ON THE LAW OF PLANETARY DISTANCES IN THE SOLAR SYSTEM

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ABSTRACT. Representations of major semiaxes of planets and distances of nearby planets by means of degrees of two and prime numbers are made. We analyse different formulas for these representations and give new formulas (3)-(4) and a system of formulas (6)-(8) which by the best appearance presents all the planets of the Planetary system, including Neptune and Pluto. Recommendations are given for the search of the tenth planet (instead of Pluto) and eleventh one.

Key words: Solar system: Titius-Bode's Law, the Law of planetary distances, its modifications

1. Introduction

The Titius-Bode's Law is an approximative empirical relationship of the planets distances from the Sun. It is a simple numerical sequence that basically predicts the spacing of the planets.

A(AU)=0.4+0.3*2^n, (n=-∞, 0, 1, 2, 3, 4, 5, 6) (1)

Chechel'nicky (1983) showed, that major semiaxes of planet orbits (A(i)) and their differences (ΔA(i) = A(i+1) - A(i)) for nearby planets normalized on specially chosen value of A* = 0.0372193 AU (astronomical unit) are near to degrees of two. He gave the formula (2) as a modification of Titius-Bode's Law that reflects regularity of planetary distances of Solar system for planets (except for Neptune and Pluto) and for the Belt of asteroids.

A(n)=A_0+2^n, (A_0=10.4, n=-∞, 3, 4, 5, 6, 7) (2)
(A_0 = 0, n = 8, 9, 10).

Also it was pointed there that all differences of major semiaxes of nearby orbits (except for the second) are near to whole numbers.

2. Results

We give all datas from above paper in Table 1, adding information (Allen, 1964) for the Belt of asteroids and Pluto. There are representations of major semiaxes of planets by means of degrees of two and prime numbers there. Representations of distances of nearby planets by means of degrees of two and prime numbers are presented in Table 2.

Table 2: Representation of distances of nearby planets by means of degrees of two and prime numbers

planets	ΔA	2^n	n	prime number
Venus-Mercury	9.034	8	3	7
Earth-Venus	7.434	8	3	7
Mars -Earth	14.072	16	4	13
Belt of aster-Mars	34.290	32	5	31
Jupiter-Belt of aster	64.554	64	6	67
Saturn-Jupiter	116.070	128	7	113 (127)
Uranium-Saturn	259.033	256	8	257
Neptune-Uranium	292.999	?	?	293
Pluto-Neptune	249.996	256	8	251

We notice that the value of A* is equal to eight radiuses of the Sun. As we can see from Table 1, Anorm (normalized values) for Saturn and Uranium are near to the values of degrees of two (256 and 512, respectively). Differences Δ=A_{norm}-A_0, where A_0 is the normalized major semiaxes of Mercury, are closed to the values of degrees of two for nearer to the Sun planets and for Belt of asteroids. However we don't see from the Table 2, that all differences of major semiaxes nearby planets are also near to the values 2^n.

For example, 116 and 293 are distant from the degrees of two: 128, 256. It is possible to notice, that both major semiaxes of planets and differences nearby ones are near to prime numbers: 7, 11, 13, 19, 29, 41, 73, 97, 113, 139, 257, 293, 509.

The normalized differences (Δ) of the observed major semiaxes of planets and calculated ones from the formulas (2) are presented in the last column of the Table 1. Evidently, that the difference (Δ) for Pluto (+33.882) is very large in comparison with differences for other planets. Thereby, the formula (2) badly represents the distance of Pluto. Pluto was excluded from the list of large planets by the decision of Congress of International Astronomical Union (IAU). So nothing frightful will happen, if we exclude it from the formula (2). Business is worse

Table 1: Representations of major semiaxes of planet's orbits by means of degrees of two and prime numbers

planet	(AU)	A_{norm}	2^n	n	prime number	Δ=A_{norm}-A(n)
Mercury	0.387097676	10.400	0	$-\infty$	11	0
Venus	0.723335194	19.434	8	3	19	+1.034
Earth	1.000007872	26.868	16	4	29	+0.468
Mars	1.523749457	40.940	32	5	41	-1.460
Belt-ast	2.8	75.230	64	6	73	+0.830
Jupiter	5.202655382	139.784	128	7	139	+1.383
Saturn	9.522688738	255.854	256	8	257	-0.146
Uranium	19.16371889	514.887	512	9	509	+2.887
Neptune	30.06894040	807.886	?	?	809	?
Pluto	39.37364135	1057.882	1024	10	1061	+33.882

Table 3: Representation of major semiaxes of planets by means of formulas

planet	A_{norm}	A(1)	A(2)	A(2b)	A(2c)	A(3)	A(4)
Mercury	10.400	10.747	10.4	10.400	10.4	10.40	10.400
Venus	19.434	18.807	18.4	18.460	18.4	18.25	18.352
Earth	26.868	26.868	26.4	26.521	26.4	26.10	26.272
Mars	40.940	42.988	42.4	42.641	42.4	41.80	42.081
Belt of aster	75.230	75.229	74.4	74.882	74.4	73.20	73.636
Jupiter	139.784	139.712	138.4	139.365	138.4	136.0	136.619
Saturn	255.854	268.677	256	268.330	266.4	261.6	262.333
Uranium	514.887	526.606	512	526.259	522.4	512.8	513.257
Neptune	807.886	-	-	-	-	-	-
Pluto	1057.882	1042.465	1024	1042.118	1034.4	1015.2	1014.103

with Neptune, as it is the major planet of the Solar system and is not represented with the formulas (1) and (2). When Titius-Bode's Law was laid down, Neptune was not discovered yet. Therefore there was no misunderstanding. When Neptune was discovered, its difference (Δ) of the observed major semiaxes and calculated ones from Law of planetary distances was explained (the book of N'etto, 1976) by different hypotheses. That the orbit of Neptune was strongly distorted by gravity influence going by an unknown major planet, or that Neptune was captured by the Solar system from the planetary system other passing by star. Anyway, but Neptune is not described by the formula (2).

One more difficulty for this formula consists in the facts that orbits of Saturn (n=8) and Uranium (n=9) are not described by the same formula with a free member, equal major semiaxes of Mercury, and require a zero free member. The formula (2), written in two lines (i. e. with two free members), actually presents two formulas. That would reflect two different histories of planets forming of Solar system, internal (including Jupiter) and external (Saturn and Uranium). The sum of squares of differences of the major semiaxes for eight planets calculated on the formula (2), including Belt of asteroids, from the observed values is $\delta(2)=\sum\Delta(i)^2=14.380$ (without Neptune and Pluto).

The formula (1), normalized on the value of A* = 0.0372193 AU looks like the folliwing one (2a):

A*(n)=10.747+8.0603*2^n, (n=$-\infty$, 0,1,2,3,4, 5, 6) (2a)

If we replace free member on more exact meaning in this formula, 10.400, equal to the orbit of Mercury, we get the formula (2b).

A*(n)=10.400+8.0603*2^n, (n=$-\infty$, 0,1,2,3,4, 5, 6) (2b)

And replacing a coefficient 8.0603 before a degree of two on integer-valued 8 =2^3, we get the formula (2c):

A*(n)=10.400+8*2^n, (n=$-\infty$, 0, 1, 2, 3, 4, 5, 6) (2c)

or, uniting the degrees of two, we get an equivalent equation (2d):

A*(n)=10.400+2^{n+3}, (n=$-\infty$, 0, 1, 2, 3, 4, 5, 6) (2d)

Designating $n+3$ through m we get formula (2e).

It coincides with the formula (2), if we write down it with only one free member A_0=10.4. This formula is true for all planets, including m=8 and 9 (Saturn and Uranium):

A*(n)=10.400+2^n, (n=$-\infty$, 3, 4, 5, 6, 7, 8, 9) (2e)

Values of $\sum\Delta(i)^2$ sums of squares of discrepancies

for these formulas are the following: δ(2a)=306.503, δ(2b)=289.23, δ(2c)=δ(2d)= δ(2e)=173.687.
The formula (2c) after varying of coefficient at 2^n (a minimum of parabolic function at a coefficient 7.85) gives the smallest δ(3)=58.543. Thereby, there can be an alternative formula (3) to the formula (2):
A*(n)=10.400+7.85*2^n, (n=-∞, 0, 1, 2, 3, 4, 5, 6) (3)
or the formula (4), which can be obtained from the formula (2) after varying of degree 2:
A*(n)=10.400+1.996^n, (n=-∞, 3, 4, 5, 6, 7, 8, 9) (4)
δ(4)=60.02 for it, i. e. rather more of the sum of squares of discrepancies δ(3) for the formula (3).

We present representations of major semiaxes of planets by means of different formulas in Table 3.

So, the formulas (2) give the least sum of squares of discrepancies ($\delta(2)=\sum \Delta(i)^2$=14.38), but its imperfection is that it consists of two lines, i.e. actually from two formulas. The first line describs six internal planets, including Belt of asteroids and Jupiter, and second one - two external planets (Saturn and Uranium). Though the formulas (3) and (4), got by us, have four times greater values of the sum of squares of discrepancy as compared to the formulas (2), however they describe uniformly and exactly enough eight objects of Solar system, from Mercury to Uranium, including Belt of asteroids.

Squares of discrepancies for every the planets are parabolic functions and have the minimum at the different coefficients q (base of power) in the vicinity of two: Venus (at q=2.0827), Earth (2.0144), Mars (1.9814), Belt of asteroids (2.0043), Jupiter (2.0031), Saturn (1.9895), Uranium (1.9967), Neptune (1.9506), Pluto (2.0045). The formula (4) has a minimum value of sum of squares of discrepancies (60.02) at q=1.996 for n=3-9 (without Pluto), Saturn and Uranium have the largest discrepancies here. Without these two planets (n=3-7) the sum of squares of discrepancies will be far less than (δ=15.386), taking on a minimum value at q=2.003 (δ=4.092). That is why Saturn and Uranium have minima of discrepancies at the values of q equal 1.9895 and 1.9967, respectively. Including them in the formula (4) lowers q to the value of 1.996. Oddly enough, but addition of Pluto with its q=2.0045, again gives the minimum sum of squares of discrepancies (δ=463.56) at the same value of q equal 2.003, i.e. Pluto compensates the contributions of Saturn and Uranium. Again the formula (5) is true with q=2.003:
A*(n)=10.400+2.003^n, (n=-∞,3,4,5,6, 7, 8, 9, 10) (5)
It shows, why the Law of planetary distances has statistical nature. It is named not a law but a rule sometimes.

Did not hurry with Pluto, depriving its status of tenth planet? It has q equal 2.0045 near to the values of q for Belt of asteroids (2.0043) and Jupiter (2.0031). Values of this parameter greater than 2 have another two planets: Earth (2.0144) and Venus (2.0827).

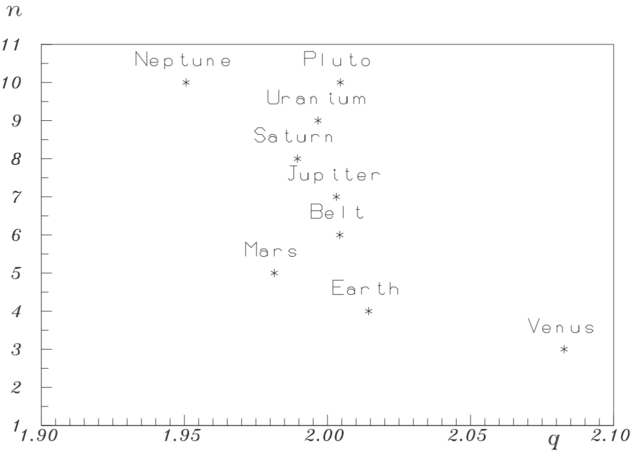

Figure 1: The dependence of a number of planet (n) from the parameter q

It gives an occasion to unite these five planets in one group, for which the formula (6) is true giving a strikingly small sum of squares of discrepancies δ(6)=9.200:
A*(n)=10.400+2.004^n, (n=3, 4, 6, 7, 10) (6)
Remaining three planets with the parameter q less than two Mars (1.9814), Saturn (1.9895) and Uranium (1.9967) give a minimum of sum of squares of discrepancies at the parameter q equal 1.996, as well as in the formula (4):
A*(n)=10.400+1.996^n, (n=5, 8, 9) (7)
The sum of squares of discrepancies δ(7) is equal 45.930 for them. Addition of another planets, except for Pluto, converts the formula (7) into the formula (4). It gives δ(4)=60.020, but addition of Pluto gives the sum of squares of discrepancies δ=1976.62! The same three planets (n= 5, 8, 9) at q equal 2.004 would increase the sum of squares of discrepancies δ(6)=9.200 in relation to the formula (6) by about 500.748! If we take a double formula - system (6)-(7) by analogy with a double formula (2), then δ(6-7)=9.200+45.930= 55.130 for eight planets (including Pluto). That is less than δ(4)=60.020 at the same coefficient q equal 1.996 for all planets (except for Pluto) and considerably less than δ(4)=1976.62, including Pluto. Value of δ(6-7)=55.130 is also much less than δ(2)=1162.37, including Pluto. Thereby, the system (6)-(7) presents all the planets of the Planetary system (except for Neptune) in the best way.

In Figure 1 we give the graph of dependence of number of planet n from the parameter q. One can see a clear peak, in the top of which is the point of Uranium (n=9). The point of Pluto (n=10, q=2.0045) is displaced to the right on 0.0045 from the integer-valued q=2. If the tenth planet (instead of eliminated Pluto) is discovered, the ideall value of q would be equal 2, to be a maximal point on the graphic. For Neptune q is equal 1.95, it differs by about 0.05 from the integer-valued q equal 2. This difference is more than ones for other planets except for Venus, for which Δ q equal 0.08 is outstandingly large, but only towards the greater value of q equal 2.08 (see Fig. 1). However Neptune follows none the formulas resulted above. The formula (8) is true for it:

$A^*(n)=10.400+1.95^n$, (n= 3, 4, 5, 10) (8)

at n=10, as well as for Pluto in the formulas resulted above. The formula (8) presents the major semiaxes of another three planets, but with larger discrepancies than Titius - Bode's Law: Venus (n=3), Earth (n=4) and Mars (n=5). The discrepancies Δ for them are equal 1.61, 1.99 and 2.30, respectively. The sums of squares of discrepancies for the formula (8) and Titius - Bode's Law are as following: 11.84 and 4.60. We will notice that at the value of parameter q equal 2.10 and n=9 the calculated value of major semiax of Neptune (804.680) will be also near to the observed one (807.886). The formula (9) with this value of parameter q presents the major semiaxes not only for Neptune (Δ=3.206) but also for Venus (Δ=-0.227) and Earths (Δ=-2.980). Mars follows (Δ=0.099) to the same formula but without a free member. Venus and Neptune are two extreme representatives on the parameter q, they both surprisingly fit the formula (9) at this parameter q equal 2.10.

$A^*(n)=10.400+2.10^n$, (n=3, 4, 9) (9)

Following these reasonings, it is possible to suppose that the graph in the Fig. 1 is periodic with the period on Δq equal 0.05. The formula (10) corresponds to a supposed peak (not supported by points) at q equal 2.05:

$A^*(n)=10.400+2.05^n$, (n= 3, 4) (10)

This formula describes the major semiaxes of Venus (Δ=0.4189) and Earths (Δ=-1.193), without a free member it describes Mars (δ=4.735) and Belt of asteroids (Δ=1.01). We remind that the formula (2) also gives smaller discrepancies for Saturn and Uranium without the free member (10.4).

According to the formula (2e) we have at integer-valued q=2 for n=10: A*(10) =1034.4 (the normalized major semiax). Multiplying it by 0.0372193 AU, we get A(10)=38.5 AU. It is an ideal value of major semiax for tenth planet. We remind for comparison that the major semiaxes of Neptune and Pluto are equal 30.07 and 39.37, respectively. For eleventh planet following this law strictly, a major semiax would be equal 76.6 AU. The formula (6) gives the maximal values of major semiaxes for the prognosis of planets discovering: A(10)=39.37 AU (equal to the value for Pluto) and A(11)=78.3 AU. The formula (7) gives analogical minimum values: A(10)=37.7 AU and A(11)=75.0 AU. According to the formula (8) A(10)= 30.07 AU (naturally coincides with the value for Neptune) and A(11)=58.1 AU.

3. Conclusions

Thereby, in the Solar planetary system we can see three sequences of planets for which the major semiaxes of their orbits are described with three formulas (6), (7) and (8). System (6)-(7) presents all the planets by the best appearance, except for Neptune. For the last the formula (8) is true. These information can be used for the search of tenth and eleventh planets and at the calculations of hydrodynamic models of the planetary system forming.

References

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