THE DETERMINATION OF METEOROIDS’ LIFE TIME UNDER ACTION OF PHOTONS AND PROTONS

E.N. Tikhomirova

Astronomical Observatory, State Pedagogical University
108 Republikanskaya Str., Yaroslavl' 150000 Russian Federation, en_tihomirova@mail.ru

ABSTRACT. The life time of meteor streams under action of photons and protons is estimated. The interval of time after which the particles of the comet 9P(1867 G1)/Tempel 1 (after bombarding the comet during the mission "Deep Impact") reach the Earth is estimated (the meteor stream is observed). The account of the action of photons only and the corpuscular analogue of Poynting-Robertson effect is made at the calculations. According to the results the intervals of time after which the particles reach the Earth can differ by 1.5 times.

Key words: Comets; meteors. Poynting-Robertson effect: life-time

1. Introduction

In works (Wyatt et al., 1950) and (Ryabova, 2005) in semi analytical form the influence of photons and the Solar Wind on motion of meteoroids is taken into account separately. Here we determine meteor shower’s life time analytically in view of simultaneous action of photons and protons.

2. The Fundamental Equation of Motion of the Meteoroids under Action of Photons

The differential equation of motion submitted in the vector form, absolutely black spherical body, isotropic re-radiating a solar energy and moving with velocity \( v \), making an angle \( u \) with a direction of a heliocentric radius - vector \( \vec{r} \) (Radzievskii, 2003) looks like:

\[
\ddot{r} = - \frac{GM'\vec{r}}{r^3} - \frac{2\pi R^2 q r^2}{Mc^2} \nu \cos u \frac{\vec{r}}{r^3} - \frac{\pi R^2 q r^2}{Mc^2} \nu \sin u \vec{e}_t
\]

(1)

\( M' \) is the reduced mass of the Sun connected to mass of the Sun \( M_S \) and mass of (spherical) particle \( M \), by the ratio:

\[
M' = M_S \frac{\pi R^2 q r^2}{GMc}
\]

(2)

The equation (1) is applicable in case \( R > \lambda \). The effect of Poynting - Robertson is characteristic for the particles with radii from 1m up to 1cm.

To apply methods of theory of perturbed motion let’s suppose that in right part of the equation (1) the first term ("photogravitational" acceleration - \( \vec{a}_0 \)) exceeds for more than tens and hundreds times the second and the third ones (\( \vec{a}_1 \) and \( \vec{a}_2 \) - perturbing accelerations).

For a case of small perturbations from the equation (1), after averaging for one orbital evolution of a meteoric particle and the subsequent integration, according to (Wyatt et al., 1950):

\[
\frac{a}{a_0} - \frac{e^{3/5}(1 - e_0^2)}{e_0^{3/5}(1 - e^2)} = 0
\]

(3)

and also a quadrature :

\[
I(e, e_0) = \int_{e_0}^{e} \frac{e^{3/5}}{(1 - e^2)^{3/2}} de
\]

\[
= - \frac{5\pi^2 R^2 q r^2 e^0}{GMc^2 T_0^{1/2} (1 - e_0^2)^2} (t - t_0)
\]

(4)

Let’s assume, that after disintegration of the cometary’s nucleus large fragments stay in the initial cometary’s orbit (its parameters are \( a_0, e_0, i_0 \)). The values of parameters of cometary’s orbit (semimajor axis \( a_c \), eccentricity \( e_c \) and inclination \( i_c \)) vary insignificantly (\( a_c \approx a_0, e_c \approx e_0, i_c \approx i_0 \)) during the intervals of time corresponding the tens and hundreds orbital periods. In this model of the cometary and meteoroidal motion the longitude of the ascending node (\( \Omega_c \)) of the plane of cometary orbit (meteoroid) and the argument of perihelion (\( \omega_c \)) can vary significantly.

We take into account that the bodies of small sizes are under action of light pressure and the Poynting - Robertson effect \( \vec{a} = a(t), e = e(t) \), we also assume \( i = i_0 \).

2.1. The Examples

Let’s estimate the time of life of meteor stream with the help of the integral 4. For example, after the
artificial explosion made July, 4, 2005 during the space mission "Deep Impact" (comet 9P (1867 G1) / Tempel 1), at density of meteoroids' substance $\rho = 1 \text{g/cm}^3$, we shall come to the following estimations of a required interval of time $\tau_{LT}$ at various values of meteoroids’ radii $R$ (see Table 1).

<table>
<thead>
<tr>
<th>Radius of meteoroids</th>
<th>100</th>
<th>10</th>
<th>1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu m$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_{LT} = t - t_0$, (years)</td>
<td>337400</td>
<td>32800</td>
<td>2200</td>
</tr>
</tbody>
</table>

Table 1: The calculation of the life time of meteoroids of various radii, with account of the action of photons; the initial parameters of the meteoroids’ orbit coincide with the parameters of the comet’s orbit (comet 9P (1867 G1) / Tempel 1).

Here $\tau_{LT}$ is the interval of time (since the disintegration of the comet’s nucleus), after which it is possible to observe the meteor stream.

3. The Account of Simultaneous Action of Photons and Protons on the Meteoroids

In (Ryabova, 2005) in semi-analytical form the influence of a solar wind on motion of meteoroids is taken into account. Mean value of velocity of the Solar Wind (in the radial direction) is $w = 400 \text{ km/s}$ (for distances $0.3\text{AU} < r < 10.4\text{AU}$). The concentration of protons $n_p$ in the Solar Wind varies as $n_p = 8.1 (r_{3.5 - 0.5} / r)^2 (400 / w) \text{ cm}^{-3}$. We also use ratios: $U = w - v$, $n_0 / n_p = 0.05$. The action of electrons and heavy ions at meteoroids is not taken into account. The parameter of the model is $\psi$, which takes on values: 1.6 (water ice), 1.4 (magnetite), 1.1 (obsidian).

Summarizing the results of the works (Wyatt et al., 1995) and (Ryabova, 2005), for the averaging equations of motion we shall find the integral of motion 5:

$$ a = \frac{(1 - e^2)}{(1 - e_0^2)} \frac{4\pi \rho^2}{M} U $$

(5)

$$ k = k_w / k_p $$

(6)

$$ k_w = 3.65 \cdot 10^3 \Psi U $$

(7)

$$ k_p = \frac{\pi q_{s - k}^2 r_{s - k} a_0^{3/2}}{\sqrt{GMc^2T_0}} $$

(8)

$a_0$ and $e_0$ are the origin values of semimajor axes and eccentricity of the meteoroid’s orbit, $U$ is the averaging value of $|U|$ at the period of time of the meteoroid’s moving.

Here $k_w$ and $k_p$ are the values proportional to the accelerations of meteoroids, which are caused by the action of protons (of the Solar Wind) and photons, correspondingly; $a_0$ and $e_0$ are the initial values of the semimajor axis and the eccentricity of the meteoroid’s orbit.

Let’s pay attention, that for possible maximal value of $k_w \left( U = 400 \cdot 10^5 \text{cm/s}, \psi = 1.6 \right)$ and possible minimal value of $k_p \left( M' = M, a_0^{3/2} / T_0 = \sqrt{GMc^2} \right)$ their ratio is not greater than 1.5. Therefore it is possible to put:

$$ 0 < k < 1.5 $$

(9)

Let’s find $k$ from (5):

$$ k = \frac{(5 \ln \frac{a(1 - e^2)}{a_0(1 - e_0^2)} - 4 \ln \frac{e}{e_0})}{(\ln \frac{e}{e_0} - 1) \left( \frac{(1 - e^2)}{(1 - e_0^2)} \right)} $$

(10)

Averaging the equation (1) and taking into account the action of the Solar Wind, we shall find the quadrature (11), analogous the quadrature (4):

$$ I(e, e_0) = - \frac{\pi q^2 r_{s - k} a_0^{3/2} R^2}{\sqrt{GMc^2T_0} M} (t - t_0) $$

(11)

3.1. The Examples

The obtained results also can be used for estimation of the life time of meteor stream associated with the comet Tempel 1. At calculations: $e_0 = 0.519$, $a_0 = 3.118 \text{ AU}$, $e = 0.243$, $a = 1.322 \text{ AU}$, where $a$ and $e$ are the semimajor axis and the eccentricity of the meteor stream’s orbit, $a_0$ and $e_0$ are the semimajor axis and the eccentricity of the orbit of Tempel 1.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\tau_{LT} = t - t_0$ (years)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2200</td>
</tr>
<tr>
<td>0.5</td>
<td>1800</td>
</tr>
<tr>
<td>0.918</td>
<td>1562</td>
</tr>
<tr>
<td>1</td>
<td>1522</td>
</tr>
<tr>
<td>1.5</td>
<td>1319</td>
</tr>
</tbody>
</table>

Table 2: The calculation of the life time of meteoroids (radius $R = 1 \mu m$) with account of the action of photons and protons ($k$ varies from 0 up to 1.5); the initial parameters of the meteoroids’ orbit coincide with the parameters of the comet’s orbit (comet 9P (1867 G1) / Tempel 1).

Then, the particles with radius $R = 1 \mu m$ reach the Earth past 1522 years after bombardment of the comet by the "Deep Impact"(if perturbing forces of photons and protons are equal) and a new meteor stream should appear.

The suggested equations can be used for the identification of meteor streams and their parent bodies.
4. Conclusion

According to the calculations particles of the comet 9P (1867 G1)/Tempel 1 reach the orbit of the Earth past 1522 years (after disintegration of the comet’s nucleus) and a new meteor stream should appear. The problem of determination of the meteoroids’ lifetime is not solved finally, so as for the problem of the discovery of meteor streams’ parent bodies. The results of this work approach the solution of indicated problems.

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References