OPEN QUESTIONS ON CEPHEIDS AND THEIR PERIOD-LUMINOSITY RELATIONSHIP

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ABSTRACT. Physical properties and pulsational behaviour of classical Cepheids can be studied in unprecedented details based on recent photometric and spectroscopic data. In spite of important new results, problems related to Cepheids still exist, such as metallicity dependence of their various properties, their mass loss and circumstellar environment, excitation of low-amplitude pulsation modes in such variables, etc.

Key words: Cepheids; Stars: binary; Stars: mass-loss; Stars: oscillations

1. Introduction

Classical Cepheids are primary distance indicators via the famous period-luminosity (*P*-*L*) relationship, as well as key objects in studying stellar structure and evolution. The main aim of the recent Cepheid studies is to make progress in understanding the evolution of intermediate mass stars and to increase the precision of the calibration of the *P*-*L* relationship, a fundamental tool in establishing the cosmic distance scale.

Major problems used to exist in explaining the behavior of Cepheids but the progress in the observational techniques and theoretical modelling of the internal structure, pulsation, and evolution of intermediate mass stars has normally led to the solution of these problems. Such long-standing problem was, e.g., the treatment of Cepheids as a homogeneous type of pulsators.

Classification into classical and Type II Cepheids solved the the controversy in their distances. Another conundrum was how to explain the characteristic phase lag between the brightness and radial velocity variations of Cepheids.

A more recent problem of the discrepancy between the evolutionary and pulsation masses of Cepheids was largely mitigated by the new opacity values in the 1990s but a partial discrepancy still exists.

Unlike these historical problems, some more recent Cepheid related issues have remained unsolved. This paper summarizes the current picture and examples are listed for the still existing problems selected from the forefront of Cepheid research.

Classical Cepheids are supergiant stars located in the Cepheid instability strip of the Hertzsprung-Russell diagram. Their main characteristics is a periodic radial oscillation of their atmospheric layers. This regularity gives rise to various relationships between the pulsation period and other physical properties (e.g., luminosity, radius, etc.) of Cepheids. The very accurate recent observational data, however, revealed that the pulsation may not be strictly regular. Deviations from perfect regularity and the individual behaviour in the pulsation of some Cepheids are topics of on-going investigations.

2. The *P*-*L* relationship

The *P*-*L* relationship has a century-long history with a permanent goal to improve its calibration. Assuming that the luminosity is a linear function of the decimal logarithm of the pulsation period, determination of two parameters is necessary for calibrating this relationship. Microlensing projects (e.g., OGLE, MACHO, and EROS) have resulted in long series of homogeneous photometric data on several thousand Cepheids in both Magellanic Clouds, therefore the slope of the *P*-*L* relationship can be conveniently determined by the Magellanic Cepheids. The zero point of the relationship is usually determined by properly selected Galactic Cepheids of known distance/luminosity (see David Turner's review in this volume).

The ridge-line linear fit is, however, an approximation. In fact, Cepheids widely scatter along the fitted line. In addition to the observational errors, there are quite a few effects that contribute to the width of the *P*-*L* plot:

– intrinsic colour of the Cepheid,
– pulsation mode,
– crossing mode,
– binarity,
– metallicity,
– helium content,
– non-linearity of the relationship,
– other effects (magnetic field, overshooting, blending, etc.).

It can be pointed out that the effective temperature of the Cepheid (corresponding to the location of the star within the instability strip) is an important parameter for the *P*-*L* relationship. Therefore, it is more appropriate to refer to *P*-*L*-colour relationship where colour means the intrinsic (i.e. dereddened) colour. For extragalactic Cepheids, the foreground (Galactic) and internal reddenings are superimposed. The effect of the average interstellar absorption is usually taken into account by calculating the Wesenheit function (Freedman & Madore 2010). The *P*-*L* plot for the Wesenheit magnitudes has smaller dispersion but its width is still non-negligible.

Magellanic Cepheids clearly show existence of separate *P*-*L* relationships for each mode. In the Magellanic Clouds, Cepheids pulsating in the fundamental mode and first overtone are ubiquitous, but single-mode Cepheids pulsating in the second overtone have also been identified (Udalski et al. 1999; Soszyński et al. 2008). In general, overtone pulsation can be readily identified in external galaxies because the luminosity of such Cepheids corresponds to the fundamental period but the actual pulsation period is shorter (the period ratio is known from double-mode Cepheids and pulsation models). In our Galaxy, however, a straightforward way for determining the pulsation mode is the Fourier decomposition of the light curve. The separation of various pulsation modes in the plots of Fourier amplitudes/phases vs. pulsation period is also demonstrated for Magellanic Cepheids (Soszyński et al. 2008b). The photometric amplitude itself is not a good indicator of the pulsation mode: although fundamental pulsators usually have a large amplitude, but companion stars decrease the observable amplitude, moreover large amplitude first overtone Cepheids are also known in the Magellanic Clouds. The most reliable indicator of the pulsation mode is the Fourier phase lag between the radial velocity and brightness variations (Szabó et al. 2007). A drawback to the application of this method is the lack of sufficiently precise radial velocity phase curves for a large percentage of Cepheids.

The *P*-*L* plot for a given pulsation mode still has a finite width. One of the causes of this dispersion is that intermediate mass stars cross the instability strip 3 (or for larger masses 5) times, and the excited pulsation period differs for each crossing. The pulsation period itself varies during crossing the instability strip, and the actual crossing mode can be inferred from the sign of the secular period change (increasing period during the 1st and 3rd crossings, decreasing period during the 2nd crossing) and the abundance of the heavy elements in the atmosphere of the given Cepheid (Turner et al. 2006).

The incidence of binarity among classical Cepheids exceeds 50% (Szabados 2003). Unresolved companions falsify the derived luminosity. When calibrating the *P*-*L* relationship using Galactic Cepheids, presence of companion(s) has to be taken into account on an individual basis. In the of case external galaxies, unresolved optical companions and blending also increase the scatter in the *P*-*L* plot.

The calibration of the Hubble constant, H_0, and the cosmic distance scale via extragalactic Cepheids was one of the Key Projects for the Hubble Space Telescope. The original intention to reach a 5% accuracy in calibrating the *P*-*L* relationship, however, could not be fulfilled mainly because of the uncertainty in the effect of metallicity on the luminosity of Cepheids. In spite of considerable theoretical and observational effort, studies of metallicity dependence of the *P*-*L* relationship for Cepheids have been still inconclusive (Marconi 2009, Romaniello et al. 2008).

Stellar luminosity also depends on the helium content of the Cepheid, especially in the case of long pulsation periods (Marconi et al. 2005). The helium abundance of Cepheids, however, can be hardly inferred from spectroscopic data.

The influence of the magnetic field and convection (including convective overshooting) on the luminosity needs thorough theoretical studies.

The effects of the interstellar extinction, binarity, and metallicity are much smaller in the infrared than in optical region. In addition, the photometric amplitude of Cepheids decreases towards longer wavelengths, so a reliable mean brightness can be determined from a few randomly obtained observations in the infrared. In view of these advantages, recently the near-infrared is the preferred region for photometry of Cepheids.

3. Other effects of metallicity

The chemical composition of the pulsating atmosphere has an influence on the Cepheid oscillations whose study was long delayed by absence of reliable spectroscopic abundance data. In the last decade, however, dedicated spectroscopic studies of Cepheids carried out by Andrievsky and his coworkers (Andrievsky et al. 2002a,b,c; 2004; 2005; Kovtyukh et al. 2005a,b) opened the possibility to detailed investigations of the effect of metallicity on pulsational properties of Cepheids.

On the one hand, the well known wavelength dependence of photometric amplitudes (decreasing amplitude towards longer wavelengths) has a metallicity dependence (Szabados & Klagyivik 2011). Similarly, the shape of the light curves is indicative of the iron content of the Cepheid via properly selected Fourier parameters (Szabados et al. 2011). On the other hand, the amplitude of the pulsational radial velocity variations itself is a function of the atmospheric iron content, especially for the short-period Cepheids (Szabados & Klagyivik 2011).

The period ratio of the excited modes in double-mode Cepheids is also sensitive to the atmospheric metal abundance. Theoretical calculations by Buchler & Szabó (2007) are in accord with the empirical formula obtained from Galactic beat Cepheids by Sziládi et al. (2007).

Recent theoretical calculations predict a larger mass loss rate via pulsation driven outflow for lower metallicity Cepheids, especially for shorter pulsation periods (Neilson & Lester 2009).

4. Problems related to the pulsation period

4.1. Period changes

Knowledge of the exact value of the actual pulsation period is essential for Baade-Wesselink type analyses to avoid phase mismatch (simultaneous V and K band photometry and radial velocity measurements seldom exist). Period changes have to be monitored, in some cases even permanently followed because of various reasons. Secular period variation may be indicative of stellar evolution. Although period noise has been superimposed on monotonous period changes during crossing the instability strip, evolutionary changes are apparent for a large number of Cepheids (see e.g., Fig. 1). Another kind of period changes is the cyclic variation in the pulsation period due to the light-time effect occurring in binary systems involving a Cepheid component.

Photoelectric and CCD photometric data obtained in the last decades are instrumental for revealing subtle period changes. Figure 2 shows an intriguing example: a phase jump ('glitch') superimposed on the monotonous period increase of Polaris (Turner et al. 2005). This phenomenon might be caused by a trigger from the physical companion of the Cepheid. Continuous time series data recently obtained by photometric space probes allow one to investigate minute cycle-to-cycle changes in the pulsation period.

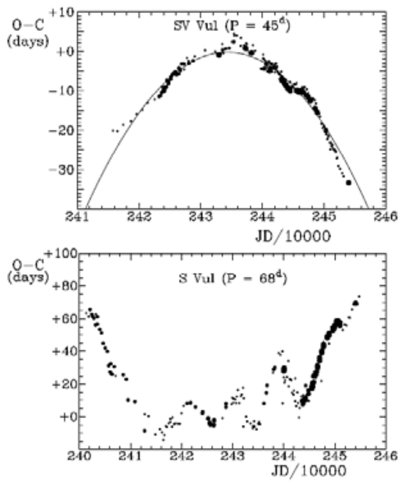

Figure 1: $O - C$ variations of long-period Cepheids SV Vul (upper panel) and S Vul (lower panel) with the size of individual residuals proportional to the weight assigned to the value (Turner et al. 2009).

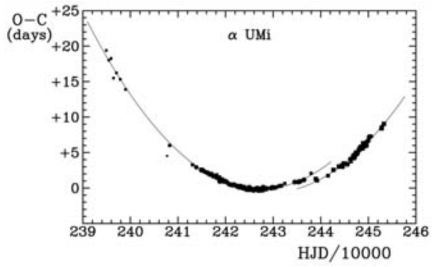

Figure 2: $O - C$ diagram of Polaris, a short period Cepheid. A sudden phase jump is clearly visible. (Turner et al. 2005).

4.2. Double and multiple periodicity

Majority of Cepheids are monoperiodic pulsators. Existence of additional periodicities is, however, important because the values of simultaneously excited frequencies bear information on the stellar interior. As an example, the dependence of the period ratio of the excited modes on the atmospheric iron abundance is mentioned (see Sect. 3).

Microlensing projects resulted in discovering more than 200 double-mode Cepheids in the LMC and over 150 such pulsators in the SMC, pulsating simultaneously in the fundamental mode and the first overtone, or in the first and second overtones. The number of the known beat Cepheids in our Galaxy (36) shows a deficiency as compared with their siblings in the Magellanic Clouds, in spite of the fact that recently Khruslov (2009a,b,c; 2010) discovered nine Galactic double-mode Cepheids analysing the ASAS photometric data base.

Double-mode Cepheids, especially those in external galaxies, are extremely important objects because the period ratio of their excited pulsation modes provides information on the metallicity of individual Cepheids without any spectroscopic observation (see Sect. 3).

Intriguing discoveries on additional periodicities were published by Moskalik & Kołaczkowski (2009). They revealed slightly excited non-radial modes in Magellanic Cepheids. These appear either very close to the primary pulsation frequency or at a much shorter period (period ratio ∼0.60-0.64). Such non-radial mode has been found in LMC Cepheids pulsating in the first radial overtone and some double-mode pulsators. No such oscillations have been observed in single-mode Cepheids performing fundamental-mode radial oscillations. Moreover, they also detected a Blazhko-type periodic modulation in 19% of double-mode Cepheids pulsating in the first two radial overtones. Both modes are modulated with a common period, always exceeding 700 d. Variations of the two amplitudes are anticorrelated. They propose that the Blazhko phenomenon in Cepheids can be explained by a resonant interaction of one of the radial modes with a non-radial oscillation mode. Triple-mode Cepheids have also been found in the LMC by Moskalik & Kołaczkowski (2009) and Soszyński et al. (2008a). These triple-mode Cepheids pulsate simultaneously either in the fundamental mode and the first two radial overtones or in the first three overtones.

5. Binarity among Cepheids

In view of the fact that at least 50% of the classical Cepheids are not solitary pulsators (Szabados 2003) (see also Fig. 3), binarity cannot be neglected when investigating Cepheid related relationships. Unresolved companions falsify the apparent brightness and the observable colour of the Cepheid, and they cause an adverse effect when determining the colour excess and deriving stellar radius by applying any version of the Baade-Wesselink method. It is, therefore, important to reveal and characterize companions to Cepheids, and correct for their effects when deriving the physical properties of individual Cepheids.

Eclipsing binaries with a Cepheid component are especially important because they provide an independent method of luminosity determination, thus facilitating the calibration of the *P-L* relationship. Cepheids in eclipsing systems have been found in both Magellanic Clouds: 4 in the LMC (Soszyński et al. 2008b), 2 in the SMC (Soszyński et al. 2010).

Companions affect astrometric measurements, as well. Though it was impossible to determine the astrometric orbit of the nearest Cepheids having a physical companion from the positional data of the Hipparcos astrometric satellite (with an angular resolution of about 1 milliarcsecond), the derived trigonometric parallaxes indicate the adverse effect of the companions: each (physically unrealistic) *negative parallax value belongs to binaries* (see Fig. 4). The ESA's next astrometric satellite, Gaia, to be launched in late 2012 will be sufficiently sensitive to resolve these binary orbits with its angular resolution of several microarcseconds.

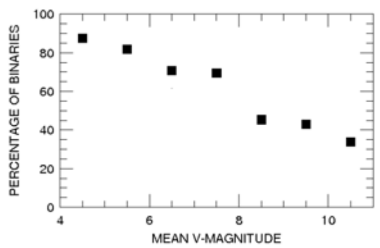

Figure 3: Frequency of known binaries (or multiple systems) among Cepheids. The diagram shows the presence of a strong observational selection effect: it is increasingly difficult to reveal companions of fainter Cepheids.

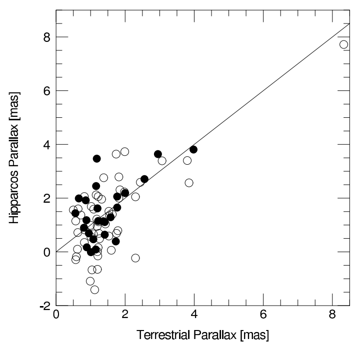

Figure 4: Hipparcos vs. 'ground-based' parallax of the nearest Cepheids. Open circles denote Cepheids with known companion(s). Cepheids without known companions are marked with black dots. Note that all negative parallax values belong to binaries.

An on-line data base on Galactic binary (and multiple) Cepheids maintained by the author is available at the URL: http://www.konkoly.hu/CEP/intro.html (Szabados 2003).

6. Mass loss and circumstellar environment

The increasing importance of the infrared spectral region in Cepheid studies is further exemplified by interferometric studies of Cepheids in the near- and mid-infrared. A surprizing result of these interferometric measurements is the discovery of circumstellar envelope around bright Cepheids. Extended envelopes surround α UMi and δ Cep (Mérand et al. 2006), Y Oph (Mérand et al. 2007), ℓ Car and RS Pup (Kervella et al. 2009). The presence of these envelopes so close to the Cepheids indicates on-going mass loss which offers an explanation for the still existing mass discrepancy (masses derived from the pulsational behaviour of Cepheids are smaller by about 15% than mass values deduced from stellar evolution – Keller 2008). However, the actual mass loss from Cepheids is unknown.

7. Determination of physical properties

The radial pulsation is instrumental in the process of determining physical properties of Cepheids. The application of the widely used Baade-Wesselink method for the determination of stellar radius and luminosity is, however, encumbered by the uncertainty in the value of the projection factor. This conversion factor between radial and pulsational velocities depends on the limb darkening effects, not properly known yet, the atmospheric velocity gradients, and the dynamical structure of the Cepheid atmosphere (Nardetto et al. 2006). In addition, when carrying out Baade-Wesselink analysis, a special care must be taken for correcting the effects of the companions on a star-by-star basis.

Availability of precise spectroscopic abundance values facilitates studies of possible metallicity dependence of various Cepheid properties (see also Sect. 3).

Furthermore, interferometric observations in the near-infrared are indispensable for determining the rate of mass loss and its effects on the evolutionary properties of Cepheids. Strangely enough, the remaining mass discrepancy depends on the stellar mass (i.e., the pulsation period of Cepheids): it vanishes at higher masses. Keller (2008) pointed out that mild internal mixing in the main-sequence progenitor of the Cepheid would be sufficient to account for this mass discrepancy.

8. Ultra-low amplitude Cepheids

Recent ground- and space based photometric equipments facilitate discovery of extremely low amplitude pulsation. Among Cepheids, such oscillations may correspond to high-overtone radial modes that are trapped in the stellar atmosphere. These acoustic surface modes ('strange' modes) have been found in classical Cepheid models (Buchler et al. 1997). The pulsation in such modes can be linearly unstable to the left of the fundamental and first overtone blue edges and causes luminosity variations in the milli-magnitude range.

An observational confirmation of this prediction is the discovery of two low-amplitude variables in one MACHO field of the LMC whose periods are 5-6 times smaller than those of fundamental mode Cepheids of equal apparent magnitude (Buchler et al. 2005). These objects are thought to be Cepheids undergoing pulsations in a surface mode. They also found 7 ultra-low amplitude Cepheids (seven of them having a Fourier amplitude less than 0.006 mag). Further ultra-low amplitude

Cepheids have been revealed in the LMC by Soszyński et al. (2008b) and Buchler et al. (2009). These stars may be just entering or about leaving the Cepheid instability strip because theoretical calculations predict extremely low amplitude pulsation near either border of the instability region (Buchler & Kolláth 2002). Further low amplitude and strange Cepheid candidates have been found in the photometric data base of CoRoT satellite (Szabó et al. 2009).

9. Extragalactic Cepheids

Thanks to the HST Key Program on the calibration of the Hubble constant, H_0, and the cosmic distance scale, Cepheid variables have been discovered in numerous galaxies. Moreover, projects aimed at revealing extragalactic Cepheids (and then calibrating the P-L relationship) have been carried out with ground based large and medium size telescopes from the 1990s. As a result, ∼12000 Cepheids are now known in 81 galaxies. Almost 40 per cent of these Cepheids are beyond the Magellanic Clouds (including ∼400 known Cepheids in galaxies belonging to the Virgo Cluster). The largest sample of Cepheids is known in the SMC (∼4600), the second largest one in the LMC (∼3400). The number of known classical Cepheids does not reach a thousand in each M31, M33, and our own Galaxy.

The period distribution of the known Cepheids is different for various galaxies. In addition to intrinsic differences (caused by e.g., the metallicity differences), an observational selection effect also contributes to this variety: in more remote galaxies only long-period Cepheids can be discovered yet.

10. New aspects of the P-L relationship

The important question whether the P-L relationship is universal has been unanswered yet. An interesting new aspect of this important relationship was revealed by Bird et al. (2009). Based on 18 Cepheids with periods exceeding 80 days, they pointed out that ultra-long period Cepheids have a relatively shallow P-L relationship, thus being more suitable standard candles than shorter period classical Cepheids. Another advantage of such long period Cepheids is their high luminosity which makes them potential stellar distance indicators to galaxies up to at least 100 Mpc. The small sample studied by Bird et al., however, involved Cepheids in only 6 metal-poor galaxies. It would be important to repeat the investigation of the luminous end of the P-L relationship based on an extended sample involving Cepheids in metal-rich galaxies, as well.

There is growing evidence that the P-L relationship is better approximated with two linear sections, instead of a single line (Sandage et al. 2004; Kanbur & Ngeow 2004). Recent theoretical models also confirm nonlinearity of the relationship (Marconi 2009). Although the break is usually assumed to be at the period of 10 days, Klagyivik & Szabados (2009) pointed out that the dichotomy occurs at the limiting period of 10.49 days. Pulsation models, however, indicate that the nonlinearity of the P-L relationship vanishes toward longer wavelengths (Marconi 2009).

11. Future tasks

Continuation of photometric observations (not only in infrared bands) are necessary to solve the problems of universality, dichotomy and fine structure of the P-L relationship. To investigate metallicity dependence of the Cepheid pulsation, accurate spectroscopic abundance determinations are necessary for Galactic as well as extragalactic Cepheids (within the Local Group). Interferometry of bright Cepheids and radial velocity measurements of faint Cepheids (including those in the Magellanic Clouds) are to be performed for revealing binarity and carrying out BW type analysis. Detailed studies of peculiar Cepheids (SU Crucis, V473 Lyrae, RS Pup, etc.) and beat (double and triple mode) Cepheids will shed light on some aspects of Cepheid pulsation to be clarified.

Ongoing space photometric studies (WIRE, CoRoT, Kepler) will certainly reveal unexpected phenomena in the behaviour of Cepheids. The Gaia astrometric satellite will result in discovery of thousands of new Galactic Cepheids, as well as determination of the astrometric orbit for binary systems involving a Cepheid component.

In addition to obtaining a three-dimensional map and kinematics of Galactic Cepheids with Gaia, the determination of cluster membership of Cepheids is also foreseen. The most important impact of the Gaia data will be undoubtedly a very accurate calibration of the *P*-*L* relationship taking into account metallicity effects.

Acknowledgements. This research was supported by the ESA and the Hungarian Space Office via the PECS programme (contract No. C98090). The author is indebted to Dr. M. Kun for her remarks on the manuscript.

References

Andrievsky, S. M., Kovtyukh, V. V., Luck, R. E., et al.: 2002a, *A&A*, **381**, 32.

Andrievsky, S. M., Bersier, D., Kovtyukh, V. V., et al.: 2002b, *A&A*, **384**, 140.

Andrievsky, S. M., Kovtyukh, V. V., Luck, R. E., et al.: 2002c, *A&A*, **392**, 491.

Andrievsky, S. M., Luck, R. E., Martin, P., & Lépine, J. R. D.: 2004, *A&A*, **413**, 159.

Andrievsky, S. M., Luck, R. E., & Kovtyukh, V. V.: 2005, *AJ*, **130**, 1880.

Bird, J. C., Stanek, K. Z., & Prieto, J. L.: 2009, *ApJ*, **695**, 874.

Buchler, J. R. & Kolláth, R.: 2002,*ApJ*, **573**, 324.

Buchler, J. R. & Szabó, R.: 2007, *ApJ*, **660**, 723.

Buchler, J. R., Yecko, P. A., & Kolláth, Z.: 1997, *A&A*, **326** 669.

Buchler, J. R., Wood, P. R., Keller, S., & Soszyński, I.: 2005, *ApJ*, **631**, 151.

Buchler, J. R., Wood, P. R., & Soszyński, I.: 2009, *ApJ*, **698**, 944.

Freedman, W. L. & Madore, B. F.: 2010, *ARA&A*, **48**, 673.

Freedman, W. L., Madore, B. F., Gibson, B. K., et al.: 2001, *ApJ*, **553**, 47.

Kanbur, S. M. & Ngeow, C.-C.: 2004, *MNRAS*, **350**, 962.

Keller, S. C.: 2008, *ApJ*, **677**, 483.

Kervella, P., Mérand, A., & Gallenne, A.: 2009, *A&A*, **498**, 425

Klagyivik, P. & Szabados, L.: 2009, *A&A*, **504**, 959.

Kovtyukh, V. V., Andrievsky, S. M., Belik, S. I., & Luck, R. E.: 2005a, *AJ*, **129**, 433.

Kovtyukh, V. V., Wallerstein, G., & Andrievsky, S. M.: 2005b, *PASP*, **117**, 1173.

Khruslov, A. V.: 2009a, *PZP*, **9**, 14.

Khruslov, A. V.: 2009b, *PZP*, **9**, 17.

Khruslov, A. V.: 2009c, *PZP*, **9**, 31.

Khruslov, A. V.: 2010, *PZP*, **10**, 16.

Marconi, M.: 2009, *Mem. SAIt*, **80**, 141.

Marconi, M., Musella, I., & Fiorentino, G.: 2005, *ApJ*, **632**, 590.

Mérand, A., Kervella, P., Coudé du Foresto, V., et al.: 2006, *A&A*, **453**, 155

Mérand, A., Aufdenberg, J. P., Kervella, P., et al.: 2007, *ApJ*, **664**, 1093

Moskalik, P. & Kołaczkowski, Z.: 2009, *MNRAS*, **394**, 1649.

Nardetto, N., Fokin, A., Mourard, D., & Matthias P.: 2006, *A&A*, **454**, 327.

Neilson, H. R. & Lester, J. B.: 2009, *ApJ*, **690**, 1829.

Romaniello, M., Primas, F., Mottini, M., et al.: 2008, *A&A*, **488**, 731.

Sandage, A., Tammann, G. A., & Reindl, B.: 2004, *A&A*, **424**, 43.

Soszyński, I., Poleski, R., Udalski, A., et al.: 2008a, *Acta Astron.*, **58**, 153.

Soszyński, I., Poleski, R., Udalski, A., et al.: 2008b, *Acta Astron.*, **58**, 163.

Soszyński, I., Poleski, R., Udalski, A., et al.: 2010, *Acta Astron.*, **60**, 17.

Szabados, L.: 2003, *IBVS*, **5394**.

Szabados, L. & Klagyivik, P.: 2011, in preparation

Szabados, L., Klagyivik, P., & Kiss, Z. T.: 2011, in preparation

Szabó, R., Buchler, J. R., & Bartee, J.: 2007, *ApJ*, **667**, 1150.

Szabó, R., Kolláth, Z., Molnár, L., et al.: 2009, *AIPC*, **1170**, 102.

Sziládi, K., Vinkó, J., Poretti, E., et al.: 2007, *A&A*, **473**, 579.

Turner D. G., Savoy, J., Derrah, J., et al.: 2005, *PASP*, **117**, 207.

Turner D. G., Abdel-Sabour, M., & Berdnikov, L. N.: 2006, *PASP*, **118**, 410.

Turner D. G., Majaess D. J., Lane D. J., et al.: 2009, *AIPC*, **1170**, 108.

Udalski, A., Soszyński, I., Szymański, M., et al.: 1999, *Acta Astron.*, **49**, 45.