MAGIC NUMBERS: SUPPLEMENT, DESCRIPTION AND CLASSIFICATION

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ABSTRACT. New magic numbers are added to the early known ones, two sequences (large and small) are distinguished which are described in recurrent formulas (1) and (2). Due to the increase of magic number quantity some of the nuclear isotopes which were not considered to be magic have been attributed to such ones. As to isotopes C(6,6), Ca(20,20), Si(14,14), Ni(28,28) and Zr(40,50) they have been referred to twice magic ones. This enabled us to qualitatively explain all the peaks in the curve of isotope abundance. Small twice peaks at magic numbers of neutrons 28, 40, 70, 112 have been added to twice peaks of r-, s-processes of neutron capture at magic numbers 50, 82, 126. Three areas of relative stability at close twice magic isotopes X(112,168), Y(126,184), Z(168,258) and (or) Z'(168,240) are predicted.

Key words: nuclear astrophysics, r-, s-processes, Mendeleev's periodic system, magic numbers

Atomic nuclei containing definite numbers (2, 8, 20, 28, 50, 82, 126) of protons (p) or neutrons (n) show an enhanced stability that makes them distinguished among other nuclei neighbouring to them. These numbers of protons and neutrons are called "magic numbers". Their explanation lies in the envilope nucleus model. In explaining the valence and other properties for atoms in the periodic system of elements the ilectron states in them can be divided into groups at filling up each of them and transferring to the next group the electron energy bond decresses. The same holds true for the nuclei wherein the nuclons form filled envelopes separately for p and n.

According to the model of nucleus envelopes the nuclon energetic levels with close values of energy are grouped into series far apart from each other which are called nuclon envelopes.

According to the Pauli priciple a certain number of nuclons of the given kind can be placed on each envelope. Whenever an envelope is filled, it corresponds to the magic nucleus formation with the respective magic number. Nuclon envelopes of protons and neutrons are filled independently. The simultaneous filling up of proton and neutron envelopes is followed by a corresponding formation of particularly stable twice magic nucleus.

In the present work, Table (117.8) from "Quantum Mechanics" (Landau and Lifshits, 1963) is analyzed. In this table (see Table 1), the nuclon states in the nucleus are distributed as the following six groups. Designations: entire numbers are the main quantum numbers of the states; letters s, p, d, f, g, h, i are meanings of the orbiting momentum l; and fractional numbers j denote spin of nuclon. For each group is given total number n of proton or neutron vacancies. Numbers n are derived as the sum for states by the formula: $n = \sum(2j+1)$. We added the following magic numbers M in the last column of the table.

In "Experimental Nucleus Physics" (Mukhin, 1974) is given a series of magic numbers with number 20 instead of 28. In literature such magic numbers as 40, 114 and 184 can be found. These divergencies make us suggest that these groups should not rather strictly correspond to nuclon envelopes in the nucleus. These groups contain envelopes as parts. As is noted by Landau and Lifshits, "though different states each of the groups enumerated consequently according to their constant filling in the series of nuclei, however, in the course of this filling considerable irregularities are noticed." They also show that state $1f_{7/2}$ are sometimes attributed to a special group. Then the remaining states in the group exhibit an envelope and magic number 20 whereas the state $1f_{7/2}$ adjoined to them gives the following stable envelope and magic number 28.

Unfortunately, in Table 1, six groups are presented and there is no possibility for us to estimate it further to see whether numbers 184 (given in literature) and 152 (was displayed from peculiatities of α-decay) are magic number indeed.

Therefore, a bypassing way had to be used: that of checking up if there is any analytical relation between members of a series of magic numbers. Two recurrent formulas have been found connecting all the set of magic numbers and isolating two their sequences: large and small ones.

$$M_i = M_{i-1} + \sum J(k) + J(i), \quad \text{for i=1,2,..., k=1,...,i-2} \qquad (1)$$

$$m_{i-2} = M_i - J(i), \quad \text{for i=3,4,...} \qquad (2)$$

We have the following relations: $J_j = 2j + 1 = 2i = J_i$ and j = $[2i - 1]/2$. Table 2 facilitates to make calculations from these formulas and straightens out the

Table 1: Groups of the nuclon states in the nucleus

No	The state in the group						n	M
1	$1s_{1/2}$						2	2
2	$1p_{3/2}$,	$1p_{1/2}$					6	8
3	$1d_{5/2}$,	$1d_{3/2}$,	$2s_{1/2}$,	$1f_{7/2}$			20	28
4	$2p_{3/2}$,	$1f_{5/2}$,	$2p_{1/2}$,	$1g_{9/2}$			22	50
5	$2d_{5/2}$,	$1g_{7/2}$,	$1h_{11/2}$,	$2d_{3/2}$,	$3s_{1/2}$		32	82
6	$2f_{7/2}$,	$1h_{9/2}$,	$2i_{13/2}$,	$2f_{5/2}$,	$3p_{3/2}$,	$3p_{1/2}$	44	126

Table 2: Calculations of magic numbers

i	1	2	3	4	5	6	7	8	9	10	11
M(i)	2	6	14	28	50	82	126	184	258	350	462
m(i-2)	-	-	8	20	40	70	112	168	240	330	440
j	1/2	3/2	5/2	7/2	9/2	11/2	13/2	15/2	17/2	19/2	21/2
J(i)	2	4	6	8	10	12	14	16	18	20	22
$\sum J(i)$	2	6	12	20	30	42	56	(72)	(90)	(110)	(132)

data.

Sums in brackets have not been used in calculating the first 16 magic numbers. As is seen from the Table 2, magic numbers M_2=6 and M_3=14 are added to the large sequence; and M_8=184 is confirmed. In the small sequence, besides magic numbers m_1=8, m_2=20, m_3=40, m_5=112 (instead of 114), numbers m_4=70, m_6=168 (not mentioned earlier) are added. All these numbers fit the peaks in the curve of abundance of isotopes with the atomic number Z. According to the above classification 152 is rather not a magic number, it is a number of α-particle configurations. Besides, it is a sum of two magic numbers of small sequence: m_3=40 and m_5=112.

Thus, in order to explain a series of magic numbers, some permutations in sequences of filling states in Table 1 are needed. If we refer the state $1d_{5/2}$ into a special subgroup, an additional magic number 14 can be obtained. In order to obtain magic number 40 which sometimes is found in literature, the states from group four $2p_{3/2}$, $1f_{5/2}$, $2p_{1/2}$ should be separated. The remaining state $1g_{9/2}$ with its $2j + 1 = 10$ gives the following magic number 50. Group five with its 32 nuclons produces magic number 50+32=82 and group six (44 nuclons) results in magic number 82+44=126. The first three members (states) of this group isolated separate envelope yield 32 nuclons and magic number 112 (instead of 114 which is found in literature).

If we permutate the state $1h_{9/2}$ and $2f_{7/2}$ in group 6 of Table 1, then after a stable state with magic number 82 a relatively stable state with Z=92 arises: M=82+10=92 (a number of α-particle configurations of uranum region). The states $2i_{13/2}$ and $3p_{1/2}$ in group 6 should be interchanged too. In group 5, the state $1h_{1/2}$ should be put in the end after the state $3s_{1/2}$. The revised Table 1 of filling the nuclon states makes a good agreement with two sequences of magic numbers. Besides, two subgroups are distinguished within each of the group starting from the second.

As the quantity of magic numbers increases, the quantity of magic and twice magic nuclei increases too. We add C(6,6), Si(14,14), Ca(20,20), Ni(28,28), Zr(40,50) and Sn(50,70) to twice magic isotopes known earlier: He(2,2), O(8,8), Pb(82,126). All they have high peaks in the curve of isotope abundance (see Fig.1).

C(6,6) corresponds to a high peak close to a twice magic oxygen O(8,8). The nucleus of magnesium Mg(12,12) with α-particle configuration gives a higher peak than aluminium Al(13,14) with its magic number N=14. Then follows a high maximum of a twice magic nucleus of silicon S(16,16) with its two numbers of α-particle combinations. After minimum of chlorine Cl and argon Ar at the ascent close to maximum of twice magic nucleus of calcium Ca(20,20) there is a magic nucleus of potassium K(19,20) with its N=20. Then a small peak of vanadium V(23,28) and chromium Cr(24,28) with magic number N=28 goes and further on - a mighty peak of iron with a magic isotope Fe(26,28). Then follows a twice magic isotope of nickel Ni(28,28), then - a small peak of germanium Ge(32,40) with its magic number N=40, and then - a peak of a magic isotopes of stroncium Sr(38,50) and ittrium Y(39,50) with N=50. Further on go small peaks of a twice magic nucleus of circonium Zr(40,50), rhutenium Ru(44,56) and palladium Pd(46,60) with their numbers of α-particle configurations. Then the peak of a twice magic isotope of tin Sn(50,70) follows, further - the peak of barium Ba(56,82) with magic number N=82 and α-particle Z=56.

After this a slow decrease in abundance isotopes of lanthanum (La), tserium (Ce), prazeodim (Pr) and neodim (Nd) follows. Then small peak of α-particle isotope of samarium Sm(62,88) appear, as well as the peak of itterbium Y(70,103) with magic number Z=70. And finally, the platinum peak Pt(78,116) with α-particle N=116 and the peak of a twice magic nucleus of lead Pb(82,126) are noticed, as well as small peaks of thorium (Th) and uranium (U) of even-numbered and α-particle nuclei. Thus, the given classification of magic numbers explains all the peaks of an isotope abundance curve.

Theoretical interpretation of elemental abundance is one of the most important problems of nuclear astrophysics. Rather high is the abundance of elements from carbon C(6,6) to calcium Ca(20,20) the nuclei of which can contain a whole number of α-particles and some of them are magic nuclei. These elements are formed as a result thermonuclear reactions (α-processes) in the interior of giant stars. The iron peak arises due to e-process - the reaction preceeding the supernovae flare. The abundance of elements heavier than iron (Fe) can be explained by processes of neutron capture by nuclei. These processes in stars can be slow (s-process) and rapid (r-process). Twice maxima in the known curve of elemental abundance (Zyuss and Yuri) at magic numbers N=50, N=82 and N=126 convincingly confirm the existence of these two capture processes. Are there any twice r- and s-peaks at other magic numbers of neutrons: 28, 40, 70 and 112? Yes, there are as is seen from the curve of isotope abundance. Twice peaks at Z=23-25 with N=28, Z=31-32 (N=40), Z=50-52 (N=70) and Z=75-78 (N=112-114) are present too.

In literature a problem is considered of isotope existence of transuranium elements and the periodical system limit. It was Fermi who calculated the limit at Z=137. With the grouth of Z because of enormous nucleus charge, an instantaneous electron capture from the envelope takes place that brings about Z decrease. These calculations were made for a nigligible nucleus size. Allowance for the nucleus size and the distribution of charges in it shift the limit to Z=150. Theoreticians put forward brave hypotheses pertaining the nucleus Z=114 and N=184 should be stable with respect to spontaneous nuclear fission. In neutron - proton (see Fig.2) diagram one can notice an area of high stability with Z=82 and N=126 (to bismuth). Here is the region of twice magic nucleus of lead (Pb).

In the area of transbismuth isotopes, two "archipelagoes" of relative stability are suggested: the first one with Z=90-96 presented with known isotopes (the region of uranium) and the second hypothetical transcurie "archipelago" with its "islands" of stability at Z=114-126 and N=184. We assume that there are two "islands" (1 and 2) in the second "archipelago" at the following pairs of magic numbers: Z=112, N=168 and Z=126, N=184. The bravest theoreticians predict one more "island" with Z=168 (without N). This "island" seems to be present at Z=168 and N=240 (or 258). Twice magic isotopes X(112,168), Y(126,184) and Z(168,258) or Z'(168,240) will correspond to "islands" 1, 2 and 3.

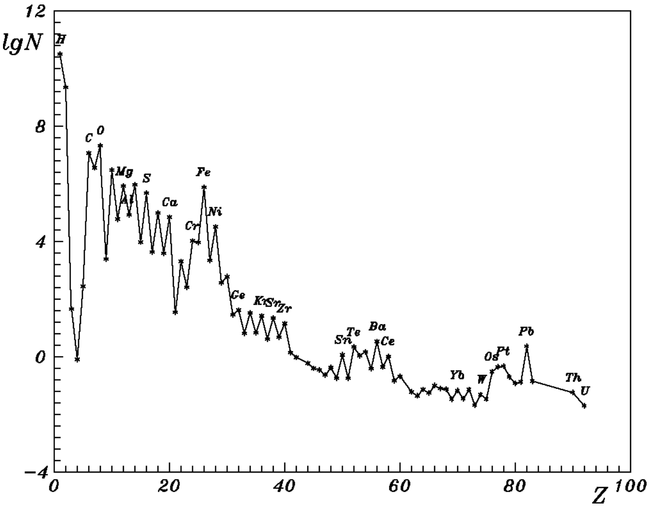

Figure 1: The curve of isotope abundance from data of sampling according to Lang (1978)

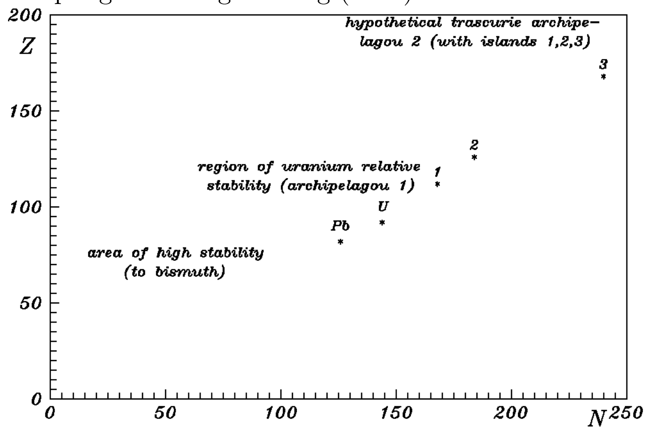

Figure 2: N-Z diagram (areas of stability)

The mean value Z=92 and N=144 correspond to "archipelago" 1. These are not magic numbers. These nuclei are apparently stable because of twice α- particle configuration. For this uranium area the ratio k_2=N_2/Z_2= 144/92=1.56; for the area of high stability of lead (Pb) k_1=126/82=1.54; and for three hypothetical "islands": k_3=168/112=1.5, k_4=184/126=1.46 and k_5=240/168=1.43 or k_5'=258/168=1.54 (coincides with k_1).

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