SECULAR EVOLUTION OF THE GALAXY

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ABSTRACT. The theory of spiral structure of rotationally supported disk-shaped galaxies has a long history, but is not yet complete. Even though no definitive answer can be given at the present time, the majority of experts in the field is yielded to opinion that the study of the stability of gravity perturbations (e.g., those produced by spontaneous disturbances) in disk galaxies of stars is the first step towards an understanding of the phenomenon. We analyse the reaction between almost aperiodically growing Jeans-unstable gravity perturbations and stars of a rotating and spatially inhomogeneous disk of highly flattened galaxies. A mathematical formalism in the approximation of weak turbulence (a quasi-linearization of the Boltzmann collisionless kinetic equation) is developed, which is a direct analogy with the plasma quasi-linear (weakly nonlinear) formalism. A diffusion equation in configuration space is derived which describes the change in the main body of equilibrium distribution of stars. The distortion in phase space resulting from such a wave-star interaction is studied. The theory, applied to the Solar neighborhood, accounts for the increase in the random stellar velocities with age and the essential radial spread of the Galaxy's disk. We argue that the Sun has migrated from its birth-place at the galactocentric radius $r = 6 - 7$ kpc in the inner part of the Galaxy outwards by $\Delta R_\odot = 2 - 3$ kpc during its lifetime of $t \approx 4.5 \times 10^9$ yr. This $\Delta R_\odot$ is in fair agreement with the estimate of Wielen et al. (1996) $\Delta R_\odot \approx 1.9$ kpc based on a radial galactic gradient in metallicity.

Key words: Galaxies: spiral; stars: individual: Sun.

1. Introduction

The bulk of the total optical mass in the Milky Way and other flat galaxies is in stars. In the spirit of Lin et al. (1969) and Shu (1970), we regard spiral structure in most galaxies of stars as a wave pattern, which does not remain stationary in a frame of reference rotating around the center of the galaxy at a proper speed, excited as a result of the gravitational Jeans-type instability. The instability is set in when the destabilizing effect of the self-gravity in the disk exceeds the combined restoring action of the pressure and Coriolis forces. The wave propagation is a process of rotation as a solid about the center of the galaxy at a fixed phase velocity, despite the general differential rotation of the system. The instability is driven by a strong nonresonant interaction of the gravity fluctuations with the bulk of the particle population, and the dynamics of Jeans perturbations can be characterized as a fluidlike wave-particle interaction. The instability represents the ability of a self-gravitating disk to relax from a nonthermal (or an almost nonthermal) state by collective collisionless processes in much less time than the binary collision time. It is our purpose to extend the investigation by studying the natural nonlinear effects. The problem is formulated in the same way as in plasma kinetic theory (Krall & Trivelpiece 1986).

2. Basic Equations

A thin rotating disk is taken as a model of the flat galaxy in many papers for analysis of the gravity perturbations. Following Morozov (1981), Griv & Peter (1996), and Griv et al. (2000, 2001, 2002), we solve the system of the collisionless Boltzmann equation and the Poisson equation describing the motion of a self-gravitating ensemble of stars in such a system within an accuracy of up two orders of magnitude with respect to small parameters $1/|k_r|r$ and $c_r/r\Omega$ for the radial wavenumber k_r, the dispersion of radial peculiar velocities c_r, and the angular velocity Ω, looking for waves which propagate in a two-dimensional galactic disk. This approximation of an infinitesimally thin disk is a valid approximation if one considers perturbations with a radial wavelength $\lambda = 2\pi/k_r$ that is greater than the typical disk thickness.

In configuration space we introduce cylindrical coordinates r, φ, z, and $0z$ axis directed along the axis of rotation. The projections of the peculiar velocity of a star on the coordinate axis is designated by v_r, v_φ, v_z, respectively. Let us assume that the stars move in the disk plane so that $v_z = 0$. This allows us to use the two-dimensional distribution function $f(r, \varphi, v_r, v_\varphi, t)$ such that $\tilde{f} = f\delta(z)\delta(v_z)$, $f = \int \tilde{f} \mathrm{d}z \mathrm{d}v_z$, and $\int f \mathrm{d}v_r \mathrm{d}v_\varphi = \sigma$, where $\sigma(\mathbf{r}, t)$ is the surface density. In galaxies the function $f(\mathbf{r}, \mathbf{v}, t)$ must satisfy the Boltzmann collisionless equation of continuity

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \frac{\partial f}{\partial \mathbf{r}} - \frac{\partial \Phi}{\partial \mathbf{r}} \cdot \frac{\partial f}{\partial \mathbf{v}} = 0 \,. \qquad (1)$$

In Eq. (1), $\Phi(\mathbf{r}, t)$ is the total gravitational potential (including a dark matter, if it exists at all) determined self-consistently from the Poisson equation

$$\frac{\partial^2 \Phi}{\partial r^2} + \frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \varphi^2} + \frac{\partial^2 \Phi}{\partial z^2} = 4\pi G \sigma \delta(z) \,, \qquad (2)$$

where $\delta(z)$ is the Dirac delta-function with respect to the spatial coordinate z. The Boltzmann and Poisson equations with appropriate boundary conditions give a complete description of the problem for disk modes of collective oscillations. The relationship between the frequency of the oscillations and the wave vector is found by equating the solutions of Eqs. (1) and (2).

Suppose that up to the time $t = 0$ the disk remains in a stationary state, i.e., for $t < 0$

$$f = f_\mathrm{e} \quad \text{and} \quad \Phi = \Phi_\mathrm{e} \,,$$

where f_e and Φ_e are the equilibrium distribution function and the equilibrium gravitation potential, respectively. At $t = 0$ the disk is perturbed in some manner, so that for $t > 0$

$$f = f_\mathrm{e} + f_1 \quad \text{and} \quad \Phi = \Phi_\mathrm{e} + \Phi_1 \,.$$

The quantities f_1 and Φ_1 characterize the deviations, or perturbations, of the distribution function and the field from the corresponding equilibrium values. We are interested in the time dependence of the perturbations, which we will assume are small.

We proceed by applying the procedure of the quasi-linear approach. In the quasi-linear theory, one may follow the standard procedure of linearization by writing $f = f_0(r, \mathbf{v}, \mu t) + f_1(\mathbf{r}, \mathbf{v}, t)$ and $\Phi = \Phi_0(r, \mu t) + \Phi_1(\mathbf{r}, t)$ with $|f_1/f_0| \ll 1$ and $|\Phi_1/\Phi_0| \ll 1$ for all $\mathbf{r}$ and t. The functions f_1 and Φ_1 are functions oscillating rapidly in space and time, while the functions f_0 and Φ_0 describe the slowly developing "background" against which small perturbations develop; $\mu \ll 1$; $f_0(t = 0) \equiv f_\mathrm{e}$ and $\Phi_0(t = 0) \equiv \Phi_\mathrm{e}$. The distribution f_0 continues to distort as long as the distribution is unstable. Linearizing Eq. (1) and separating fast and slow varying variables one obtains the equation for the fast developing distribution function

$$\frac{\mathrm{d}f_1}{\mathrm{d}t} = \frac{\partial \Phi_1}{\partial r}\frac{\partial f_0}{\partial v_r} + \frac{1}{r}\frac{\partial \Phi_1}{\partial \varphi}\frac{\partial f_0}{\partial v_\varphi} \,, \qquad (3)$$

where $\mathrm{d}/\mathrm{d}t$ means total derivative along the star orbit and f_0 is a given equilibrium distribution function determined from the following equation (see Eq. (1)):

$$\mathbf{v} \cdot \frac{\partial f_0}{\partial \mathbf{r}} - \frac{\partial \Phi_0}{\partial \mathbf{r}} \cdot \frac{\partial f_0}{\partial \mathbf{v}} = 0 \,.$$

The equation for the slow part of the distribution function is

$$\frac{\partial f_0}{\partial \mu t} = \left\langle \frac{\partial \Phi_1}{\partial r}\frac{\partial f_1}{\partial v_r} + \frac{1}{r}\frac{\partial \Phi_1}{\partial \varphi}\frac{\partial f_1}{\partial v_\varphi} \right\rangle , \qquad (4)$$

where $\langle \ldots \rangle$ denotes a time average over the fast oscillations,

$$f_0 = \langle f \rangle = \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} f \mathrm{d}t \quad \text{and} \quad \langle f_1 \rangle = \langle \Phi_1 \rangle = 0 \,,$$

and $\mathcal{T}$ is the characteristic time of the quasi-linear relaxation, i.e., the time during which the oscillations influence the equilibrium state.

It is useful to define a generalized entropy function (Krall & Trivelpiece 1986, p. 364)

$$S_\mathrm{gen} = - \int f_0 \ln f_0 \mathrm{d}\mathbf{r} \mathrm{d}\mathbf{v} \,. \qquad (5)$$

Contrary to the case of the true entropy $S_\mathrm{true} = -\int f \ln f \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}$, which is constant in the absence of collisions, with this definition the "entropy" is not constant, and can be used to measure the increase of disorder (e.g., temperature) of the system. It is, incidentally, just such a *coarse-grained* single-particle distribution function f_0 that is determined in practice, particularly in stellar dynamics, where it is determined in a fairly large region of phase space.

3. Perturbation

In the familiar Wentzel-Kramers-Brillouin (WKB) approximation in Eqs. (3) and (4), assuming the weakly inhomogeneous disk, each perturbation of equilibrium parameters is selected in the form of a plane wave (in the circular rotating frame)

$$X_1(\mathbf{r}, \mathbf{v}, t) = \sum_{\mathbf{k}} X_\mathbf{k} e^{ik_r r + im\varphi - i\omega_{*,\mathbf{k}} t} + \text{c.c.} \,, \qquad (6)$$

In Eq. (6), $X_\mathbf{k}$ is an amplitude that is a constant in space and time, m is the nonnegative azimuthal mode number (= number of spiral arms), $\omega_{*,\mathbf{k}} = \omega_\mathbf{k} - m\Omega$ is the Doppler-shifted wavefrequency, $r \sim R$, $|k_r| r \gg 1$, $|\mathrm{d} \ln k_r / \mathrm{d} \ln r| \ll 1$, suffixes $\mathbf{k}$ denote the $\mathbf{k}$th Fourier component, and "c.c." means the complex conjugate. Evidently, in Eq. (6) X_1 is a periodic function of φ, and hence m must be an integer. The criteria for stability differ for each m, and must be determined by a detailed analysis. The assumption that $X_\mathbf{k}$ has a weak spatial dependence corresponds to the quasiclassical approximation in quantum mechanics and to the approximation of geometrical optics in the propagation of light in an inhomogeneous medium. It is convenient to write the eigenfrequency $\omega_{*,\mathbf{k}}$ in a form of the sum of the real part $\Re\omega_{*,\mathbf{k}}$ and the imaginary part $i\Im\omega_{*,\mathbf{k}}$. The imaginary part of $\omega_{*,\mathbf{k}}$ corresponds to a growth ($\Im\omega_{*,\mathbf{k}} > 0$) or decay ($\Im\omega_{*,\mathbf{k}} < 0$) of the components in time, $f_1, \Phi_1 \propto \exp(\Im\omega_{*,\mathbf{k}} t)$, and the real part to a rotation with angular velocity

$$\Omega_{\rm p} = \frac{\Re\omega_{*,\mathbf{k}}}{m} \,. \tag{7}$$

Thus, when $\Im\omega_{*,\mathbf{k}} > 0$, the medium transfers its energy to the growing wave and oscillation buildup occurs. A galaxy is considered as a superposition of different oscillation modes. A disturbance in the disk will grow until it is limited by some nonlinear effect.

In the linear theory, one can select one of the Fourier harmonics:

$$X_1(\mathbf{r}, \mathbf{v}, t) = X_{\mathbf{k}} e^{ik_r r + im\varphi - i\omega_* t} + \text{c.c.} \,. \tag{8}$$

The solution in such a form represents a spiral wave with m arms (or a ring, $m = 0$) whose shape Φ_1 in the plane is determined by the relation

$$k_r(r - r_0) = -m(\varphi - \varphi_0) \,.$$

With φ increasing in the rotation direction, we have $k_r > 0$ for trailing spiral patters, which are the most frequently observed among spiral galaxies. A change of the sign of k_r corresponds to changing the sense of winding of the spirals, i.e., leading ones. With $m = 0$, we have the density waves in the form of concentric rings that propagate away from the center when $k_r > 0$ or toward the center when $k_r < 0$.

Paralleling the analysis leading to Eq. (13) of Griv & Peter (1996), from Eqs. (3) and (8) it is straightforward to show that the perturbed distribution function is

$$f_1 = -\Phi_1 \sum_{l=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} \frac{\exp[i(n-l)(\phi - \zeta)]}{\omega_* - l\kappa} \times J_l(\chi) J_n(\chi) \left[l\kappa \frac{\partial f_0}{\partial(v_\perp^2/2)} + \frac{2m\Omega}{r\kappa^2} \frac{\partial f_0}{\partial r} \right] , \tag{9}$$

where $J_l(\chi)$ is the Bessel function of the first kind of order l with its argument $\chi = k_* v_\perp/\kappa$, k_* is the effective wavenumber, $v_\perp^2 = v_r^2 + (2\Omega/\kappa)^2 v_\varphi^2$, and $\kappa \sim \Omega$ is the epicyclic frequency. In Eq. (9) the denominators vanish when $\omega_* - l\kappa = 0$. This occurs near corotation and other resonances. The above resonances take place where the frequency with which a star crosses the peaks and dips of the wave potential, $|\omega - m\Omega|$, is either zero (i.e., the star is always in phase with the wave) or equal to the oscillation frequency of the star about a circular orbit. The corotation resonance occurs at a radius where $l = 0$ in Eq. (9). The Lindblad resonances occur at radii where the field $(\partial/\partial\mathbf{r})\Phi_1$ resonates approximately with the harmonics $l = -1$ (inner resonance) and $l = 1$ (outer resonance) of the epicyclic (radial) frequency of equilibrium oscillations of stars $\kappa(r)$. Clearly, the location of these most important resonances depends on the rotation curve and the spiral pattern speed $\Omega_{\rm p}$; the higher the m value, the closer in radius the resonances are located (Lin et al. 1969). Resonances are places where linearized equations describing the motion of particles do not apply. In the vicinity of the resonances it is necessary to use nonlinear equations, or to include terms of higher orders into the approximate form of the equations (Griv et al. 2000). In this work only the main part of the disk is considered, which lies sufficiently far from the resonances: in all equations $\Re\omega_* - l\kappa \neq 0$ (Lin et al. 1969; Griv et al. 2002). The distortion of the wave packet due to the disk inhomogeneity is included through the second term in the brackets on the right-hand side in Eq. (9).

4. Diffusion Equations

We anticipate that the fluidlike Jeans-unstable oscillations must influence the distribution function of the main part of stars in such a way as to hinder the wave excitation, i.e., to increase the peculiar velocity spread ultimately at the expense of circular motion and gravitational energy. This is because the Jeans instability, being essentially a gravitational one, tends to be stabilized by chaotic motions of stars. Simultaneously, unstable perturbations effectively transfer angular momentum outward to the outer parts of the system, as mass flows both inward to the growing center mass concentration and outward to the outer regions through gravitational torques. Eventually the disk evolves toward a quasi-stationary Jeans-stable distribution.

Let us suppose that the nonlinear effects in galactic disks are small, so that the linear theory is a good first approximation. Next, we substitute the solution (9) into Eq. (4) and average the latter over time. After averaging over $\phi = \arctan(2\Omega/\kappa)v_\varphi/v_r$, the equation for the slow part of the distribution function is obtained:

$$\frac{\partial f_0}{\partial \mu t} \approx \sum_{\mathbf{k}} \mathcal{E}_{\mathbf{k}} \frac{\partial}{\partial v_\perp} \frac{k_*\kappa}{v_\perp \chi} \frac{J_1^2(\chi)}{\kappa^2} \Im\omega_* \frac{\partial f_0}{\partial v_\perp} + \sum_{\mathbf{k}} \mathcal{E}_{\mathbf{k}} \frac{k_*\kappa}{v_\perp^2 \chi} \frac{J_1^2(\chi)}{\kappa^2} \Im\omega_* \frac{\partial f_0}{\partial v_\perp} + \sum_{\mathbf{k}} \mathcal{E}_{\mathbf{k}} \frac{2\Omega}{\kappa^2} \frac{m}{r} \frac{\partial}{\partial r} \frac{2\Omega}{\kappa^2} \frac{m}{r} \frac{J_1^2(\chi)}{\kappa^2} \Im\omega_* \frac{\partial f_0}{\partial r} \,, \tag{10}$$

where $\mathcal{E}_{\mathbf{k}} = 8\pi|\Phi_{\mathbf{k}}|^2 \exp(2\Im\omega_* t)$, and we considered both $\omega_* = \Re\omega_* + i\Im\omega_*$ and the complex conjugate frequency of excited waves $\omega_*^* = \Re\omega_* - i\Im\omega_*$.

As usual in the quasi-linear theory, in order to close the system one must engage an equation for $\mathcal{E}_{\mathbf{k}}$. Averaging over the fast oscillations, we obtain

$$(\partial/\partial t)\mathcal{E}_{\mathbf{k}} = 2\Im\omega_* \mathcal{E}_{\mathbf{k}} \,. \tag{11}$$

Equations (10) and (11) form the closed system of quasi-linear equations for Jeans oscillations of the rotating inhomogeneous disk of stars, and describe a *diffusion* in both velocity and coordinate space.

4.1 Velocity Diffusion

Growing density waves (spiral arms) excite random motions parallel to the equatorial plane. According to Eq. (10), the heating efficiency of unstable density wave features depends on their spatial and temporal form. Let us evaluate the law for the age-velocity dispersion rate $\langle v_\perp^2 \rangle(t)$ and the heating $\Delta v_\perp$ for a realistic model of the disk of the Galaxy in the Solar neighborhood ($\langle v_\perp^2 \rangle$ is the averaged squared velocity dispersion in the $z = 0$ equatorial plane). In accordance with the theory as developed above, we consider the fastest growing mode with $\Im\omega_* \approx \Omega$. According observations, in the Solar vicinity $\mathcal{E}_{\mathbf{k}}/\Phi_0^2 \approx 10^{-2}$ (Lin et al. 1969), $\Phi_0 \approx 0.5 r_\odot^2 \Omega^2$, $r_\odot \approx 8.5$ kpc, $c_r(t = 0) \approx 10$ km s^{-1}, and $\kappa \approx 1.5\Omega$. From Eq. (10), one easily obtains

$$\langle v_\perp^2 \rangle \propto t \,, \tag{12}$$

and $\Delta v_\perp = 20 - 30$ km s^{-1}, where $t = 10^9$ yr (Griv et al. 2001, 2002). These values of $\langle v_\perp^2 \rangle$ and $\Delta v_\perp$ are in agreement with both estimates based on the observed stellar velocities (Wielen 1977; Grivnev & Fridman 1990; Dehnen & Binney 1998) and N-body simulations (e.g., Liverts et al. 2003). Thus already in the first 3-4 galactic revolutions, in say about 10^9 yr, the stellar populations see their epicyclic energy vary by a factor of ten. de Souza & Teixeira (2007) have detected such a velocity variation in 10^9 yr by considering the kinematic segregation of nearby disk stars.

4.2 Migration of the Sun's Guiding Center

As we have mentioned above, the amplification of spiral gravitational instabilities produces not only heating but also redistribution of matter in the disk. In this connection, there is considerable scatter in the metallicities of stars that have a common guiding center and age (Edvardsson et al. 1993). On the other hand, it is widely believed that all interstellar material at a given time and radius has a common metallicity. The paradox can be resolved if one assumes that these stars were born at different radii and then migrated to its present locations as a result of a series of uncorrelated scattering events (Wielen et al. 1996).

The migration may be explained naturally by "collisions" of stars with the Jeans-unstable density waves. Let us estimate the scale of radial migration $\Delta R_\odot$ of the Sun's guiding center. According to observations, we adopt the ratio $\mathcal{E}_{\mathbf{k}}/\Phi_0^2 \approx 10^{-2}$, $m \approx 1$, $r_\odot = 8.5$ kpc, and $\Im\omega_* \approx \Omega$. Then from Eq. (10) we obtain $\Delta R_\odot = 2 - 3$ kpc (Griv et al. 2002). This $\Delta R_\odot$ is in fair agreement with the estimate of Wielen et al. (1996) $\Delta R_\odot \approx 1.9$ kpc based on a radial galactic gradient in metallicity. We conclude that the Sun has migrated from its birth-place at $r \approx 7$ kpc in the inner part of the Galaxy outwards by approximately 2 kpc during its lifetime of $t \approx 4.5 \times 10^9$ yr.

5. Summary

With passage of time as the perturbation energy increases, the initial distribution spreads ($f_0(v_\perp^2)$ becomes less peaked) and the temperature grows (Eq. (12)). In other words, the relaxation of the stars takes place. Formally, the relaxation corresponds to the presence of the collision integral on the right-hand side of Eq. (10). In addition, the radial spread of the disk is increased. The diffusion in configuration space is due entirely to the growth of the Jeans-unstable modes ($\Im\omega_* > 0$) in a self-gravitating collisionless system subject to a time-dependent potential.

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