological constant in accordance with the present day observations.

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