ON MASS LOSS FROM EVOLVED MASSIVE STARS

A.V. Dorodnitsyn, G.S. Bisnovatyi - Kogan

Space Research Institute
Profsoyuznaya st. 84/32, Moscow, Russia,
dora@mx.iki.rssi.ru,
gkogan@mx.iki.rssi.ru

ABSTRACT. We consider stationary outflowing stellar envelopes accelerated by a radiation flux pressure. A method is developed describing a spherically symmetric flow in radiational hydrodynamics in regions with arbitrary optical depth (τ). The solution of the derived system of differential equations is obtained numerically. It proceeds through the singular point, where a velocity is eqial to the isothermal sound speed, and satisfies zero temperature and pressure boundary conditions at the infinity. Method is discussed of self- consistent evolutionary calculations for massive stars on the stage of yellow and red supergiants, with a mass loss determined unambiguously.

Key words: Evolution of starswith mass loss: outflowing envelopes.

1. Introduction

Evolution of massive stars ($M >\sim 20M_\odot$) is accompanied by a mass loss, initiated by a high luminosity and high radiation pressure. In blue supergiants, situated near the main sequence, mass loss rate is moderate $M \sim 10^{-6} M_\odot$/yr, and is connected with the outflow of layers having small optical depth. This mass loss rate is determined by radiation pressure in lines, where spectral absorption coefficient may be very high. Evolved massive stars may lose mass with much higher rate than the blue supergiants. Formation of single Wolf-Rayet stars probably took place as a result of such intense mass loss. The main goal of the theory of mass loss from stars is to determine the mass loss rate as an eigenfunction of the problem, together with its luminosity and radius. The evolutionary scenario of the WR star formation as a result of the intensive mass loss on the stage after finishing of a hydrogen burning in the core, was first suggested in the paper of Bisnovatyi-Kogan and Nadyozhin (1972) on the base of a crude calculations of self-consistent evolutionary models of mass-losing massive stars. The main shortcoming of this paper was connected with ignoring of the transition to a small optical depth in the outer regions of the flow, and with using everywhere of the equations with the equilibrium radiation pressure and energy density. In subsequent papers of Zytkow (1972,1973), Kato (1985), Kato and Iben (1992) different types of simplifications were used, which do not take into account the difference of the outflowing envelopes from the static ones.

The goal of the present paper is to derive equations, which are approximately valid at all optical depths, giving exact limiting equations for the case of very large and very small τ. Solution of these equations is obtained at correct boundary conditions at large r (infinity), where gas density ρ and gas temperature T tend to zero. Such procedure after fitting the solution to the stellar core will give self-consistent values of $\dot{M}$, as well as the parameters in the critical point and τ_{ph}.

We derive relations for pressure, energy density, and energy flux of radiation, which describe smoothly the transition of the flow between optically thick and optically thin regions. In the limiting cases they reduce to corresponding solutions of the radiative transfer equations in Eddington approximation. These relations are used in the equations of the radiative hydrodynamics with a constant total energy flow. Singular points of the equations are analyzed, and expansion in the isothermal sonic point is obtained, necessary for obtaining a numerical solution. Then the numerical solution which satisfies the boundary conditions at infinity is obtained. The parameters characterizing the properties of the underlying star have been prescribed.

2. Basic equations

A system of equations of radiation hydrodynamics describing continuous transition between optically thick and optically thin regions for the stationary outflow is written as:

$$u\frac{du}{dr} = -\frac{1}{\rho}\frac{dP_g}{dr} - \frac{GM(1-\tilde{L}_{th})}{r^2}, \quad (1)$$

$$\tilde{L}_{th} = \frac{L_{th}(r)}{L_{ed}}, \qquad L_{ed} = \frac{4\pi cGM}{\kappa},$$

$$L = 4\pi\mu\left(E + \frac{P}{\rho} - \frac{GM}{r} + \frac{u^2}{2}\right) + L_{th}(r) \quad (2)$$

$$L_{th} = -\frac{4\pi r^2 c}{\kappa\rho}\left(\frac{dP_r}{dr} - \frac{E_r\rho - 3P_r}{r}\right), \quad (3)$$

$$\frac{\dot{M}}{4\pi} \equiv \mu = \rho u r^2, \quad (4)$$

$$P = \frac{aT^4}{3}\left(1-e^{-\tau}\right) + \frac{L_{th}^{\infty}}{4\pi r^2 c} + P_g, \quad (5)$$

$$E\rho = aT^4\left(1-e^{-\tau}\right) + \frac{L_{th}^{\infty}}{4\pi r^2 c} + E_g\rho, \quad (6)$$

$$P_g = \rho\mathcal{R}T, \qquad E_g = \frac{3}{2}\mathcal{R}T, \quad (7)$$

$$\tau = \int_r^{\infty}\kappa\rho dr, \quad (8)$$

where L - is a constant total energy flux consisting of the radiative energy flux together with the energy flux of the matter flow, u is a rate of the outflow, κ is an opacity, assumed to be constant, a is the constant of a radiative energy density, $\mathcal{R}$ is a gas constant. E_r - part of energy density, that is due to radiation.P_r - pressure due to radiation. We consider here the flow in the gravitational field of a constant mass M, neglecting self-gravity of the outflowing envelope . This system of equations provides a description of a stationary outflowing envelope accelerated by a radiative force at arbitrary optical depth, where continuum opacity prevails. In the optically thick limit $\tau \to \infty$, when terms with L_{th}^{∞} are negligible, and $E_r\rho = 3P_r$, a solution of this system was obtained by Bisnovatyi-Kogan (1967). In the case of a small τ for the anisotropic radiation flux we find: $E_r\rho \simeq P_r$, what follows from the solution of the transfer equation in Eddington approximation (Sobolev, 1967).

When optical depth is becoming small, separation of radiation and matter should be taken into consideration. It means that only a part of radiation is determined by the outflowing gas. For this part of quanta we assume LTE to be valid, what means that the mean energy of such quanta is characterized by the temperature of the outflowing gas For the rest part of radiation, another "temperature"- mean energy of quanta should be introduced. This part of radiation transfers momentum to the outflowing matter (pushes it) and thus produces only the anisotropic part of the pressure, determined by the term L_{th}. The separation of radiation into two different parts occurs near the photosphere and the mean energy of the free propagating quanta are characterized by the effective temperature of the photosphere.

The solution passing continuously this critical point, which is of a saddle type (Parker, 1963). This point corresponds to the "isothermal sonic" point where

$$u^2 = u_s^2 \equiv \left(\frac{\partial P}{\partial\rho}\right)_T$$

.

The second singular point of the system of equations is situated at infinity $r \to \infty$, where

$$T = 0, \quad \rho \sim \frac{1}{r^2} \to 0, \quad u \to \text{const} = u_{\infty}. \quad (9)$$

This condition is related to the fact that far from the star the density in the stellar wind is very small.

Let us introduce nondimensional variables

$$\tilde{T}(r) = \frac{T(r)}{T_{cr}}, \quad \tilde{\rho}(r) = \frac{\rho(r)}{\rho_{cr}}, \quad \tilde{x} = \frac{r_{cr}}{r}. \quad (10)$$

After transformations we obtain a dimensionless system of equations

$$\frac{d\rho}{dx} = \left(\frac{x^4}{\rho^3} - \frac{T}{\rho}\right)^{-1}\left\{\frac{dT}{dx}\left(1 + A_1(1-e^{-\tau})\frac{T^3}{\rho}\right)\right. \quad (11)$$

$$\left. -A_3 + \frac{1}{4}\frac{A_1 e^{-\tau}}{A_5}\frac{T^4}{x^2} + 2\frac{x^3}{\rho^2}\right\} \quad ,$$

$$\frac{dT}{dx} = -\left(\frac{5}{2}T - A_3 x + \frac{1}{2}\frac{x^4}{\rho^2} + (1-e^{-\tau})A_1\frac{T^4}{\rho}\right. \quad (12)$$

$$\left. + \frac{e^{-\tau}}{4A_2A_5}\frac{T^4}{x^2} + 2L^{\infty}A_3A_5\frac{x^2}{\rho} - \frac{A_4}{A_2}\right)\frac{A_2\rho}{T^3(1-e^{-\tau})} \quad ,$$

$$\frac{d\tau}{dx} = \frac{\rho}{A_5 x^2} \quad . \quad (13)$$

Where $L^{\infty} \equiv \tilde{L}_{th}^{\infty}$. To simplify writing here and further we omit tilde. Dimensionless coefficients A_i are the same as in (Bisnovatyi-Kogan and Dorodnitsyn, 1999)

Additional fifth parameter A_5 is not independent,

$$A_5 = \left(\frac{3}{4}\right)^{1/5}\frac{A_3^{1/5}\mathcal{R}^{4/5}}{A_1^{3/5}A_2^{4/5}\kappa^{1/5}a^{1/5}c^{4/5}(GM)^{1/5}}. \quad (14)$$

It is of the order of the reciprocal optical depth in the critical point. Condition of a continuous transition of the solution through the critical point reduces the number of independent dimensionless parameters.

In addition to coefficients A_i, we have independent nondimensional papameters L^{∞}, and the optical depth in the critical point τ_{cr}, so before satisfying the boundary conditions at infinity, we have 5 "independent" nondimensional parameters of the problem.

Sample numerical solution

In order to satisfy boundary conditions far from the star we need to integrate (11)-(13) from the critical point outward to the infinity. We exit the critical point by means of expansion formulae. Expanding the solution in the vicinity of the critical point $x = T = \rho = 1$ in powers of $(1 - x)$ we have

$$T = 1 + a(1 - x), \quad \rho = 1 + b(1 - x), \tag{15}$$

$$e^{-\tau} \simeq e^{-\tau_{cr}}(1 + \frac{y}{A_5}), \tag{16}$$

where $y = 1 - x$. a and b coefficients are given in (Bisnovatyi-Kogan and Dorodnitsyn, 1999)

A numerical integration is started from the critical point making the first step by means of the expansion formulas. Integrating outward to the infinity we satisfy the boundary conditions. To satisfy condition of zero T at infinity we find a unique dependence $A_3(A_1, A_2, L_{th}^{\infty}, \tau_{cr})$. After that the value of τ_{cr} is found uniquely for the given values of A_1, A_2 and L_{th}^{∞} by matching the condition v^{∞} =const. When $r \to \infty$ velocity tends to a constant and thus $\rho \sim 1/r^2$. Only a unique value of τ_{cr} allows to obtain the proper behavior of u (and ρ) at the infinity.

The obtained solution corresponds to the following dimensionless parameters: $A_1 = 50$, $A_2 = 10^{-4}$, $A_3 = 43.88$, $\tau_{cr} = 125$, $L_{th}^{\infty} = 0.6$. This set of parameters corresponds to the following values in the critical point: $T_{cr} = 1.4 \cdot 10^4 K$, $r_{cr} = 2.6 \cdot 10^{13}$cm, $\rho_{cr} = 6.6 \cdot 10^{-12}$ g/cm^3. We accepted here $M = 2 \cdot 10^{34}$ g. The velocity of the flow in the critical point is $v_{cr} \approx 11$ km/s, and the mass loss rate $\dot{M} \approx 9 \cdot 10^{-3} M_{\odot}$/yr.

A behavior of the solution with a Mach number rapidly decreasing inside at $r < r_{cr}$, gives a possibility to match it to a static solution in the core. In reality the opacity peak is situated near the critical point, the opacity inside is decreasing and the velocity drops inside more rapidly, then in the case of κ=const (Bisnovatyi-Kogan and Nadyozhin, 1972). Deep inside the star, all hydrodynamical solutions converge to a static one. In the static atmosphere, at L =const, and M =const, we have $\rho \sim T^3$ and $T \sim 1/r$ (Bisnovatyi-Kogan, 1973). Since $u \sim 1/(\rho r^2)$, the velocity in the subsonic region tends to zero $\sim r$. In reality, deeper in the star $\dot{M}$ decreses, tending to zero. That means, that the velocity goes to zero faster, than $\sim r$. The Mach number M_a, is defined by the relation $M_a = u/\sqrt{\gamma P/\rho}$, where P is taken without the anisotropic term.

The effective temperature of the photosphere is obtained from the relation: $L_{th}^{\infty}/4\pi r^2 = \sigma T^4$. For the given set of parameters we get $x_{ph} = 0.03$, $\tau_{ph} = 4$, $T_{eff.} = 0.06$, and $\tilde{\tau}_{ph} = 3.75$. That corresponds to $r_{ph} = 8.6 \cdot 10^{14}$ cm, $T = 840$ K, what corresponds to a very luminous infrared star with an extended outflowing atmosphere. It is possible that on the stage of a very intensive mass loss the massive star is transformed into an infrared object. We should have in mind, that for realistic functions $\kappa(\rho, T)$, $P(\rho, T)$, the observed quantities should be different. Below the temperature of several thousands Kelvin the opacity drops (Iglesias and Rogers, 1996; Seaton, 1996), the photosphere approaches the star, and the effective temperature is greater.

The condition of matching of the solution of the outflowing envelope to the static core determine uniquely the values of r_{cr} and L, leading to the unique solution of the mass-losing star with a self-consistent mass loss rate.

References

Bisnovatyi–Kogan G.S.: 1967, *Prikl. Mat. Mech.*, **31**, 762.

Bisnovatyi–Kogan G.S., Nadyozhin D.K.: 1972, *Ap. Space Sci.*, **15**, 353.

Bisnovatyi-Kogan G.S., Dorodnitsyn A.V.: 1999, *As.Ap.*, **344**, 647.

Chandrasekhar S.: 1939, *Stellar structure*, Chicago.

Kato M.: 1985, *PASJ*, **37**, 19.

Kato M., Iben I.: 1992, *ApJ*, **394**, 879.

Parker E.N.: 1963, *Interplanetary dynamical processes.* Interscience Publishers, New York - London.

Sobolev, V.V.: 1967, *Course of theoretical astrophysics.* Moscow, Nauka Publ..

Żytkow A., 1972.: *Acta Astron.*, **22**, 103.

Żytkow A., 1973.: *Acta Astron.*, **23**, 121.