ON THE STABLE SPHERICALLY-SYMMETRIC CHARGED DUST CONFIGURATIONS IN GENERAL RELATIVITY

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ABSTRACT. The radial motion of a self-graviting charged dust and stability condition of the static charged dust spheres are considered. The stability is possible for the bound states of the weakly charged layer with abnormal charge with respect to the active mass.

Key words: charged dust; classification of motions; stability conditions.

1. Introduction

The collapse of a spherically-symmetric charged dust cloud is the important problem of a General Relativity. In the papers of Vickers P.A. (1973), Markov M.A. and Frolov V.P., (1970,1972), Bailyn M. and Eimerl D. (1972), Ivanenko D.D., Krechet V.G. and Lapchinski V.G. (1973) the solution of the Einstein-Maxwell equations is reduced to the first integrals. The exact solutions for charged dust spheres are obtained in the works of Hamoui A. (1969), Bekenstein J.D. (1971), Bailyn M., Eimerl D. (1972), Shikin I.S. (1974), Khlestkov Yu.A.(1975), Pavlov N.V. (1976), Ori A. (1990). The main goal for researches is to find the conditions under which the gravitational collapse of the charged mediums is impossible. Let us note the problem of shell crossing, which arises here. In works of Ori A. (1991) and Goncalves S.M. (2001) it is shown that shell crossing is inevitable in the gravitational collapse of weakly charged dust spheres. The important problem is also to find stability conditions of charged dust spheres. The equilibrium of a charged dust sphere with extremal charge distribution was considered by Bonnor W.B (1965). The similar problem was considered by Bonnor W.B (1993), Gladush V.D. and Galadgyi M.V. (2007) for charged particles in the Reissner-Nordström field. In this paper we introduce the classification of charged spherically-symmetric configurations, and further we find the stability conditions for them.

2. The equations of motion for spherical layers of the charged dust

The space-time metric has the form

$$ds^2 = \gamma_{ab} dx^a dx^b - R^2 d\sigma^2 \,, \qquad (1)$$

where $d\sigma^2 = d\theta^2 + \sin^2\theta d\alpha^2$, $\gamma_{ab} = \gamma_{ab}(x^a)$ and $R = R(x^a)$, x^a - time-radial coordinates $(a, b = 0, 1)$.

For dynamics description of a charged dust sphere we shall consider small region without not shell crossing. The evolution of a spherical layers of Lagrangian radius r $(dr/ds = 0)$, which bounds of the sphere of the total mass $M_{tot}(r)$ and charge $Q(r)$, can be described by the equation (see, for example, Ori A. (1991)).

$$\begin{aligned} \left(\rho c^2 \frac{dR}{ds}\right)^2 &= -U \equiv \varepsilon_{tot}^2 - \rho^2 c^4 + \\ &+ (\kappa\rho^2 c^2 M_{tot} - \rho_e \varepsilon_{tot} Q)\frac{2}{R} - (\kappa\rho^2 - \rho_e^2)\frac{Q^2}{R^2} \,. \end{aligned} \qquad (2)$$

Here $U = U(R, M_{tot}, Q)$ is an "effective velocity potential", ρ and ρ_e are the dust and charge densities, ε_{tot} is the energy density. In this case the following relations take place:

$$\alpha(r) = \frac{\rho_e}{\rho c^2} = \frac{dQ(r)}{c^2 d\mathcal{M}(r)} \,, \quad \mathcal{H}(r) = \frac{\varepsilon_{tot}}{\rho c^2} = \frac{dM_{tot}(r)}{d\mathcal{M}(r)} \,,$$

$$\varepsilon_u = \frac{\varepsilon_{tot}}{\rho c^2} - \frac{\rho_e}{\rho c^2}\frac{Q(r)}{R} = \mathcal{H}(r) - \alpha(r)\frac{Q(r)}{R} \,,$$

where $\mathcal{M}(r)$ is a total rest-mass of a dust.

3. Classification of charged dust spherically-symmetric configurations

The character of evolution of spheres is determined by the motion of layers

The regions of the admissible motions of the layers are defined by the inequality $U(R) \le 0$, the equality $U = 0$ gives turning points R_m. The object of our classification is a sphere with its boundary layer. The potential U here is not suitable, as it depends on parameter of classification – the total energy of the sphere $\mathcal{E}_{tot} = M_{tot} c^2$. From the equation (2) it follows that $M_{tot} \ge U_M$. We shall accept this condition as a basis of classification. Here

$$\begin{aligned} U_M = \frac{1}{2\kappa\rho^2 c^2} \big(&(\rho^2 c^4 - \varepsilon_{tot}^2) R + \\ &+ 2\rho_e \varepsilon_{tot} Q + \left(\kappa\rho^2 - \rho_e^2\right) \frac{Q^2}{R} \Big) \end{aligned} \qquad (3)$$

is the "effective mass potential". This potential depends both on global magnitudes – the charge Q and radius R of the sphere, and on local ones — the densities $\rho\,,\rho_e$ and ε_{tot}. Solutions of the equation $M_{tot} = U_M$ define the turning points R_m of the layer. The asymptotics of the mass potential define the character of the layers motions. Under $R \to 0$ we have the following cases:
1.1. $U_M \to +\infty, \quad \kappa\rho^2 > \rho_e^2$ – the case for weakly charged layer;
1.2. $U_M \to \rho_e\varepsilon_{tot}Q/\kappa\rho^2, \kappa\rho^2 = \rho_e^2$ – the case for the layer with an extremal density of charge;
1.3. $U_M \to -\infty, \kappa\rho^2 < \rho_e^2$ – the case for the layer with an abnormal density of charge. These conditions define the type of behaviour of a layer depending on its local electrical characteristics. On the other hand, under $R \to \infty$ we have:
2.1. $U_M \to +\infty, \text{if } \rho^2c^4 > \varepsilon_{tot}^2$ – the case for the bound states of a dust;
2.2. $U_M \to \rho_e\varepsilon_{tot}Q/\kappa\rho^2, \text{if} \rho^2c^4 = \varepsilon_{tot}^2$ – the case for the critical density of a dust;
2.3. $U_M \to -\infty, \text{if } \rho^2c^4 < \varepsilon_{tot}^2$ – the case for the unbound states of a dust. These conditions define the type of behaviour of a layer depending on its local energy characteristics. Thus there are nine basic types of behaviour of the mass potential U_M which are determined by local characteristics of a sphere. Besides, the sphere of Lagrangian radius is characterized by integral magnitudes – $M_{tot}(r)$ and $Q(r)$. For the given total charge $Q(r)$ the value of a total mass $M_{tot}(r)$ determines three types of the sphere:
3.1. $M_{tot}\sqrt{\kappa} > |Q|$ – the weakly charged sphere;
3.2. $M_{tot}\sqrt{\kappa} = |Q|$ – the sphere with an extremal charge;
3.3 $M_{tot}\sqrt{\kappa} < |Q|$ – the sphere with an abnormal charge.

As a result the charged layers have 27 variants of motion. In the dimensionless coordinates $\{V_M = U_M\sqrt{\kappa}/Q,\ x = c^2R/Q\sqrt{\kappa}\}$, these cases are illustrated at Fig. 1-9. The regions of the admissible motions are determined by segments of an axis x, for which the line $V_M = M_Q$ lays above the curve $V_M = V_M(x)$. The turning points x_m (R_m) can be found as abscissae of intersection points of the curve $V_M = V_M(x)$ and of the line $V_M = M_Q$. The segment of the dashed line $V_M = 1$ corresponds to the motion of a layer in the field of the sphere with the extremal charge. The segments of the dotted lines with $V_M > 1$ and $V_M < 1$ correspond to the motions of the layers in a field of the weakly and abnormally charged spheres, accordingly.

4. The stability conditions for spherically-symmetric configurations of the charged dust

The stable static state of the layer is possible for bound states of the weakly charged layer (fig. 3), when $\kappa\rho^2 > \rho_e^2\,,\ \rho^2c^4 > \varepsilon_{tot}^2$. If the layer is at the bottom of the potential well the conditions of the stationarity

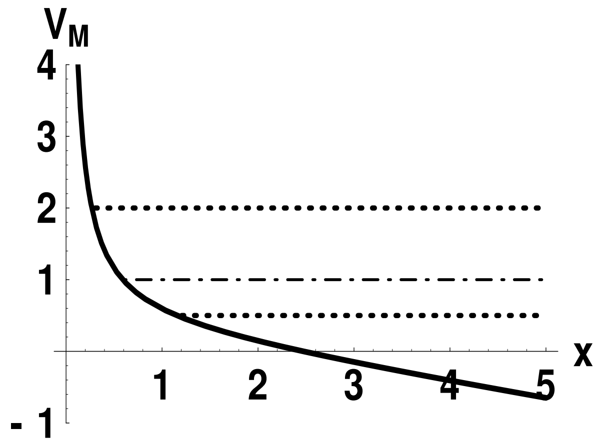

Figure 1: The unbound states of the weakly charged layer: $\kappa\rho^2 > \rho_e^2\,,\ \rho^2c^4 < \varepsilon_{tot}^2$.

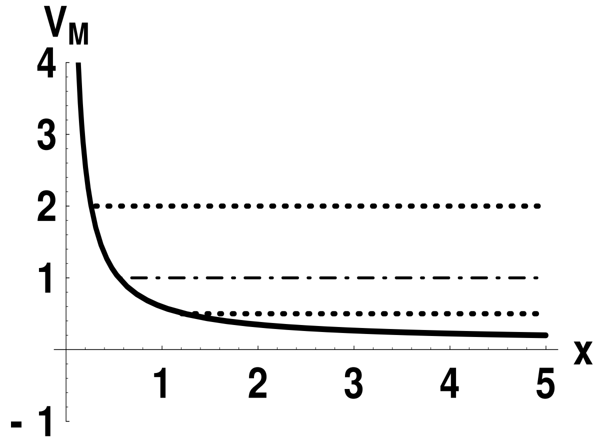

Figure 2: The weakly charged layer with the critical density of the dust: $\kappa\rho^2 > \rho_e^2\,,\ \rho^2c^4 = \varepsilon_{tot}^2$.

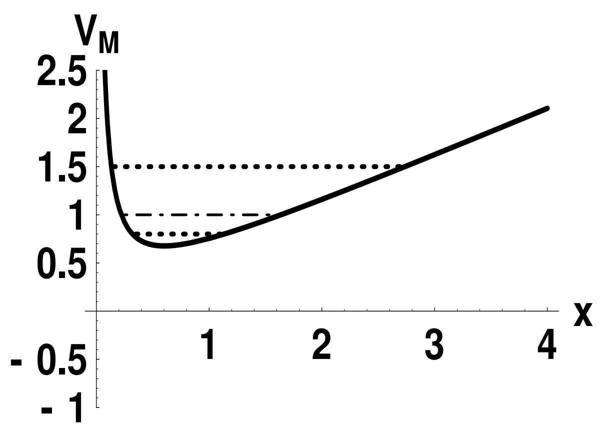

Figure 3: The bound states of the weakly charged layer: $\kappa\rho^2 > \rho_e^2\,,\ \rho^2c^4 > \varepsilon_{tot}^2$.

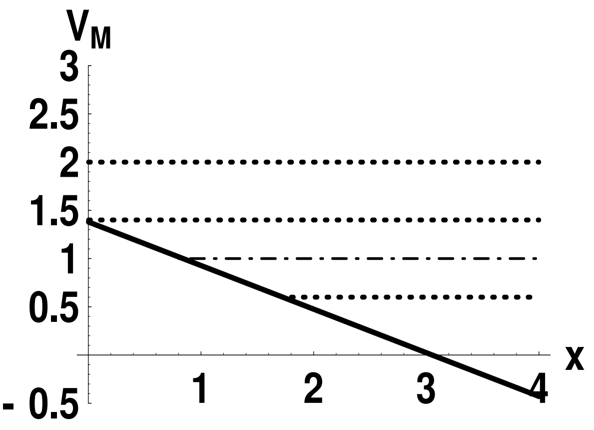

Figure 4: The unbound states of the layer with extremal density of the charge: $\kappa\rho^2 = \rho_e^2\,,\ \rho^2 c^4 < \varepsilon_{tot}^2$.

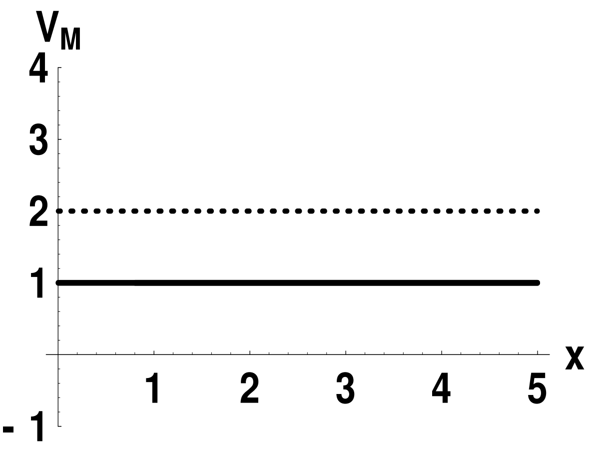

Figure 5: The layer with extremal density of the charge and critical density of the dust: $\kappa\rho^2 = \rho_e^2\,,\ \rho^2 c^4 = \varepsilon_{tot}^2$.

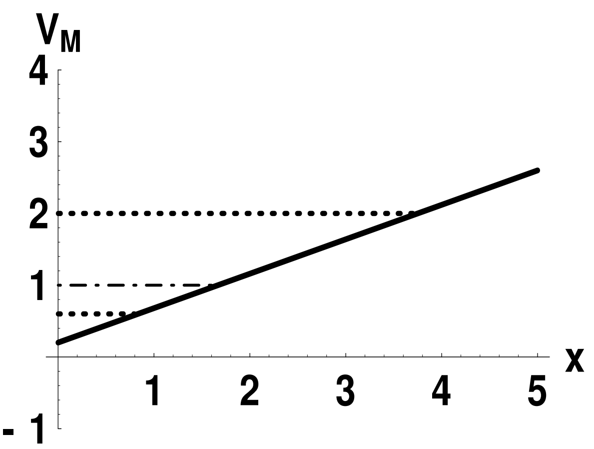

Figure 6: The bound states of the layer with extremal charge density: $\kappa\rho^2 = \rho_e^2\,,\ \rho^2 c^4 > \varepsilon_{tot}^2$.

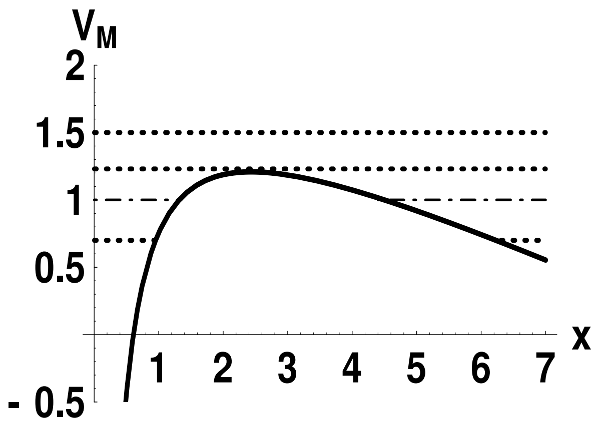

Figure 7: The unbound states of the layer with the abnormal density of charge: $\kappa\rho^2 < \rho_e^2\,,\ \rho^2 c^4 < \varepsilon_{tot}^2$.

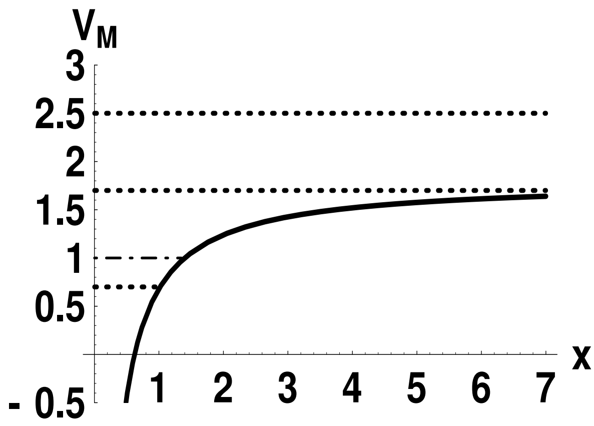

Figure 8: The layer with the abnormal density of the charge and critical density of the dust: $\kappa\rho^2 < \rho_e^2$, $\rho^2 c^4 = \varepsilon_{tot}^2$.

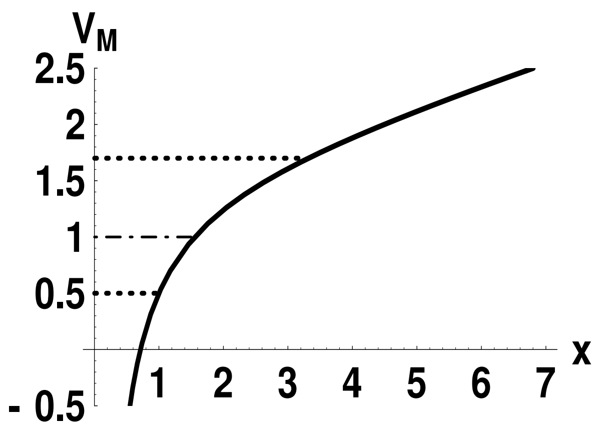

Figure 9: The bound states of the layer with the abnormal charge density: $\kappa\rho^2 < \rho_e^2\,,\ \rho^2 c^4 > \varepsilon_{tot}^2$.

and equilibrium are satisfied:

$$\left(\frac{dR}{ds}\right)^2 = \left(\mathcal{H}(r) - \alpha(r)\frac{Q(r)}{R}\right)^2 - \\ - 1 + \frac{2\kappa M_{tot}(r)}{c^2 R} - \frac{\kappa Q^2(r)}{c^4 R^2} = 0, \quad (4)$$

$$\frac{d^2 R}{ds^2} = \left(\mathcal{H}(r) - \frac{\alpha(r)Q(r)}{R}\right)\frac{\alpha(r)Q(r)}{R^2} - \\ - \frac{\kappa M_{tot}(r)}{c^2 R^2} + \frac{\kappa Q^2(r)}{c^4 R^3} = 0. \quad (5)$$

Hence we obtain the following equilibrium condition for the layers

$$\left(\frac{\kappa M_{tot}(r)}{Q(r)} - \alpha c^2 \mathcal{H}(r)\right)^2 = \\ = \left(1 - \mathcal{H}^2(r)\right)\left(\kappa - \alpha^2 c^4\right). \quad (6)$$

Thus for radius of an equilibrium layer we have

$$R = \frac{\kappa M_{tot}(d\mathcal{M})^2 - QdQdM_{tot}}{c^2((d\mathcal{M})^2 - (dM_{tot})^2)}, \quad (7)$$

that corresponds to the minimum of potential M_{tot}. Using integral magnitudes we obtain necessary and sufficient conditions of a stable equilibrium of the charged dust configuration:

$$|d\mathcal{M}| > |dM_{tot}|, \quad \sqrt{\kappa}|d\mathcal{M}| > |dQ|, \quad (8)$$

$$\kappa\Phi(dM_{tot}, dQ) \equiv \kappa Q^2 (dM_{tot})^2 - \\ -2\kappa Q M_{tot} dQ dM_{tot} + Q^2(dQ)^2 = \\ = \kappa\left(Q^2 - \kappa M_{tot}^2\right)(d\mathcal{M})^2. \quad (9)$$

Than it follows that

$$\kappa\left(QdM_{tot} - M_{tot}dQ\right)^2 = \\ = \left(Q^2 - \kappa M_{tot}^2\right)\left(\kappa(d\mathcal{M})^2 - (dQ)^2\right) > 0. \quad (10)$$

In virtue of (8) one can formulate the following theorem: the stable static states are possible only for the bound states of the weakly charged layer with the abnormal charge Q with respect to the active mass M_{tot}:

$$|d\mathcal{M}| > |dM_{tot}|, \ \sqrt{\kappa}|d\mathcal{M}| > |dQ|, \ Q^2 > \kappa M_{tot}^2. \quad (11)$$

It follows from here, that for the stable static sphere of the charged dust the quadratic form $\Phi(dM_{tot}, dQ)$ in (10) is positively defined $\Phi(dM_{tot}, dQ) > 0$.

From (10) we obtain one more relation for stable static states of the charged dust sphere:

$$\frac{\sqrt{\kappa}\, dM_{tot}}{dQ} = \frac{\sqrt{\kappa}\, M_{tot}}{Q} \pm \\ \pm \sqrt{1 - \frac{\kappa M_{tot}^2}{Q^2}}\sqrt{\frac{\kappa(d\mathcal{M})^2}{(dQ)^2} - 1}. \quad (12)$$

This equation is invariant with respect to scale transformation

$$M_{tot}(r) = aM'_{tot}(r), \ Q(r) = aQ'(r), \ \mathcal{M}(r) = a\mathcal{M}'(r).$$

Here $R = aR'$. Thus we have a scaling law. If we multiply the distribution functions of the charge, total and proper mass by factor of a the new configuration remains stable, and its radius will grow by the same factor a.

As an example, let us consider the particle-like configuration with parameters of an electron (mass $m_e = 9,109 \cdot 10^{-28}\,\mathbf{g}$, charge $e = 4,803 \cdot 10^{-10}\,\mathbf{cm}^{3/2}\,\mathbf{g}^{1/2}\,\mathbf{s}^{-1}$) and with the geometrodynamical charge $q_{m_e} = \sqrt{\kappa} m_e = 23,53 \cdot 10^{-32} \mathbf{cm}^{3/2}\mathbf{g}^{1/2}\mathbf{s}^{-1}$. Since $e/q_{me} = 2 \cdot 10^{21} \gg 1$, we deal here with the abnormally charge object. Let us consider now point object with parameters of an electron and the exterior gravitational field of the Reissner-Nordström. In virtue of inequality $e > \sqrt{\kappa} m_e$ a naked singularity takes place. It contradicts to the cosmic censorship hypothesis of Penrose (1969), according to which the singularity should be hidden by horizon. Thus it follows, that the electron can not be the point object. If we neglect an intrinsic moment, the simplest classical model of such object can be particle-like spherical configuration of the charged dust with the total mass $M_{tot} = m_e$ and charge $Q = e$, which satisfies the indicated stability conditions.

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