# DE SITTER METRIC WITH MAGNETIC FIELD

M.P. Korkina, V.S. Kazemir

Department of Physic, Dniepropetrovsk National University,
Naukova str, 13, Dniepropetrovsk, 49050, Ukraine, *korkina@ff.dsu.dp.ua, kazemirws@ukr.net*

ABSTRACT. De Sitter model with magnetic field was considered. The exact solution was obtained. The properties of the model in comparison with empty de Sitter model were analyzed.

**Key words**: de Sitter, model; vacuum; magnetic, field; anisotropic, Universe.

## 1. Introduction

There is the considerable interest arose in cosmological models describing magnetic field in the Universe (Шикин, 1966; Vajk and Eltgroth, 1970; Banerjee and Sanyal, 1986; Giovannini, 2000). At the initial stage of evolution of the Universe such magnetic field was much strong and played the important role in expansion of the Universe. According to the present observations (Riess, 2004; Virey J.-M., 2005) the Universe expands with acceleration. Accelerated expansion of the Universe now is explained by presence of cosmological vacuum with a state equation:

$$\varepsilon_\Lambda + p_\Lambda = 0, \tag{1}$$

where $\varepsilon_\Lambda = const$. The vacuum energy density dominates over energy densities of all other matter (Perlmutter S., 1998). De Sitter model describes the Universe with matter with the state equation (1). From Einstein equations we obtain de Sitter metric in the form:

$$ds^2 = (1 - \frac{r^2}{a_\Lambda})dt^2 - \frac{dr^2}{(1 - \frac{r^2}{a_\Lambda})} - r^2 d\sigma^2, \tag{2}$$

where $d\sigma^2 = d\theta^2 + \sin^2\theta d\varphi^2$, $a_\Lambda^2 = \frac{3c4}{8\pi G\varepsilon_\Lambda}$. Let us take for simplicity system of units in which Newton gravitational constant $G = 1$ and light velocity $c = 1$. This metric contains R and T-regions. There is T-solution under $r < a_\Lambda$. Under $r > a_\Lambda$ this solution describes R-region of the manifold. Recently many original papers and reviews have been devoted to the cosmological vacuum (Sahni and Starobinsky, 1999; Carrol M., 2000; Чернин, 2001; Sahni, 2004). In this article we propose to consider de Sitter model with a frozed magnetic field for the early Universe.

## 2. Solution

Let's consider the homogeneous but anisotropic magnetic field which depends on time only and choose an axis along magnetic field. In synchronous coordinate system we consider the spherically symmetric metric in the form:

$$ds^2 = dt^2 - e^{\lambda(t)}dR^2 - r^2(t)d\sigma^2. \tag{3}$$

The nonzero components of Maxwell tensor for the frozed magnetic field are $F_{23} = -F_{32} = H_1$ where $H_1$ is magnetic field strength. Than from Maxwell equations it follows that $\frac{\partial H_1}{\partial t} = 0$ (Коркина, Мартыненко, 1977). Therefore stress-energy tensor of magnetic field can be written in the following form:

$$T^0_{0mag} = T^1_{1mag} = -T^2_{2mag} = T^3_{3mag} = \frac{H_1^2(\theta)}{4\pi r^2 sin^2\theta}. \tag{4}$$

Cosmological vacuum has constant energy density and pressure. For the chosen metric nonzero components of stress-energy tensor are:

$$T^0_0 = \varepsilon_\Lambda, T^1_1 = T^2_2 = T^3_3 = p_\Lambda. \tag{5}$$

We assume the independence of the dust and magnetic field. Then from conservation equation

$$T^\mu_{\nu;\mu} = 0 \tag{6}$$

for magnetic field it follows that:

$$H_1 = qsin\theta, \tag{7}$$

where $q = const$. Than we take the stress-energy tensor as a sum of stress-energy tensors of vacuum and magnetic field.

$$8\pi T^0_0 = 8\pi T^1_1 = \frac{q^2}{r^4} + \frac{3}{a_\Lambda} \; and \; 8\pi T^2_2 = -\frac{q^2}{r^4} + \frac{3}{a_\Lambda}. \tag{8}$$

Then taking (8) into account we obtain Einstein field equations in the form:

$$\frac{\dot{\lambda}\dot{r}}{r} + \frac{\dot{r}^2}{r^2} + \frac{1}{r^2} = \frac{q^2}{r^4} + \frac{3}{a_\Lambda}, \tag{9}$$

$$\frac{2\ddot{r}}{r}+\frac{\dot{r}^2}{r^2}+\frac{1}{r^2}=\frac{q^2}{r^4}+\frac{3}{a_\Lambda}, \quad (10)$$

$$\frac{\dot{\lambda}\dot{r}}{2r}+\frac{\ddot{\lambda}}{2}+\frac{\dot{\lambda}^2}{4}+\frac{\ddot{r}}{r}=-\frac{q^2}{r^4}+\frac{3}{a_\Lambda}, \quad (11)$$

where a dot $\dot{}$ denotes differentiation with respect to $t$. Integrating the equation (9) on time, we obtain:

$$\dot{r}^2=\frac{r^2}{a_\Lambda^2}-1-\frac{q^2}{r^2}+C. \quad (12)$$

Without a magnetic field we must obtain de Sitter solution. We take an integration constant $C=0$. Than integrating (12) under $q=0$ we obtain $r=a_\Lambda ch(t/a_\Lambda)$. The metric coefficient $e^\lambda$ we find from (9):

$$e^\lambda=sh^2(t/a_\Lambda). \quad (13)$$

The obtained metric is the de Sitter solution in synchronous coordinates:

$$ds^2=dt^2-sh^2(t/a_\Lambda)dx^2-a_\Lambda^2ch^2(t/a_\Lambda)d\sigma^2. \quad (14)$$

Integrating the equation (12) for case $q\neq 0$ we obtain:

$$\frac{r^2}{a_\Lambda^2}-\frac{1}{2}+\sqrt{\left(\frac{r^2}{a_\Lambda^2}\right)-\frac{1}{4}-\frac{q^2}{a_\Lambda^2}}=C_1e^{2t/a_\Lambda}, \quad (15)$$

where $C_1$ is an integration constant. As far as the obtained solution (15) must turn into de Sitter one, thus $C_1=\frac{1}{2}$. Finally we have for $r^2(t)$:

$$r^2(t)=a_\Lambda^2ch^2(t/a_\Lambda)+q^2e^{-2t/a_\Lambda}. \quad (16)$$

After integration of (9) we obtain the metric coefficient $e^{\lambda(t)}$ in the form:

$$e^\lambda=\frac{r^2}{a_\Lambda^2}-1-\frac{q^2}{r^2}, \quad (17)$$

Equations (16) and (17) determine de Sitter solution with magnetic field. In expression (16) the magnetic field contribution gives an item $q^2e^{-2t/a_\Lambda}$, therefore influence of a magnetic field on expansion in a radial direction weakens exponentially. Dependences $r(t)$ and $e^{\lambda(t)}$ in comparison with de Sitter metric are shown at figures 1, 2. Under $t\to\infty$ the obtained solution turns into de Sitter solution.

As for as $\dot{r}=0$ corresponds to a minimum $r(t)$ than metric coefficient $e^\lambda=\dot{r}^2$ turn into zero under $r=r_{min}=a_\Lambda\sqrt{\frac{1}{2}+\frac{1}{2}\sqrt{1+\frac{4q^2}{a_\Lambda^2}}}$ . However energy densities of a magnetic field and vacuum are finite, the obtained critical point is not the space-time singularity.

In curvature coordinates obtained metric have the form:

$$ds^2=e^\nu dt^2-e^\lambda dR^2-e^\mu d\sigma^2, \quad (18)$$

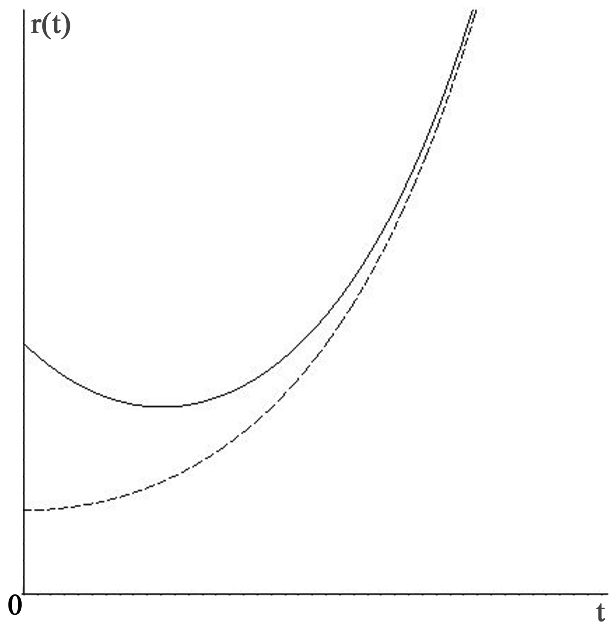


Рис. 1: Dependence $r^2(t)$. Dash curve is for $r^2(t)$ for de Sutter metric, Solid curve is for de Sitter metric with magnetic field.

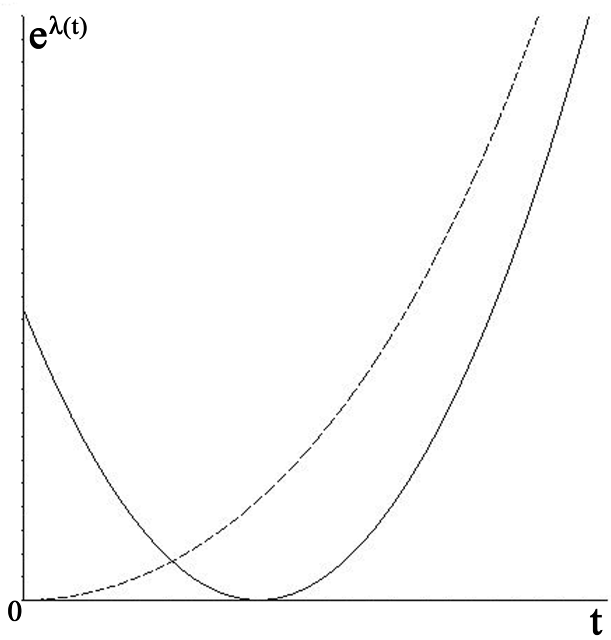


Рис. 2: Dependence $e^\lambda(t)$. Dash curve is for $e^\lambda(t)$ for de Sutter metric, Solid curve is for de Sitter metric with magnetic field.

where $e^{\nu} = 1 - \frac{x^2}{a_\Lambda^2} + \frac{q^2}{x^2}$, $e^{\lambda} = e^{-\nu}$, $e^{\mu} = x^2$. The event horizon between R- and T-regions is determined by expression:

$$e^{\lambda}\dot{\mu}^2 = e^{\nu}\acute{\mu}^2, \qquad (19)$$

where an $\acute{\mu}$ denotes differentiation with respect to $x$. For the obtained solution from (19) we have an equation:

$$1 - \frac{x^2}{a_\Lambda^2} + \frac{q^2}{x^2} = 0. \qquad (20)$$

This equation has one positive solution:

$$x_1 = a_\Lambda \sqrt{\frac{1}{2} + \frac{1}{2}\sqrt{1 + \frac{4q^2}{a_\Lambda^2}}}. \qquad (21)$$

Under $x > x_1$ there is T-solution, under $x < x_1$ there is R-solution. As far as constant $q$ characterizes a magnetic field, than the presence of a magnetic field changes boundary between R- and T-regions of de Sitter metric.

## 3. Conclusion

New solution with cosmological vacuum and magnetic field has been obtained and the properties of the model have been analysed. R- and T-region of the model have been considered. It was shown, that the influence of magnetic field decreases with time and when $t \to \infty$ de Sitter solution take place.

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