ACCRETION DISCS AROUND BLACK HOLES: DEVELOPEMENT OF THEORY

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ABSTRACT. Standard accretion disk theory is formulated which is based on the local heat balance. The energy produced by a turbulent viscous heating is supposed to be emitted to the sides of the disc. Sources of turbulence in the accretion disc are connected with nonlinear hydrodynamic instability, convection, and magnetic field. In standard theory there are two branches of solution, optically thick, and optically thin. Advection in accretion disks is described by the differential equations what makes the theory nonlocal. Low-luminous optically thin accretion disc model with advection at some suggestions may become advectively dominated, carrying almost all the energy inside the black hole. The proper account of magnetic filed in the process of accretion limits the energy advected into a black hole, efficiency of accretion should exceed $\sim 1/4$ of the standard accretion disk model efficiency.

Key words: Stars: accretion discs; black holes.

1. Introduction

Accretion is a main source of energy in binary X-ray sources, quasars and active galactic nuclei (AGN). The intensive development of accretion theory began after discovery of X-ray sources and quasars. Accretion into stars is ended by a collision with an inner boundary, which may be a stellar surface, or outer boundary of a magnetosphere for strongly magnetized stars. All gravitational energy of the falling matter is transformed into heat and radiated outward.

In black holes matter is falling to the horizon, from where no radiation arrives. All luminosity is formed on the way to it. The efficiency of accretion is not known from the beginning, and depends on angular momentum of the falling matter, and magnetic field embedded into it. It was first shown by Schwartsman (1971) that during spherical accretion of nonmagnetized gas the efficiency may be as small as 10^{-8} for sufficiently low mass fluxes. He had shown that presence of magnetic field in the accreting matter increase the efficiency up to about 10%, and account of heating of matter due to magnetic field annihilation rises the efficiency up to about 30% (Bisnovatyi-Kogan and Ruzmaikin, 1974) In the case of a thin disc accretion, when matter has large angular momentum, the efficiency is about 1/2 of the efficiency of accretion into a star with a radius equal to the radius of the last stable orbit. In the case of geometrically thick and optically thin accretion discs the situation is approaching the case of spherical symmetry, where presence of a magnetic field plays a critical role.

Advection dominated accretion flow (ADAF) was suggested by Narayan and Yu (1995), and used as a solution for some astrophysical problems. The suggestions underlying ADAF: ignorance of the magnetic field annihilation in heating of a plasma flow, electron heating only due to binary collisions with protons (ions) had been critically analyzed in papers of Bisnovatyi-Kogan and Lovelace (1997, 1999), Bisnovatyi-Kogan (1999), Quataert (1997). There are contradictions between ADAF model and observational data in radioemission of elliptical galaxies (Di Matteo et al., 1999), and X-ray emission of Seyfert galaxy NGC4258 (Cannizzo, 1998). Account of processes connected with a small-scale magnetic field in accretion flow, strongly restricts solution. Namely, the efficiency of the accretion flow cannot become less then about 1/4 of the standard accretion disc value.

2. Development of the standard model

Matter falling into a black hole forms a disc when its angular momentum is sufficiently high. It happens when matter comes from the star companion in the binary, or from a tidal disruption of the star which trajectory approaches close to the black hole. The first situation is observed in many galactic X-ray sources (Cherepashchuk, 1996). A tidal disruption happens in quasars and active galactic nuclei (AGN) in the model of supermassive black hole surrounded by a dense stellar cluster.

Equations of a standard accretion disk theory had been first formulated by (Shakura, 1972); some corrections and generalization to general relativity (GR) had been done by Novikov and Thorne (1973). Observational aspects of accretion disks have been analyzed by Shakura and Sunyaev (1973). Note, that all authors of the accretion disc theory from USSR were students (N.I.Shakura) or collaborators (I.D.Novikov and R.A.Sunyaev) of academician Ya.B.Zeldovich, who was not among the authors, but whose influence on them hardly could be overestimated. The main idea of this theory is to describe a geometrically thin non-self-gravitating disc of a mass M_d, much smaller then the mass of a black hole M, by hydrodynamic equations averaged over the disc thickness $2h$.

2.1. Equations

The small thickness of the disc in comparison with its radius $h \ll r$ means small importance of the pressure gradient ∇P in comparison with gravity and inertia forces. Radial equilibrium equation in a disc is a balance between the last two forces with an angular velocity equals to the keplerian one $\Omega = \Omega_K = \left(\frac{GM}{r^3}\right)^{1/2}$. Note, that just before a last stable orbit around a black hole this suggestion fails, but in the "standard" accretion disc model this relation is supposed to hold all over the disc, with an inner boundary at the last stable orbit. The equilibrium equation in the vertical z-direction is determined by a balance between the gravitational force and pressure gradient $\frac{dP}{dz} = -\rho\frac{GMz}{r^3}$. For a thin disc this differential equation is substituted by an algebraic one, determining the half-thickness of the disc in the form

$$h \approx \frac{1}{\Omega_K}\left(2\frac{P}{\rho}\right)^{1/2}. \qquad (1)$$

The balance of angular momentum, related to the ϕ component of the Euler equation has an integral in a stationary case, written as

$$\dot{M}(j - j_{in}) = -2\pi r^2\, 2h t_{r\phi}, \quad t_{r\phi} = \eta r\frac{d\Omega}{dr}. \qquad (2)$$

Here $j = v_\phi r = \Omega r^2$ is a specific angular momentum, $t_{r\phi}$ is a component of the viscous stress tensor, $\dot{M} > 0$ is a mass flux per unit time into a black hole, j_{in} is equal to the specific angular momentum of matter falling into a black hole. In the standard theory the value of j_{in} is determined from physical considerations. For accretion into a black hole it is suggested, that on the last stable orbit the gradient of the angular velocity is zero, corresponding to zero viscous momentum flux. In that case $j_{in} = \Omega_K r_{in}^2$, corresponding to the Keplerian angular momentum of the matter on the last stable orbit.

The choice of the viscosity coefficient is the most speculative problem of the theory. In the laminar case at low microscopic (atomic or plasma) viscosity the stationary accretion disc must be very massive and thick, and before its formation the matter is collected by disc leading to a small flux inside. It contradicts to observations of X-ray binaries, where a considerable matter flux along the accretion disc may be explained only when viscosity coefficient is much larger. It was suggested by Shakura (1972), that matter in the disc is turbulent, what determines a turbulent viscous stress tensor, parametrized by a pressure

$$t_{r\phi} = -\alpha\rho v_s^2 = -\alpha P, \qquad (3)$$

where v_s is a sound speed in the matter. This simple presentation comes out from a relation for a turbulent viscosity coefficient $\eta_t \approx \rho v_t l$ with an average turbulent velocity v_t and mean free path of the turbulent element l. It follows from the definition of $t_{r\phi}$ in (2), when we take $l \approx h$ from (1)

$$t_{r\phi} = \rho v_t h r\frac{d\Omega}{dr} \approx \rho v_t v_s = -\alpha\rho v_s^2, \qquad (4)$$

where a coefficient $\alpha < 1$ is connecting the turbulent and sound speeds $v_t = \alpha v_s$. Presentations of $t_{r\phi}$ in (3) and (4) are equivalent, and only when the angular velocity differs considerably from the Keplerian one the first relation to the right in (4) is more preferable. That does not appear in the standard theory, but may happen when advective terms are included.

Development of a turbulence in the accretion disc cannot be justified simply, because a Keplerian disc is stable in linear approximation to the development of perturbations. It was suggested by Ya.B.Zeldovich, that in presence of very large Reynolds number $\mathrm{Re} = \frac{\rho v l}{\eta}$ the amplitude of perturbations at which nonlinear effects become important is very low, so a turbulence may be developed due to nonlinear instability even when the disc is stable in linear approximation. Viscous stresses may arise from a magnetic field, it was suggested by (Shakura, 1972), that magnetic stresses cannot exceed the turbulent ones. It was shown by Bisnovatyi-Kogan and Blinnikov (1976), that inner regions of a highly luminous accretion discs where pressure is dominated by radiation, are unstable to vertical convection. Development of this convection produce a turbulence, needed for a high viscosity.

With alpha- prescription of viscosity the equation of angular momentum conservation is written in the plane of a disc as

$$\dot{M}(j - j_{in}) = 4\pi r^2\alpha P_0 h. \qquad (5)$$

When angular velocity is far from Keplerian one the relation (2) is valid with a coefficient of a turbulent viscosity $\eta = \alpha\rho_0 v_{s0} h$, where values with the index "0" denote the plane of the disc.

In the standard theory a heat balance is local, all heat produced by viscosity in the ring between r and

$r+dr$ is radiated through the sides of disc at the same r. The heat production rate Q_+ related to the surface unit of the disc is written as

$$Q_+ = h\, t_{r\phi} r \frac{d\Omega}{dr} = \frac{3}{8\pi}\dot{M}\frac{GM}{r^3}\left(1 - \frac{j_{in}}{j}\right). \quad (6)$$

Heat losses by a disc depend on its optical depth. The standard disc model (Shakura, 1972) considered a geometrically thin disc as an optically thick in a vertical direction. That implies energy losses Q_- from the disc due to a radiative conductivity, after a substitution of the differential equation of a heat transfer by an algebraic relation

$$Q_- \approx \frac{4}{3}\frac{acT^4}{\kappa\Sigma}. \quad (7)$$

Here a is a constant of a radiation energy density, c is a speed of light, T is a temperature in the disc plane, κ is a matter opacity, and a surface density $\Sigma = 2\rho h$. Here and below ρ, T, P without the index "0" are related to the disc plane. The heat balance equation is represented by a relation $Q_+ = Q_-$. A continuity equation in the standard model is used for finding of a radial velocity v_r

$$v_r = \frac{\dot{M}}{4\pi r h\rho} = \frac{\dot{M}}{2\pi r\Sigma}. \quad (8)$$

Completing these equations by an equation of state $P(\rho, T)$ and relation for the opacity $\kappa = \kappa(\rho, T)$ we get a full set of equations for a standard disc model. For power low equations of state of an ideal gas $P = P_g = \rho\mathcal{R}T$ ($\mathcal{R}$ is a gas constant), or radiation pressure $P = P_r = \frac{aT^4}{3}$, and opacity in the form of electron scattering κ_e, or Karammers formulae κ_k, the solution of a standard disc accretion theory is obtained analytically. Checking the suggestion of a large optical thickness confirms a self-consistency of the model. Note that solutions for different regions of the disc with different equation of states and opacities are not matched to each other.

2.2. Optically thin solution

It was found by Shapiro et al. (1976) that there is another branch of the solution for a disc structure with the same input parameters M, $\dot{M}$, α which is also self-consistent but has a small optical thickness. That implies another equation of energy losses, determined by a volume emission $Q_- \approx q\,\rho\, h$. The emissivity of the unit of a volume q is connected with a Planckian averaged opacity κ_p by a relation $q \approx acT_0^4\kappa_p$. In the optically thin limit the pressure is determined by a gas $P = P_g$. In the optically thin solution the thickness of the disc is larger then in the optically thick one, and density is lower.

While heating by viscosity is determined mainly by heavy ions, and cooling is determined by electrons, the rate of the energy exchange between them is important for a structure of the disc. The energy balance equations are written separately for ions and electrons. For small accretion rates and lower matter density the rate of energy exchange due to binary collisions is so slow, that in the thermal balance the ions are much hotter then the electrons. That also implies a high disc thickness. In the highly turbulent plasma the energy exchange between ions and electrons may be strongly enhanced due to presence of fluctuating electrical fields, where electrons and ions gain the same energy. In such conditions difference of temperatures between ions and electrons may be negligible. The theory of relaxation in the turbulent plasma is not completed, but there are indications to a large enhancement of the relaxation in presence of plasma turbulence, in comparison with the binary collisions (Galeev and Sagdeev, 1983; Quataert, 1997).

2.3. Accretion disc structure from equations describing continuously optically thin and thick regions

Equations of the disc structure smoothly describing transition between optically thick and optically thin disc, had been obtained using Eddington approximation. The expressions had been obtained (Artemova et al., 1996) for the vertical energy flux from the disk F_0, and radiation pressure in the symmetry plane

$$F_0 = \frac{2acT_0^4}{3\tau_0\Phi}, \quad P_{rad,0} = \frac{aT_0^4}{3\Phi}\left(1 + \frac{4}{3\tau_0}\right), \quad (9)$$

where $\tau_{\alpha 0} = \kappa_p\rho h = \frac{1}{2}\kappa_p\Sigma_0$, $\tau_* = (\tau_0\tau_{\alpha 0})^{1/2}$, $\Phi = 1 + \frac{4}{3\tau_0} + \frac{2}{3\tau_*^2}$. At $\tau_0 \gg \tau_* \gg 1$ we have (7) from (9). In the optically thin limit $\tau_* \ll \tau_0 \ll 1$ we get

$$F_0 = acT_0^4\tau_{\alpha 0}, \quad P_{rad,0} = \frac{2}{3}acT_0^4\tau_{\alpha 0}. \quad (10)$$

Using F_0 instead of Q_- and equation of state $P = \rho\mathcal{R}T + P_{rad,0}$, the equations of accretion disc structure together with equation $Q_+ = F_0$, with Q_+ from (6), have been solved numerically by Artemova et al. (1996). Two solutions, optically thick and thin, exist separately when luminosity is not very large. They intersect at $\dot{m} = \dot{m}_b$ and there is no global solution for accretion disc at $\dot{m} > \dot{m}_b$. It was concluded by Artemova et al. (1996) that in order to obtain a global physically meaningful solution at $\dot{m} > \dot{m}_b$, account of advectivion is needed. For the calculated case $M_{BH} = 10^8\, M_\odot$, $\alpha = 1.0$ at $\dot{m} = \dot{m}_b$ luminosity of the accretion disk is less than the critical Eddington one.

3. Accretion discs with advection

Standard model gives somewhat nonphysical behavior near the inner edge of the accretion disc around a black hole, with a zero heat production at the inner edge of the disk. It is clear from physical ground, that in this case the heat brought by radial motion of matter along the accretion disc becomes important. In presence of this advective heating (or cooling) term, depending on the radial entropy S gradient, written as $Q_{adv} = \frac{\dot{M}}{2\pi r} T \frac{dS}{dr}$, the equation of a heat balance is modified to

$$Q_+ + Q_{adv} = Q_-. \quad (11)$$

In order to describe self-consistently the structure of the accretion disc we should also modify the radial disc equilibrium, including pressure and inertia terms

$$r(\Omega^2 - \Omega_K^2) = \frac{1}{\rho}\frac{dP}{dr} - v_r \frac{dv_r}{dr}. \quad (12)$$

Appearance of inertia term leads to transonic radial flow with a singular point. Conditions of a continuous passing of the solution through a critical point choose a unique value of the integration constant j_{in}. First approximate solution for the advective disc structure have been obtained by Paczyński and Bisnovatyi-Kogan (1981). Attempts to find a solution for advective disc had been done by Abramovicz et al. (1988), Matsumoto et al. (1984). For moderate values of $\dot{M}$ a unique continuous transonic solution was found, passing through singular points, and corresponding to a unique value of j_{in}. The number of critical point in the radial flow with the gravitational potential ϕ_g (Paczyński and Wiita, 1980) $\phi_g = \frac{GM}{r - r_g}$, $r_g = \frac{2GM}{c^2}$. may exceed unity. Appearance of two critical points for a radial flow in this potential was analyzed by Chakrabarti and Molteni (1993). Using of equations averaged over a thickness of the disc changes a structure of hydrodynamic equations, leading to a position of singular points not coinciding with a unit Mach number point.

4. Amplification of the magnetic field at a spherical accretion

A matter flowing into a black hole is usually magnetized. Due to more rapid increase of a magnetic energy the role of the magnetic field increases when matter flows inside. It was shown by Schwartsman (1971), that magnetic energy density E_M approaches a density of a kinetic energy E_k, and he proposed a hypothesis of *equipartition* $E_M \approx E_k$, supported by continuous annihilation of the magnetic field in a region of the main energy production. This hypothesis is usually accepted in the modern picture of accretion (Narayan and Yu, 1995). Account of the heating of matter by magnetic field annihilation was done by Bisnovatyi-Kogan and Ruzmaikin (1974). For a spherical accretion with $\mathbf{v} = (v_r, 0, 0)$ the equations describing a field amplification in the ideally conducting plasma reduce to (Bisnovatyi-Kogan, Ruzmaikin, 1974)

$$\frac{d(r^2 B_r)}{dt} = 0, \quad \frac{d(r v_r B_\theta)}{dt} = 0, \quad \frac{d(r v_r B_\phi)}{dt} = 0, \quad (13)$$

where $\frac{d}{dt} = \frac{\partial}{\partial t} + v_r \frac{\partial}{\partial r}$ is a full Lagrangian derivative. Consider a free fall case with $v_r = -\sqrt{\frac{2GM}{r}}$. The initial condition problem is solved separately for poloidal and toroidal fields. For initially uniform field $B_{r0} = B_0 \cos\theta$, $B_{\theta 0} = -B_0 \sin\theta$ we get the solution (Bisnovatyi-Kogan, Ruzmaikin, 1974)

$$B_r = \frac{B_0 \cos\theta}{r^2} \Phi_1^{4/3}, \quad B_\theta = -\frac{B_0 \sin\theta}{\sqrt{r}} \Phi_1^{1/3}, \quad (14)$$

where $\Phi_1 = r^{3/2} + \frac{3}{2} t \sqrt{2GM}$. The radial component of the field is growing most rapidly. It is $\sim r^{-2}$ for large times, $\sim t^{4/3}$ at given small radius, and is growing with time everywhere. For initially dipole magnetic field

$$B_{r0} = \frac{B_0 \cos\theta}{r^3}, \quad B_{\theta 0} = -\frac{B_0 \sin\theta}{2r^3}$$

we obtain the following solution

$$B_r = \frac{B_0 \cos\theta}{r^2} \Phi_1^{-2/3}, \quad B_\theta = -\frac{B_0 \sin\theta}{2\sqrt{r}} \Phi_1^{-5/3}. \quad (15)$$

Here the magnetic field is decreasing everywhere with time, tending to zero. That describes a pressing of a dipole magnetic field to a stellar surface. The azimuthal stellar magnetic field if confined inside the star. When outer layers of the star are compressing with a free-fall speed, then for initial field distribution $B_{\phi 0} = B_0 r^n f(\theta)$ the change of B_ϕ with time is described by a relation $B_\phi = -\frac{B_0 f(\theta)}{\sqrt{r}} \Phi_1^{n+1/3}$.

5. Two-temperature advective discs

In the optically thin accretion discs at low mass fluxes the density of the matter is low and energy exchange between electrons and ions due to binary collisions is slow. In this situation, due to different mechanisms of heating and cooling for electrons and ions, they may have different temperatures. First it was realized by Shapiro et al. (1976), where advection was not included. It was noticed by Narayan and Yu (1995), that advection in this case is becoming extremely important. It may carry the main energy flux into a black hole, leaving rather low efficiency of the accretion up to $10^{-3} - 10^{-4}$ (advective dominated accretion flows

- ADAF). This conclusion is valid only when the effects, connected with heating of matter by magnetic field annihilation are neglected.

In the ADAF solution the ion temperature is about a virial one $kT_i \sim GMm_i/r$, what means that even at high initial angular momentum the disc becomes thick, forming a quasi-spherical accretion flow. When energy losses by ions are low, some kind of a "thermo-viscous" instability is developed, because heating increases a viscosity, and viscosity increases a heating. Development of this instability leads to formation of ADAF.

A full account of the processes, connected with a presence of magnetic field in the flow, is changing considerably the picture of ADAF. It was shown by Schwarzman (1971), that in the region of the main energy production, the condition of equipartition takes place, and efficiency of a radiation increase enormously from $\sim 10^{-8}$ up to ~ 0.1 due to magneto-bremstrahlung. To support the condition of equipartition a continuous magnetic field reconnection and heating of matter due to Ohmic dissipation takes place. It was shown by Bisnovatyi-Kogan and Ruzmaikin (1974), that due to Ohmic heating the efficiency of a radial accretion into a black hole may become as high as $\sim 30\%$. The rate of the Ohmic heating in the condition of equipartition was obtained in the form

$$T\frac{dS}{dr} = -\frac{3}{2}\frac{B^2}{8\pi\rho r}. \qquad (16)$$

In the supersonic flow of the radial accretion equipartition was suggested in a form (Schwartsman, 1971) $\frac{B^2}{8\pi} \approx \frac{\rho v_r^2}{2} = \frac{\rho GM}{r}$. For the disc accretion equipartition between magnetic and turbulent energy was suggested by Shakura (1972), what reduces with account of "alpha" prescription of viscosity to a relation $\frac{B^2}{8\pi} \sim \frac{\rho v_t^2}{2} \approx \alpha_m^2 P$, where α_m characterizes a magnetic viscosity in a way similar to the turbulent α viscosity. It was suggested by Bisnovatyi-Kogan and Ruzmaikin (1976) the similarity between viscous and magnetic Reynolds numbers, or between turbulent and magnetic viscosity coefficients $Re = \frac{\rho v l}{\eta}$, $Re_m = \frac{\rho v l}{\eta_m}$, where the turbulent magnetic viscosity η_m is connected with a turbulent conductivity $\sigma = \frac{\rho c^2}{4\pi\eta_m}$. Taking $\eta_m = \frac{\alpha_m}{\alpha}\eta$, we get a turbulent conductivity

$$\sigma = \frac{c^2}{4\pi\alpha_m h v_s}, \quad v_s^2 = \frac{P_g}{\rho} \qquad (17)$$

in the optically thin discs. For the radial accretion the turbulent conductivity may contain mean free path of a turbulent element l_t in (17) instead of h. In ADAF solutions, where ionic temperature is of the order of the virial one two above suggestions for magnetic equipartition almost coincide at $\alpha_m \sim 1$.

The heating of the matter due to an Ohmic dissipation may be obtained from the Ohm's law in radial accretion

$$T\frac{dS}{dr} = -\frac{\sigma\mathcal{E}^2}{\rho v_r} \approx -\sigma\frac{v_E^2 B^2}{\rho v_r c^2} = -\frac{B^2 v_E^2}{4\pi\rho\alpha_m v_r v_s l_t}, \qquad (18)$$

what coincides with (16) when $\alpha_m = \frac{4rv_E^2}{3v_r v_s l_t}$, or $l_t = \frac{4rv_E^2}{3v_r v_s \alpha_m}$. Here a local electrical field strength in a highly conducting plasma is of the order of $\mathcal{E} \sim \frac{v_E B}{c}$, $v_E \sim v_t \sim \alpha v_s$ for a radial accretion.

Equations for a radial temperature dependence in the accretion disc, separately for the ions and electrons are written as

$$\frac{dE_i}{dt} - \frac{P_i}{\rho^2}\frac{d\rho}{dt} = \mathcal{H}_{\eta i} + \mathcal{H}_{Bi} - Q_{ie}\ , \qquad (19)$$

$$\frac{dE_e}{dt} - \frac{P_e}{\rho^2}\frac{d\rho}{dt} = \mathcal{H}_{\eta e} + \mathcal{H}_{Be} + Q_{ie} - \mathcal{C}_{ff} - \mathcal{C}_{cyc}\ , \qquad (20)$$

A rate of a viscous heating of ions $\mathcal{H}_{\eta i}$ is obtained from (6) as

$$\mathcal{H}_{\eta i} = \frac{2\pi r}{\dot M}Q_+ = \frac{3}{2}\alpha\frac{v_K v_s^2}{r}, \quad \mathcal{H}_{\eta e} \le \sqrt{\frac{m_e}{m_i}}\mathcal{H}_{\eta i}. \qquad (21)$$

Combining (1),(8),(5), we get

$$v_r = \alpha\frac{v_s^2}{v_K\mathcal{J}}, \quad h = \sqrt{2}\frac{v_s}{v_K}r, \quad \rho = \frac{\dot M}{4\pi\alpha\sqrt{2}}\frac{v_K^2\mathcal{J}}{r^2 v_s^3}, \qquad (22)$$

where $v_K = r\Omega_K$, $\mathcal{J} = 1 - \frac{j_{in}}{j}$. The rate of the energy exchange between ions and electrons due to the binary collisions was obtained by Landau (1937). Neglecting pair formation for a low density accretion disc, we may write an exact expression for a pressure $P_g = P_i + P_e = n_i kT_e + n_e kT_p = n_e k(T_e + T_i)$, and an approximate expression for an energy, containing a smooth interpolation between nonrelativistic and relativistic electrons. The bremstrahlung $\mathcal{C}_{ff}$ and magneto-bremstrahlung $\mathcal{C}_{cyc}$ cooling of maxwellian semi-relativistic electrons, with account of free-bound radiation in nonrelativistic limit, may be written as interpolation of limiting cases (Bisnovatyi-Kogan and Ruzmaikin, 1976).

In the case of a disk accretion there are several characteristic velocities, v_K, v_r, v_s, and $v_t = \alpha v_s$, all of which may be used for determining "equipartition" magnetic energy, and one characteristic length h. Consider three possible choices of $v_B^2 = v_K^2$, v_r^2, and v_t^2 for scaling $B^2 = 4\pi\rho v_B^2$. The expression for an Ohmic heating in the turbulent accretion disc also may be written in different ways, using different velocities v_E in the expression for an effective electrical field $\mathcal{E} = \frac{v_E B}{c}$. A self-consistency of the model requires, that expressions for a magnetic heating of the matter $\mathcal{H}_B$, obtained from the condition of stationarity of the flow (16), and from the Ohm's law (18), should be identical. It happens at $\frac{\alpha}{\mathcal{J}\alpha_m}\frac{v_E^2}{v_r^2} = \frac{3\sqrt{2}}{4}$. That implies $v_E \sim v_r \sim \frac{\alpha v_s^2}{v_K\sqrt{\mathcal{J}}} \simeq \frac{v_t v_s}{v_K\sqrt{\mathcal{J}}} < v_t$. In the advective models $\mathcal{J}$ is substituted by a function which is not zero at the inner edge of the disc. The heating due to magnetic field reconnection $\mathcal{H}_B$ in the equations (19), (20) may be written with account of (21) as $\mathcal{H}_B = \frac{3}{16\pi}\frac{B^2}{r\rho}v_r = \frac{1}{2\mathcal{J}}\mathcal{H}_{\eta i}\left(\frac{v_B}{v_K}\right)^2$. At $v_B = v_K$ the expressions for viscous and magnetic heating are almost identical. Observations of the magnetic field reconnection in the solar flares show (Tsuneta, 1996), that electronic heating prevails over the ionc one. Transformation of the magnetic energy into a heat is connected with the change of the magnetic flux, generation of the vortex electrical field, accelerating the particles.

The equations (19), (20) have been solved by Bisnovatyi-Kogan and Lovelace (1997) for nonrelativistic electrons, at v_B=v_K. The combined heating of the electrons and ions were taken as $\mathcal{H}_e = (2-g)\mathcal{H}_{\eta i}$, $\mathcal{H}_e = g\mathcal{H}_{\eta i}$. The results of calculations for $g = 0.5 \div 1$ show that almost all energy of the electrons is radiated, so the relative efficiency of the two-temperature, optically thin disc accretion cannot become lower then 0.25. Increase of the term Q_{ie} due to plasma turbulence may restore the relative efficiency to a value, corresponding to the optically thick disc.

6. Discussion

Observational evidences for existence of black holes inside our Galaxy and in the active galactic nuclei (Cherepashchuk, 1996; Ho, 1999) make necessary to revise theoretical models of the disc accretion. The improvements of a model are connected with account of advective terms and more accurate treatment of the magnetic field effects. Account of the effects connected with magnetic field annihilation does not permit to make a relative efficiency of the accretion lower then ~ 0.25 from the standard value. Strong relaxation connected with the plasma turbulence may increase the efficiency, making it close to unity. For explanation of underluminous galactic nuclei two possible ways may be suggested. One is based on a more accurate estimations of the accretion mass flow into the black hole, which could be overestimated. The second is based on existence of another mechanisms of the energy losses in the form of accelerated particles, like in the radiopulsars, where these losses exceed strongly a radiation. This is very probable to happen in a presence of a large scale magnetic field which may be also responsible for a formation of the observed jets (Bisnovatyi-Kogan, 1999; Blandford and Begelman, 1999). We may suggest, that underlumilnous AGN loose main part of their energy to the formation of jets, like in SS 433. The search of the correlation between existence of jets and lack of the luminosity could be very informative.

Acknowledgements. The author is grateful for partial support to RFBR, grant 99-02-18180, and GPNT "Astronomy", grant 1.2.6.5.

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