

MULTI-TIME-SCALE VARIABILITY OF STARS

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ABSTRACT. Physical mechanisms and corresponding mathematical models for different types of stellar variability are reviewed with applications to concrete stars. Special attention is attributed to the following topics: *cataclysmic variables* (synchronous and synchronizing magnetic systems, non-magnetic nova-like stars, dwarf novae; magnetic activity of the red companion, third body around), *pulsating variables* (additional classification criteria of variability, multi-parameter correlation analysis of the characteristics of the mean light curves of groups of long-period stars and of the multi-parameter correlation analysis of the individual characteristics of their pulsations), *mathematical models* (multi-periodic, multi-harmonic, multi-shift variations with possible trends, mono- and multi-cyclic variations of low coherence and the frequency changing/switching signals, "red noise" variability. Some original papers and links may be found at <http://ila.webjump.com>

Key words: stars: cataclysmic variables; stars: pulsating: Mira, semi-regular; stars: individual: TT Ari, UV Aur, BY Cam, PZ Cas, EM Cyg, AF Cyg, V792 Cyg, V1329 Cyg, AM Her, DQ Her, V533 Her, S Per, RX J0558.0+5353, RX J2107.9-0518, QQ Vul; data analysis.

Introduction

This self-review is based on the highlights of the results obtained during recent years in the group "Periodic and aperiodic processes in variable stars" in the main directions: cataclysmic variables at different stages of the influence of the magnetic field onto accretion; pulsating variables showing complicated multi-periodic and quasi-periodic behaviour; photometric classification of newly discovered or poorly studied variables; methods of mathematical modeling adequately satisfying the statistical conditions of the input data and approaching the asymptotic "signal/noise" ratio.

In *magnetic cataclysmic variables*, the variability is present at the time scale of seconds ("boiling column"), dozens of seconds ("shot noise", flare), hours (spin), days (spin-orbit beat, switches between a high and

low states), years ("swinging dipole" and other mechanisms); $10^2 - 10^5$ years - spin period variations of the white dwarf;

non-magnetic nova-like: seconds to dozens of minutes (red noise); minutes to hour (quasi-periodic oscillations); hours (positive and negative superhumps, orbital variations); days (superhump-orbit beat); years (luminosity switches);

non-magnetic dwarf novae: cycle-to-cycle and season-to-season changes of characteristics of outbursts;

magnetic activity of the red companion: year-scale variations of the luminosity of systems with magnetic (polars and intermediate polars) and non-magnetic (nova-likes of VY Scl and UX UMa subtypes) white dwarfs; brightness variations at low states in dwarf novae and old classical novae; smooth variations and abrupt switches of the seasonal outburst cycle length in dwarf novae;

Asynchronous magnetic cataclysmic variables

The magnetic field of the white dwarf, if strong enough, leads to the synchronization of the spin with the orbital motion with a characteristic time $\tau = (P_{orb} - P_{spin})/\dot{P}_{spin} \lesssim 10^3$ yrs (Andronov, 1987). As this stage is very short as compared with the evolution time, only 3 nearly synchronous systems are discovered yet with a period difference about one per cent. This model predicted also a switching of the accretion stream from one pole to another in the asynchronous polars.

To check the observational evidence of the asynchronism between the spin and orbital motion, the international campaign "The Noah project" was organized (Silber et al., 1997) for observations of BY Cam. This title based on the Bible was preferred, as the duration of the project was suggested to be 40 days with a huge "flow" of the data. Further more detailed analysis of the results was published by Mason et al (1998).

The photometric variability has few periods. The largest amplitude corresponds to the sidelobe frequency $f = 2\omega - \Omega$, where ω is the spin frequency and Ω is the orbital frequency. This corresponds to the model that the accretion switches from one pole to another

during "idling" of the white dwarf in respect to the secondary. The accretion columns at both poles are nearly equal, as the amplitude of the variations with the spin frequency is ≈ 4 times smaller than that with the sidelobe (primary) frequency. The amplitude of the orbital variability is ≈ 2.5 times smaller than that of the primary variations.

The observed characteristic time of the synchronization obtained from the period derivative (Mason et al., 1998) is $\tau \approx 880 \pm 34$ yrs in an excellent agreement with the theoretical expectation (Andronov, 1987).

This behaviour is distinctly different from that of the intermediate polar RX J0558.0+5353 which shows variations of the circular polarization with a period twice the photometric period (Shakhovskoy and Kolesnikov 1997). This again argues for the magnetic axis of the white dwarf being relatively close to the spin equator. Another intermediate polar showing circular polarization is RX J2107.9-0518 (Shakhovskoy and Kolesnikov 1993).

Synchronous magnetic cataclysmic variables

After synchronization, the white dwarf becomes phase-locked to the red dwarf, and the spin and orbital periods become equal. However, there may be excited some orientation changes of the magnetic axis in respect to the rotating binary system, as was predicted by the "swinging dipole" model (Andronov, 1987). Such "swingings" are seen in the O-C variations of the extrema of two systems being studied for decades - AM Her and QQ Vul. Several mechanisms affect the accelerating/decelerating torque with a complicated nonlinear overlap.

This torque is dependent on the orientation of the magnetic axis of the white dwarf and on the accretion rate. The latter is dependent on the irradiation of the secondary by the flux from the accretion column with variable orientation, magnetic and spot activity of the red dwarf itself, minor variations of the orbital separation owed to the third body. Even in a simplest case of the accretionless dipole-dipole interaction, the orientation changes are very complicated, and even in this case different magnetic poles may be seen from the red dwarf.

This makes variations of the luminosity and the orientation very complicated and not periodic, with few characteristic time scales. This is well seen both in the long-term light curve (Andronov et al., 1997) and the changes of the longitude and latitude of the magnetic axis (Shakhovskoy et al., 1992).

The accreted plasma blobs become longer and more thin while falling onto the red dwarf causing few types of instability of the accretion column (cf. Andronov, 1987). One of the most important characteristic of this "rain" of plasma "spaghetties" is the characteristic time τ_e of penetration of such a blob through the shock

front above the white dwarf. Usually this parameter is in the range from 40 to 120 seconds.

However, an interesting phenomenon was detected in BY Cam. It shows a dependence between the τ_e and the mean polarization which is obviously dependent on the orientation of the accretion column. This dependence is splitted into two parts possibly indicating a switch of the accretion pole from one to another. No splitting is seen for the same diagram for synchronous system QQ Vul (Shakhovskoy et al., 1999).

A comparison of the properties of the unprecedented UV Cet-type flare of AM Her (Shakhovskoy et al., 1993) with the small outbursts owed to accretion of plasma blobs was recently discussed by Bonnet-Bidaud et al. (2000).

The temperature estimate of the flare region at the red dwarf is $T \approx 12000\text{K}$ with an effective radius of $R \approx 9000\text{km}$ which is comparable with the radius of the white dwarf but the corresponding surface is only 0.5 per cent of the surface of the red dwarf. The modeling of the energy distribution of the white dwarf allowed to estimate its temperature of 20000K (Silber et al., 1996), whereas the temperature of the accretion column is of $\approx 10^6 - 10^8\text{K}$.

Slow variations of the characteristics of cataclysmic binaries

Cataclysmic variables of different types show variations of characteristics with cycles of few years (cf. Bianchini, 1990; Andronov and Shakun, 1990). They may be seen in the luminosity changes between the "high" and "low" states of nova-like variables, polars and intermediate polars; in the lower amplitude changes of luminosity of these objects and in the variability of the cycle length between the outbursts of the dwarf novae.

In the addition to the usual interpretation of this variability as the accretion rate changes owed to a magnetic activity of the convective red dwarf companion, one may expect significant changes of the accretion rate caused by minor changes of the orbital separation owed to the gravitational interaction with a third body (light star or a heavy planet).

Chinarova et al. (1996) have found a 3000^d cyclicity of the mean brightness which is highly correlated with the effective amplitude of the outbursts. The brightness at the seasonal maximum is not statistically dependent on the mean brightness, thus the changes of the luminosity are caused mainly by the variability of the depth of the minima. Unexpectedly, no correlation was found between the mean seasonal outburst cycle length and luminosity or amplitude. However, the additional wavelet analysis of the individual cycles shows similar 3000^d wave in the duration.

Another example of drastic change of the seasonal mean cycle length shows V 792 Cyg seen on the SAI

photographic plate collection. This characteristic has switched from $25\text{--}29^d$ to $\approx 39^d$ between 1967 and 1973 yrs (Chinarova and Andronov, 2000). Unfortunately, the available data are not sufficient to study the possible cyclicity of the outburst behaviour.

Chinarova and Andronov (1999) reported on the possible waves of low coherency in RU Peg. The wavelet analysis shows characteristic times of 800^d , 2000^d and 22000^d either in the peak outburst brightness and in the interval between the successive outbursts.

The luminosity variability at a similar time scale of few years was found in the old novae V533 Her and DQ Her in their inactive state (Andronov et al., 1998).

The model of the modulation of the accretion rate by changing orbital separation in a triple system was indirectly confirmed by North (1999), who suggested a third body in EM Cyg, the star from our sample. Thus one may suggest that this mechanism may be one of complementary acting in at least some cataclysmic variables.

Positive and negative superhumps in nova-like variables

The SU UMA-type stars exhibit so-called superhumps during their superoutbursts which are interpreted as the precession of the eccentric accretion disk. The photometric period P_{sh} is larger than the orbital one P_{orb} in these stars, thus $P_{sh} - P_{orb} > 0$. For some nova-like variables (VX Scl-type stars), the difference $P_{sh} - P_{orb} < 0$. These objects may be called "negative superhumps" (cf. Skillman et al., 1998). The most studied object of this type in our sample is TT Ari (Tremko et al., 1996; Andronov et al., 1999b) which recently underwent an unprecedented switch from a negative to positive superhump. Results of the international campaign "TT Ari-88" are published separately in this volume (Andronov et al., 1999a).

Variations of the light curves of symbiotic binaries

Symbiotic stars show variations of the phase light curves either if they belong to the group with a pulsating Mira-type component (UV Aur, Chinarova et al., 1994, Chinarova, 1998; R Aqr, Chinarova et al., 1996) or to the group of symbiotic novae (V 1329 Cyg, Chochol et al., 1999).

In UV Aur, a significant decrease both of the mean brightness and of the amplitude occurred near JD 2444900. The periodogram analysis showed the characteristic time of variations of 6800 ± 46^d with an approximation $dm_{max}/dm_{min} = 2.3 \pm 0.5$ which is opposite in sign as compared with the dwarf nova EM Cyg (see above). One of the possible explanations of the long-term light curve variations is the possible beat between the pulsations and the orbital motion.

In the symbiotic nova V 1329 Cyg, the secondary

period of 553 ± 2^d and a longer cycle of 5300 ± 160^d are suggested which could be explained by systematic changes in geometry and location of the emitting region.

Wavelet analysis of irregularly spaced data

Irregularity of many astronomical signals leads to drastic deformations of a wavelet map making impossible the inverse wavelet transform and thus making lost the initial sense of the wavelet transform. A comparative analysis of the different modifications of the wavelet analysis based on the Morlet function was presented by Andronov (1998). An algorithm was proposed for use of the wavelet analysis based on the least squares method with supplementary weights, which extends the "weighted wavelet transform" proposed by Foster (1996). Spectral and statistical properties of the test- and smoothing functions were studied, the optimal values of the argument and frequency step were proposed. For the argument step, one may use a shift, at which the autocorrelation function corresponding to the response function crosses zero. Results are illustrated by application to continuous functions, numerical models of regular and irregular signals, including autoregressive processes, observations of the dwarf nova star SS Cygni.

The optimal wavelet smoothing algorithm was proposed by Andronov (1999b). For fixed time, the peak corresponding to the maximum of the test function WWZ is determined. Then the period is corrected to maximize the correlation coefficient between the data and the local wavelet fit. Then the corresponding test-function $S(f)$, mean value of the harmonic function, amplitude and smoothing value are computed.

Precise analytic expressions for these parameters as well as for the weighted wavelet transform are derived and illustrated on numerical examples of the harmonic, multi-frequency signals, autoregressive models, real data and the "running parabola" fits. The response functions corresponding to different basic (ordinary and trigonometric polynomials) and weight (rectangular, Gaussian and intermediate $p(z) = (1 - z^2)^2$) functions are compared by Andronov (1999b).

The wavelet analysis is effective for studies of signals with significantly variable period, mean value and amplitude, i.e. quasi-periodic oscillations in cataclysmic binaries, mode switchings in semi-regular variables etc.

The effective width of the Morlet-type wavelet corresponding to the width of a rectangular filter producing the same frequency resolution, is equal to $\Delta T = (3/8\pi^2c)^{1/2}P \approx 1.74(1/80c)^{1/2}P$, where c is the only free parameter in this type of wavelet, and is set to $c = 1/80$ by default (cf. Foster, 1996).

For the first-order autoregressive model, the corrected shape of the autocorrelation function is presented.

The are applied to the stars of different types - cataclysmic, semi-regular, symbiotic - as well as to the

numerical models.

Scalegram analysis of the "Red noise"

The method of "running parabolae" is an effective tool for smoothing of aperiodic astronomical signals. The statistical and spectral properties of the fit for arbitrary weight and basic functions were described by Andronov (1997).

However, this method has an important application to irregular signals which are characterized by the "red noise" (cf. Andronov 1999a) caused either by the "shot noise" or by the dynamical chaos.

Such type of variability may be effectively described by the test function σ_{O-C} (i.e. the unbiased estimate of the data from the fit corresponding to the filter half-width Δt). For the noise superimposed onto the smooth variations, this test function has a standstill while Δt is small enough to produce systematic differences between the signal and the fit. With increasing systematic difference, the test-function σ_{O-C} increases with Δt to some asymptotic value corresponding to the parabolic fit.

For the flickering in cataclysmic variables, there is no standstill at the dependence $\sigma_{O-C}(\Delta t)$. Moreover, this dependence has a linear branch in a double logarithmic scale, which corresponds to a power law $\sigma_{O-C} \propto (\Delta t)^{\gamma_\sigma}$. This is caused by a presence of systematic differences at all characteristic times and by the accumulation of these differences with increasing Δt .

It may be recommended to introduce more complicated characteristics of fast variability, e.g. σ_{O-C} and the slope γ_σ at a fixed argument Δt . If computing σ_{O-C} for 3 fixed values of Δt , one may additionally determine the contribution of the uncorrelated observational noise.

If the power spectrum $S(f) \propto f^{\gamma_S}$, then one may expect that $\gamma_\sigma = (\gamma_S - 1)/2$. For the signals with a fractal dimension D , one may obtain $\gamma_\sigma = 0.5 - D$.

For AM Her, Andronov et al. (1997) have obtained $\gamma_\sigma = 0.18$, in excellent agreement with previously found value $\gamma_S = 1.36$. The corresponding value $D = 0.32$.

Periodogram analysis of the Hipparchos-Tycho suspected variables

The search for possible periodicities in 455 newly discovered suspected variables from the Hipparchos-Tycho observations was carried out by using several complementary methods. The main method was to compute the periodograms $S(f)$ corresponding to trigonometric polynomial (TP) fits of orders 1, 2 and 3 separately for B and V observations. Then the combined periodograms $S = (S_B S_V)^{1/2}$ have been computed indicating the peaks occurring at both colors. Results of this preliminary photometric classification are pre-

sented by Andronov et al. (1999).

To increase the "signal/noise" ratio, the periodic splines with *variable* degree have been proposed. To find periods of stars with asymmetric one-wave light curves, we have applied the "RR-catcher", which is a 5-parameter fit: the descending branch is fitted by a parabola, and the ascending branch by a cubic parabola, keeping the smoothing curve and its first derivative continuous.

For the Algol-type variables ("EA-catcher"), the phase diagram is splitted into two minima of equal duration opposite in phase with an unknown depth, and a constant level outside minima. Both these methods need non-linear 3D optimization for each candidate period (characteristic phase width, phase shift and period correction). Then the stars are classified according to the phase light and color curves corresponding to the best "TP", "EA" and "RR" candidates.

Nonparametric methods of periodogram analysis

These methods are slow, but effective for the period search if there are the branches of fast signal variations with phase, e.g. in EA or RR- type stars. General approach to these methods based on the effective proximity of the points subsequent in phase was reviewed by Pel't (1980) and Terebizh (1992). Andronov and Chinarova (1997) studied statistical properties of 9 modifications of non-parametric methods and concluded that they may be subdivided into two main groups - that similar to the method by Lafler and Kinman (1965) and that by Deeming (1970).

The method has been extended to data with different weights by replacing the difference $|x_{k+1} - x(k)|$ in the test function by the weighted difference $|x_{k+1} - x(k)|/(\sigma_{k+1}^2 + \sigma_k^2)^{1/2}$. The corresponding computer programs have been applied to the Hipparchos-Tycho observations of known and suspected variable stars.

Effective colors of correlated flickering

For cataclysmic variables, we approximate the signal by the sum of a slow non-linear trend (orbital or superhump variations), correlated flickering or quasi-periodic oscillations and uncorrelated photon counting noise. Initially we have applied the method to TT Ari (Tremko et al., 1996) where the U-B colors correspond to increasing temperature from the mean flux through superhump variability to 15-25 min QPOs.

For highly noisy signals, the method should be improved in the following way: the covariation matrix $R_{ij} = \langle (O-C)_i (O-C)_j \rangle$ is computed for different color pairs i, j for the residuals $(O-C)$ of the data from the "orbital/superhump" curve, than the mathematical expectation may be written as $R_{ii} = \sigma_i^2 = \sigma_{si}^2 + \sigma_{ni}^2$ for $i = j$ and $R_{ij} = \sigma_{si}\sigma_{sj}$ for $i \neq j$. Here σ_{si}^2 and σ_{ni}^2 are $2m$ variances of correlated signal and of an uncor-

related noise, respectively, where m is the number of simultaneously used filters. The number of independent equations is equal to $m(m+1)/2$, because the matrix is symmetrical: $R_{ij} = R_{ji}$.

For 3 colors, the solution is simple: $\sigma_{s1}^2 = R_{12}R_{13}/R_{23}$, $\sigma_{n1}^2 = R_{11} - R_{12}R_{13}/R_{23}$, and for the rest colors by using a cyclical index shift.

For more colors, one could minimize the weighted sum of the residuals from the equation $\rho_{ij} = R_{ij}/(R_{ii}R_{jj})^{1/2} = \rho_i\rho_j$, where $\rho_i = \sigma_{si}/\sigma_i$ is the correlation coefficient between the deviation from the fit of the correlated signal and of the observed (noisy) signal. Here the unknowns are ρ_i . The final parameters are $\sigma_{si} = R_{ii}^{1/2}\rho_i$ and $\sigma_{ni} = (R_{ii}(1 - \rho_i^2))^{1/2}$.

This approach is slightly different from the minimization of the residuals of the equation $\eta_j\eta_j = \rho_{ij}^{-2}$ with the weights $(1 - \rho_{ij}^2)^{-1}$, as was proposed by Tremko et al. (1996). Here the unknowns are $\eta_i = 1 + \sigma_{ni}^2/\sigma_{si}^2$. Obviously, both solutions for σ_{ni} and σ_{si} are equal for 3 colors.

The relative effective amplitudes of the correlated signal may be converted to stellar magnitudes by using a differential color $\Delta(m_i - m_j) = 2.5 \lg(\sigma_j/\sigma_i)$ which is valid for $\sigma_i \ll 1$. However, for relatively large amplitudes, this "color correction" may also characterize an effective color of the flickering/QPO.

Long-period variables: additional classification criteria of variability

The studies of the pulsating variables are carried out in few directions.

At first, the multi-parameter correlation analysis of the characteristics of the *mean* light curves of groups of Mira-type stars. Results are briefly summarized by Kudashkina and Andronov (1996). The study of the "main sequences" at the diagrams of the parameters of the mean light curves and of the "outstanding objects" is still in progress.

Second, the multi-parameter correlation analysis of the *individual* characteristics of long-period stars. The methods and 35 parameters of the individual light curve were described by Marsakova and Andronov (1998). Based on these methods, the Mira-type stars best covered by the observations from the AFOEV and VSOLJ databases have been analyzed.

The "Catalogue of main characteristics of individual pulsational cycles of 35 Mira-type stars" was published by Marsakova and Andronov (1998). The catalogue contains only the moments and brightness of the extrema and the inverse slopes dt/dm of the light curves and the corresponding error estimates. One of the stars, Y Per, showed intervals of variability of different types - Mira or semiregular. Four stars show a secular period variations in a good agreement with the theoretical evolutionary track at the helium flash stage. Other stars show cycle-to-cycle variations, and very often

the characteristic time is of order 15 000–20 000 days. The stars are clustered into groups showing correlations between different groups of parameters.

Some results of the analysis of this catalogue were published separately by Marsakova (1999, 2000).

At third, pulsating variables are studied as components of *symbiotic* variables, as was mentioned above.

The fourth direction is the study of *semiregular* pulsating variables based on own photographic observations and on the AFOEV and VSOLJ databases. Among most recent results one may note the necessity of the subdivision of the type SRc into at least two subtypes (Kudashkina and Andronov, 2000): a) SRca – supergiant stars with multi-periodic pulsations and regular light curve similar to the Mira-type, sometimes disturbed by the switching mode intervals (prototype S Per); b) SRcb – supergiant stars with a quasi-periodic light curve and intervals of brightness constancy (PZ Cas).

A phenomenological classification of semi-regular stars was proposed by Andronov et al. (1998). It is based on the arbitrary *stability* of their (multi-periodic) components of variability and, particularly, on the *switches* of pulsation mode in these stars (eg. AF Cyg). The used method is the periodogram analysis by using the least squares harmonic approximation with subsequent prewhitening (cf. Andronov, 1994). The classification was made according to the relative frequencies and number of the peaks at the periodograms. Despite the cycle-to-cycle changes of the light curve, some stars show only one main frequency; second subgroup consists of "multiharmonic" stars with frequencies close to integer multiplies of the main frequency; the third group collects the "multiperiodic" stars with 2 or more periods; the fourth group may be called "chaotic": after removal of the waves corresponding to 3 highest peaks at the periodogram, the periodogram of the residuals still contains a lot of peaks being statistically significant. The distribution of the SR stars in these groups does not usually coincide with the subtypes from the "General catalogue of variable stars". This study is continued for more wide sample of stars.

Other mathematical models:

The software package has been elaborated to study *multi-periodic* variations with possible periodic and aperiodic *trends* in the data and *shifts* between the sets owed to difference in the instrumental systems or run-to-run slow changes. The main expressions are described by Andronov (1994). The methods allow to study *mono-* and *multi-* cyclic variations of low coherence and the *mode switching* stars; aperiodic variability, e.g. corresponding to the "shot noise" and "red noise" types; secular period variations.

The algorithms for "running approximations" are based on the extension of the method of least squares

res to the case of *unequal weights* of the data and an additional weight (*filter*) function making the models wavelet-like, being both dependent on time and a characteristic time scale. *Non-parametric* periodograms are extended to a case of unequal weights. These algorithms are compared according to a statistical significance of the results. More detailed descriptions and some papers may be found at our WWW pages <http://ila.webjump.com> and (briefly mirrored) <http://paco.odessa.ua/~il-a>.

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