THE NON-RELATIVISTIC LIMIT OF THE RANDALL-SUNDRUM BRANE WORLD MODEL: SOLUTIONS AND APPLICATIONS

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ABSTRACT. The non-relativistic limit of the Randall-Sundrum model with a single brane is investigated. Both exact and approximate expressions for the potential of a delta-shaped material source are used, some geometric configurations of the sources are considered. Formulae for the potential energy and the force of the gravitational interaction are derived, including the most important case of two balls. The constraints on the model's parameter are obtained.

Nowadays the brane world models are becoming increasingly popular. Despite the four-dimensional nature of the directly observed physical world, extra dimensions of the space-time can be macroscopic and even noncompact. Then the four-dimensionality of our world is achieved by localizing matter in multidimensional space-time on its four-dimensional submanifolds called branes. There is an assumption that, unlike the gravitational field that freely lives and propagates in the multidimensional volume, ordinary fields of matter are localized on branes and, at the fundamental level, are four-dimensional and not multidimensional objects. The multidimensional gravitational field becomes effectively four-dimensional in the low-energy region despite the macroscopic and even infinite length of extra dimensions [1].

The most famous and popular contemporary model of the world on the brane is the Randall-Sundrum model [4, 5]. It describes the five-dimensional gravitational field with the cosmological constant Lambda, interacting with the four-dimensional brane. Tension Sigma is characteristic of the brane. From the viewpoint of a four-dimensional observer the brane's tension can be viewed a four-dimensional cosmological constant. The brane is a time-like plane and multidimensional space-time is Z_2 symmetric about it. As the brane is a delta-shaped distribution of matter and tension from the standpoint of the five-dimensional space-time, the solution of the corresponding multidimensional Einstein equations is not smooth: normal to the brane derivatives of the metric coefficients undergo a jump on the brane. Israel's matching conditions are met on the brane [3]. Here is the metrics of this model:

$$ds^2 = \exp\left(-\frac{2|\xi|}{l}\right)\eta_{\mu\nu}dx^{\mu}dx^{\nu} - d\xi^2 .$$

The gravitational field potential created by a single particle of mass m at rest is expressed in terms of the zero and massive modes.

$$\varphi(r) = -\frac{G_5 m}{r}\varphi_0^2(l) - \frac{G_5 m}{r}\int_0^{\infty} d\tilde{m}\,\varphi_{\tilde{m}}^2(l)\exp(-\tilde{m}r),$$

$$\varphi_0(z) = \left(\frac{1}{l}\right)^{1/2}\left(\frac{l}{z}\right)^{3/2},$$

$$\varphi_{\tilde{m}}(z) = \left(\frac{\tilde{m}z}{2}\right)^{1/2}\frac{Y_1(\tilde{m}l)J_2(\tilde{m}z) - J_1(\tilde{m}l)Y_2(\tilde{m}z)}{\left(J_1^2(\tilde{m}l) + Y_1^2(\tilde{m}l)\right)^{1/2}}.$$

The zero mode is concentrated in the vicinity of the brane, and massive modes in the form of oscillating stationary waves go to infinity along the fifth coordinate [1, 6]. The asymptotic behaviour of the gravitational field potential can be seen on the Fig.1.

The smooth curve describes the exact solution for the gravitational potential and the dashed curves describe its asymptotics.

The potential of the gravitational field induced by a single particle of mass m at rest can be expressed as a product of the Newtonian gravitational potential and a function f.

$$\varphi(r) = -\frac{G_N m}{r}\cdot f(r),$$

$$f(r) = 1 + \underbrace{\frac{l^2}{2}\int_0^{\infty} d\tilde{m}\tilde{m}\left[\frac{Y_1(\tilde{m}l)J_2(\tilde{m}l) - J_1(\tilde{m}l)Y_2(\tilde{m}l)}{\left(J_1^2(\tilde{m}l) + Y_1^2(\tilde{m}l)\right)^{1/2}}\right]^2 e^{-\tilde{m}r}}_{\text{exact correction}}$$

$$\text{or } f(r) = 1 + \underbrace{\frac{\alpha}{r^2}}_{\text{approximate correction}} .$$

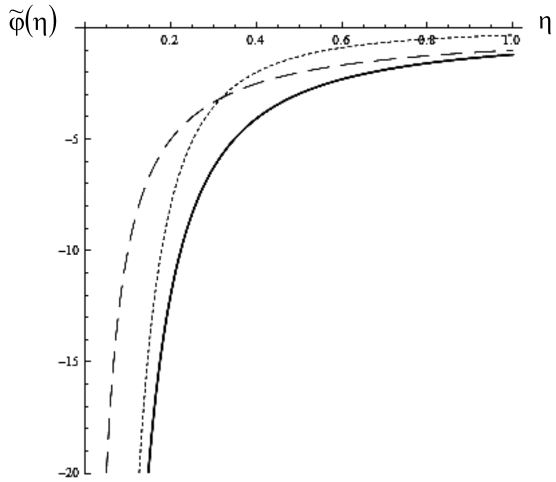

Fig.1. Gravitational field potential and its asymptotic behaviour

We introduce a new parameter Alpha that is related with the Randall-Sundrum model parameter l by the following ratio: $\alpha = \frac{l^2}{2}$.

We use the above-mentioned exact correction to describe the potential as well as the approximate correction. Within this approach we determine the analytic expressions for some values. Particularly we define the sphere field potential, the ball layer field potential, ball field potential, accelerations of a test body in these fields. In addition to gravitational potentials we also define the potential energy and the absolute value of the force of the gravitational interaction of two ball layers and two balls . For the expressions taken with the approximate correction their asymptotic behaviour is being considered.

But the results for the absolute value of the force of the gravitational interaction of two balls are the most interesting and challenging.

$$F(r) = \frac{\gamma m m'}{r^2}\left(1 + \delta_F\right),$$

where in the case of the approximate correction the relational force correction reads

$$\delta_F = -\frac{9\alpha}{8R^3R'^3}\left\{ \ln\frac{r^2-(R'+R)^2}{r^2-(R'-R)^2}\left(-\frac{1}{4}r^4+\frac{1}{2}r^2\left(R'^2+R^2\right)-\frac{1}{4}\left(R'^2-R^2\right)^2\right) - \right.$$
$$\left. -r^2R'R + R'^3R + R'R^3 \right\}.$$

The expression takes the approximate form $F(r) = \frac{G_N m m'}{r^2}\left(1 + \frac{3\alpha}{r^2}\right)$ up to the order $1\!\!/_{r^4}$ when $r \gg R, R'$.

In the case of the exact correction

$$\delta_F = \frac{9}{R^3R'^3}\cdot\frac{l^2}{2}\int_0^\infty d\tilde{m}\left[\frac{Y_1(\tilde{m}l)J_2(\tilde{m}l) - J_1(\tilde{m}l)Y_2(\tilde{m}l)}{\left(J_1^2(\tilde{m}l)+Y_1^2(\tilde{m}l)\right)^{1/2}}\right]^2 \times$$
$$\times\frac{1}{\tilde{m}^5}e^{-\tilde{m}r}\left(1+\tilde{m}r\right)\left[\tilde{m}R\cosh(\tilde{m}R) - \sinh(\tilde{m}R)\right]\times\left[\tilde{m}R'\cosh(\tilde{m}R') - \sinh(\tilde{m}R')\right].$$

The smooth curve describes the exact solution for the relational force correction and the dasheded curve describes the approximate solution. In the Tables 1, 2 there are some characteristic values we can get using the formulas for the interaction force calculated in this paper. For example, the relational force correction assumes the value about 1% when the model parameter is one fifth of the millimeter.

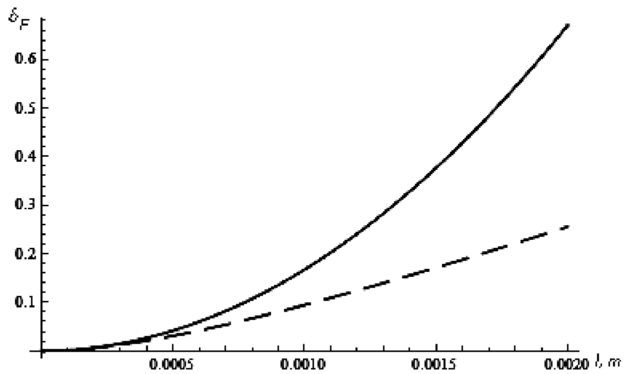

Fig. 2. The relational correction to the force of the gravitational interaction

Table 1. The model parameter and both the exact and the approximate relational force corrections

l, model parameter (m)	δ_F, exact correction	δ_F, approx. correction
10^{-3}	$9{,}46366\cdot10^{-2}$	$1{,}67664\cdot10^{-1}$
10^{-4}	$1{,}61438\cdot10^{-3}$	$1{,}67664\cdot10^{-3}$
10^{-5}	$1{,}67528\cdot10^{-5}$	$1{,}67664\cdot10^{-5}$
10^{-6}	$1{,}67662\cdot10^{-7}$	$1{,}67664\cdot10^{-7}$
10^{-7}	$1{,}67664\cdot10^{-9}$	$1{,}67664\cdot10^{-9}$

Table 2. The relational force correction and both the exact and the approximate values of the model parameter

δ_F, force correction	l, exact value (m)	l, approximate value (m)
0,01%	0,0000244692	0,0000244219
0,1%	0,0000782407	0,0000772289
1%	0,000262267	0,000244219
10%	0,00103709	0,000772289

The results for the gravitational force value provide some restrictions on the model parameter proceeding from the modern experimental data for testing the Newtonian inverse square gravitational law on the short distances. If the accuracy of determining the gravitational constant in the Washington and Zurich experiments [2, 7] is taken as a tentative force correction, the upper limit for the value of the Randall-Sundrum model's parameter is about 10^{-6} m.

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