SCALAR FIELDS AND MULTIDIMENSIONAL MODELS IN GRAVITY AND COSMOLOGY

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Introduction

In the development of relativistic gravitation and dynamical cosmology after Einstein and Friedmann one may see 3 stages:

1. Investigation of models with matter sources as perfect and viscous fluids.
2. Studies of models with sources as different physical fields, starting from electromagnetic and scalar ones, both in classical and quantum cases (our program from 1968 [1]).
3. Application of ideas and results of unified theories of physical interactions (UT) to treating fundamental problems of cosmology (dark energy and dark matter) and BH physics, especially in high energy regimes, using extra dimensions and p-branes (our program from 1987[2-4]).

The study of relations between gravitation and microphysics became one of the most actual and developing branches of modern physics since 60's of the previous century. Among basic trends there were quantization of the gravitational field, of other physical fields in a given gravitational background, super-gravity, quantization of model systems - quantum cosmology, self-consistent treatment of quantum-gravitational effects (spontaneously broken symmetries in a gravitational field, particle production, vacuum polarization, etc.), particle like solutions, unification of gravitation and other physical interactions (for details see [1]).

These investigations became possible due to the progress of quantum field theory. Their aim is the solution of such vital problems as birth of the Universe and creation of its mass, avoidance of the initial singular state in cosmology and the final one during the collapse, isotropization of the Universe expansion, explanation of its entropy, charge asymmetry and so on.

Much attention was paid also to modifications of GR, which is caused by several reasons. First, GR is known to be nonrenormalizable. This means that we must modify the classical theory of gravitation, or change the quantization procedure, or consider the gravitational field to be classical in its nature. Second, there is a possibility of slow variations of the effective gravitational coupling less than $5\cdot 10^{-13}$ per year. Finally, generalizations of GR are motivated by attempts to surmount such difficulties as singularities, the energy problem, etc.

Here, we briefly dwell upon our earlier and recent studies on the role of gravitation in micro- and macrophysics through the study of exactly solvable self-consistent gravitational and cosmological model systems in GR and many dimensions (for details see [1-4], in particular in 4 dimensions:

– exact solutions of classical field systems and their applications to collapse, cosmology and elementary particles,
– quantization of cosmological models with the cosmological constant and fields as matter sources and applications of the results to the problems of initial singularity, transitions between metrics with different topology and the birth of the Universe ;
– self-consistent treatment of quantum-gravitational effects (back-reaction) such as spontaneous symmetry breaking in the gravitational field, particle production and vacuum polarization;
– analysis of relations among the fundamental constants and their possible variations;
– construction of the theory of gravitation with a conformal Higgs field, giving models free of initial cosmological singularity and admitting variations of the effective gravitational coupling;
– and afterwards on similar problems in many dimensions.

On the first stage classical gravitational and other physical fields are used as sources of classical GR equations. Then, in quantum cosmology we studied as quantum ones both the material and the simplest gravitational (cosmological) fields. On the next stage quantum effects of matter and its back reaction on the gravitational field were calculated. Further, based on the previous analyses and a study of relations between macro- and microphysical constants, we suggested one of the variants of generalised gravitational theory with conformal scalar field [5].

Then, exact solutions of the system of scalar, electromagnetic and gravitational fields were obtained and studied both in the case of spherical symmetry and in the metrics of homogeneous and isotropic models [6]. Criteria of particle-like solutions of fields systems in gravitational field were formulated and applied to different classes of solutions [7].

Quantization of a cosmological model with the conformal scalar field was done. The result was obtained on null probability of a singular state which is of pure quantum nature. When the cosmological constant was taken into account, we got the possibility of transition between different types of models. As a limiting case of this process we had an explanation of birth of the Universe as a penetration through a potential barrier into the observable region and as a particular case - its creation from vacuum [8,9]. It was the first demonstration of the creation of the Universe from "nothing" as a quantum vacuum effect.

In the self-consistent approach the effect of gauge SSB of a conformal Higgs field was treated. In the open Friedmann model it leads to the non-singular solution [10,11]. This was also the first demonstration of the non-singular solution due to quantum vacuum effect. Some self-consistent models with particle creation were constructed, explaining the observable mass of the Universe.

We suggested and studied the theory of gravitation with the conformal Higgs field which is free of the initial cosmological singularity and is compatible with the mentioned above variations of fundamental constants data [10].

These results led to a conclusion about a regularizing role of gravitation both in the micro-world and in cosmology. Now, we will dwell upon these studies in more detail.

Scalar and electromagnetic fields in GR

Treatment of classical fields as matter sources in the Einstein-Hilbert equations was and still is an important and necessary step in the study of relations between gravitation and micro-physics. It allows one to include a wider class of equations of state than those used in the hydrodynamical approach in GR. The reason is that self-consistent systems of gravitational and other physical fields imply equations of state through the corresponding dynamical equations. Besides, this approach is better suited to the analysis of such extreme processes as the collapse of massive bodies, early stages of the evolution of the Universe and also for the study of the role of gravitation at small distances and in particular in the formation of particle-like configurations.

Investigation of self-consistent model systems of gravitational and other fields was first carried out in [5] starting from conformal scalar and gravitational fields in connection with the hypothesis about the existence of superheavy particles - planckeons (or maximons) whose parameters are the Planck mass m_L, length L and time t_L. In that paper for the first time classical and quantized scalar fields with conformal coupling were used in a self-consistent manner. Exact static solutions were obtained as closed Einstein universes. It was proved that their size could not be less than L. Einstein equations were used with the cosmological constant which for this solution took the value of the order $1/L^2$. For this solution the energy density within the planckeon and the vacuum energy density defined via the cosmological term are of the same order. This establishes a certain relation between the planckeons and the "gravitational vacuum".

Stability of the planckeons was investigated under time-dependent, maximally symmetric perturbations. These were shown to increase with time that means a finite life of the planckeons as cells of the gravitational vacuum. This decay of super massive particles later was used in the inflaton decay idea in the early universe and still is being used nowadays.

Solutions for N quantum scalar particles in the closed Einstein model were also obtained . This solution led to relations connecting hadron mass, Λ-term and the observable radius and particle number of the Universe. When any two of the parameters are taken from the experiment, these relations give the other two parameters. The maximal number of any scalar particles in the Universe nowadays is of the order 10^{120} according to this solution and their mass is equal to the graviton mass.

Nonstatic solutions of interacting massless conformal scalar field and gravitation with a nonzero Λ-term in the Friedmann metric were obtained. A nonsingular solution, stable according to the Lyapunov criterion, was singled out; for late times this solution tends to the static Einstein model. This solution also gives the maximal particle number: $N_{max} \sim 10^{120}$ when $\Lambda \sim 10^{-56}$ cm^{-2}, which corresponds to its present value .

Another application of self-consistent systems of gravitational and conformal scalar fields is the description of spherically symmetric systems, e.g. in the collapse problem. Exact solutions were found in [12]. They consist of three classes, one of which is rather simple: the metric is

$$ds^2 = (1 - r_Q/r)^2 dt^2 - (1 - r_o/r)^{-2} dr^2 - r^2 d\Omega^2, \; r_o = \text{const.}$$

It has an event horizon and realizes the Reissner-Nordstrom black hole solution with an external massless scalar field. The third term in the expansion of g_{00} for $m = m_{pl}$, corresponds to the quantum-mechanical repulsion potential $(h/r)^2 /2m_{pl}$, connected with the momentum uncertainty h/r caused by the scalar field in the spherically symmetric gravitational field. Analysis of test particle motions in this metric has shown that there always exist some r_{min} at which a bounce takes place. Similar solutions for scalar, electromagnetic and gravitational fields in the spherically symmetric case were obtained and discussed in detail in [6].

Particle-like solutions of fields in GR

The necessity of extended particle models treatment is dictated, on one hand, by modern experimental data on complicated internal structure of particles and, on the other hand, by the difficulties of theories operating with point objects. Classical particle models (instantons, solitons, etc.) based on nonlinear field equations are at present widely discussed. However, in these investigations the gravitational field is, in general, not taken into account. A self-consistent inclusion of gravity is important due to the universal character of gravitational interaction which is always present and cannot be shielded.

In [7] exact solutions of interacting scalar, electromagnetic and gravitational fields were discussed. Two criteria of particle-like solutions with gravity were formulated, the strong one and the weak one. First requires the solution to be (i) asymptotically flat (with material fields also properly decreasing), (ii) static or stationary, (iii) singularity-free.

The regularity requirement combined with (i) guarantees both finiteness of the total field energy, of the inertial mass m_I. and the equality $m = m_I$, where m is the active gravitational (Schwarzchild) mass. Besides, existence of singularities manifests a limitation on the validity domain for a given model, or even on the classical concept of space-time. However, requirement (iii) cannot be fulfilled not only by systems with linear matter fields, but also by a broad class of systems containing directly interacting fields. Therefore, along with the strong criterion, we formulated a weak one, in which the requirement (iii) is replaced by the requirements (iii.a) $E_f < \infty$; (iii.b) $m = m_I$. Then, some singular configurations will be particle-like in the sense of the weak criterion. The solutions with linear massless scalar, linear electromagnetic and gravitational fields were analysed on the basis of these criteria. It was shown that all the solutions are singular and thus violate the strong criterion. Besides, in all the cases when horizons are absent, the total material fields energy is not finite and therefore the weak criterion is also violated. When there is a horizon (the Reissner-Nordstrom type solution), E_f integrated over static region is finite and formally the weak criterion is fulfilled. Non-linear electrodynamics was also investigated with the Lagrangian depending arbitrary on the electromagnetic field invariant. It was proved that the strong criterion cannot be fulfilled, whatever this dependence is, since such a configuration cannot have a regular center. General conditions were given for the fulfilment of the weak criterion. The fact that it can be really satisfied was demonstrated explicitly for the solution of the GR-Born-Infeld system. Solutions for various types of direct interaction of massless scalar and electromagnetic fields in GR were also obtained and analysed. In the case of $L_{int} = \Psi(\varphi) F_{\alpha\beta} F^{\alpha\beta}/16\pi$, $\Psi(\varphi) = \exp(\sigma \varphi) - 1$, $\sigma = const$ the solution satisfies the weak criterion though the metric and the scalar field are singular at the centre. The field energy is finite both in the flat-space limit and for $\sigma \to O$. In the latter an event horizon of the Reissner-Nordstrom type appears and the total energy is of the Planck order. For real particles mass and charge the direct field interaction dominates in the field energy and the contribution of gravity is extremely small. A regular model, satisfying the strong criterion was constructed as well. In this model charge and mass densities in the centre are zero and take the maximum values when $r = r_{cl}/4 = e^2 /(4\ mc^2)$. This picture corresponds to the real proton charge and mass distributions obtained experimentally.

Quantum cosmological models

Methods of quantum cosmology (QC} became especially popular after DeWitt quantized the Friedmann universe filled with dust. QC occupies an intermediate position between different approaches to the unification of gravity with quantum physics: quantization of material fields on the gravitational background and quantization of the gravitational field itself. From the first one it differs in that the quantization of a simple gravitational field (cosmological) is performed along with the material fields quantization in a self-consistent manner. As for the second one, QC is a simpler task since only a finite number of degrees of freedom is quantized and the problem effectively becomes quantum-mechanical. The relative simplicity of QC gives one a possibility to solve exactly some problems both on the classical and quantum levels. Nevertheless, the results of the QC are not exact results of a complete quantum theory of gravitation. Indeed, there must be vacuum fluctuations in those degrees of freedom which are suppressed, otherwise the uncertainty principle would be violated for the corresponding coordinates and momenta. But one may hope that the main features of QC remain in the complete quantum gravity and we shall be able to find ways of unification of micro- and macroworld phenomena.

Quantization of a closed Friedmann model filled with a conformal scalar field by the Dirac-DeWitt method was first performed. It was proved that the probability of a singular initial state is equal to zero, this result being of purely quantum nature. It was also shown there that the localization domain of the solution in the ultra-relativistic case (small scale factor) is of the order of L. In the classical limit the usual Friedmann singular solution was obtained. It was also shown that at small times the conformal scalar field behaves like an ultra-relativistic gas and at the present epoch - like dust. So, it represents a good model for matter with variable equation of state in cosmology. Other aspects of QC, namely, a comparison of the Dirac-DeWitt and ADM methods in the case of geometrical optics, the uncertainty relation in isotropic cosmology, quantization of the Friedmann metric matched with the Kruscal one, etc., were studied .

The role of the cosmological constant in QC was first studied in [8] . It was shown that for $\Lambda > 0$ there exist quasi-stationary states of the system and a finite probability of tunneling of the Universe from one type of model to another. This solution describes birth of the Universe with both finite and null energies of matter. Exact vacuum solutions (i.e. without matter) in QC for closed and open models were obtained and investigated later. These general solutions were shown to be non-singular for DeWitt's operator ordering. An example of a singular general solution (that with a finite probability of zero scale factor) was also given, though particular solutions may of course be non-singular. This example refers to another operator ordering. Quantization of an open Friedmann model with dust, cosmological costant and conformal scalar field was also obtained and studied [1].

Self-consistent treatment of quantum-gravitational effects

Another approach to the problem of singular states in cosmology is a self-consistent treatment of quantum effects calculated in a given gravitational field (back reaction). With quantum effects the conditions of classical theorems on singularities can be violated and as a result non-singular solutions in cosmology can be obtained even though the gravitational field remains classical. The most interesting effects treated that may in principle lead to avoidance of singularities, are spontaneously breaking of symmetry (SSB) of physical fields filling the Universe and the vacuum polarization and particle production in a non-stationary cosmological field. Back reaction of the gauge SSB of massless conformal Higgs field was first treated in [10,11]. It was shown that in the open Friedmann model a non-singular solution of the form $a = (L^2 + t^2)$ exists, where a is the scale factor and $L = (\kappa/2\lambda)^{1/2} \sim 10^{-33}$cm for the self-interaction parameter of the order unity. Non-singular solutions for a massive Higgs field at small times were also obtained [13]. There it was shown that at times of the order of $a \sim m^{-1}$ the symmetry is restored and the solution becomes that of the usual Friedmann-type obtained earlier. Later it was proved also that the conclusion of singularity avoidance remains valid if radiation and nonzero cosmological constant are taken into account, see [1]. The same happens if combined quantum effects such as SSB and vacuum polarization (including the cosmological term) are included in a self-consistent manner. SSB in an anisotropic cosmological model with two Higgs fields, one of which describes a vacuum domain structure in agreement with Bogoliubov's ideas, was investigated. Analytical and computer calculations gave some non-singular solutions for a certain range of the parameters involved. The states of maximum compression and symmetry restoration in the massive case were obtained.

Conformal SSB of a massless Higgs field was also [10] treated. In this case the gravitational constant is expressed as a function of the scalar field vacuum average and a cosmological constant appears in the Lagrangian. The most important fact is the appearance of the Einstein-Hilbert gravitational term in the action, $R/2\kappa$, the new macroscopic quantity, as a result of conformal SSB in a curved space-time. In other words, gravitation may be treated as a classical manifestation of quantum vacuum properties. In this respect one may not exclude the possibility that the gravitational field is purely classical and must not be quantized.

Several self-consistent cosmological models with particle creation were constructed [1] using the variable-frequency oscillator quantization method. The present number density of nucleons and the possible number density of gravitons are obtained as a result of particle production in the gravitational field of the expanding Universe. Self-consistent models with incoherent creation of fermions have been obtained, using the method of Hamiltonian diagonalization. The observational parameters are calculated, in particular, the present matter density and the age of the Universe. A model is singled out in which the so called Mach relation $2GM/a \sim 1$ is fulfilled.

Theories with variations of fundamental physical constants

As a result of analyses of hypotheses and theories predicting variations of the constants along with the corresponding observational and experimental, astronomical, astrophysical and geophysical data a conclusion was drawn that possible variations of the atomic constants are excluded at a rather high level. Variations of the gravitational coupling on the level $5\cdot10^{-13}$ per year in the atomic system of units (c, h, m = const) or of all the masses in the gravitational system of units (c, h, G = const) do not contradict modern experimental data. From the present viewpoint any variations of the constants (if they exist) are not absolute: they depend on the system of measurements used, i.e. on the standards based on a particular type of interaction.

The theory of gravitation with a Higgs conformal field, generalizing the one with linear conformal field used first in [5] was suggested in [10]. In this theory GR Lagrangian appears as a consequence of conformal SSB of the nonlinear scalar field on the gravitational background. An advantage of this theory is the absence of an initial singular state in isotropic cosmology as a result of a self-consistent treatment of gauge SSB in a cosmological field. Besides, in the atomic system of units it admits variations of the effective gravitational coupling allowed by the modern experiments. In the gravitational system of units this corresponds to variations of all masses. Transitions from one system of units to another are performed through conformal transformations. Different aspects of this theory are studied: cosmological solutions, the role of conformal transformations etc. Cosmological solutions with local inhomogeneities were investigated. The general method of construction and study of topological transitions models is formulated. Their local properties including asymptotic behaviour in the singularity neighbourhood are studied and applications to the problems of gravitation and cosmology as well (singular state and exponential expansion). The method is shown to lead to scalar-tensor theories of topological transitions.

We estimated also the possible variations of the gravitational constant G in the framework of a generalized (Bergmann-Wagoner-Nordtvedt scalar-tensor theory of gravity on the basis of the field equations, without using their special solutions. Specific estimates were essentially related to the values of other cosmological parameters (Hubble and acceleration parameters, dark matter density etc.), but the values of $\dot{G}/G$ compatible with modern observations did not exceed $5\cdot10^{-13}$/year.

Thus, the method of exactly solvable self-consistent model systems of interacting fields allows us to get rather important results in gravitation, cosmology and astrophysics and to find some relations between different physical interactions.

Multidimensional gravitational and cosmological models

The necessity of studying multidimensional models of gravitation and cosmology [2-4] is motivated by several reasons. First, the main trend of modern physics is the unification of all known fundamental physical interactions: electromagnetic, weak, strong and gravitational ones. During the recent decades there has been a significant progress in unifying weak and electromagnetic interactions, some more modest achievements in GUT, supergravity, string and super-string theories. But, we still have no good model unifying all 4 interactions. All present unified models use different number of extra dimensions: 10, 11, 26, 5, 6, 7 etc.

Second, multidimensional gravitational models, as well as scalar-tensor theories of gravity, are theoretical frameworks for describing possible temporal and range variations of fundamental physical constants [15-17]. These ideas have originated from the earlier papers of E. Milne (1935) and P. Dirac (1937) on relations between the phenomena of micro- and macro-worlds, and up till now they are under thorough study both theoretically and experimentally. The possible discovery of the fine structure constant variations is now at a critical further investigation.

Lastly, applying multidimensional gravitational models to basic problems of modern cosmology and black hole physics, we hope to find answers to such long-standing problems as:

singular or nonsingular initial states,

creation of the Universe,

creation of matter and its entropy,

cosmological constant,

coincidence problem, origin of inflation and specific scalar fields which may be necessary for its realization, isotropization and graceful exit problems,

stability and nature of fundamental constants,

possible number of extra dimensions, their stable compactification,

new revolutionary data on present acceleration of the Universe, dark matter and dark energy nature etc.

Bearing in mind that multidimensional gravitational models are certain generalizations of general relativity which is tested reliably for weak fields up to 0.0001 and partially in strong fields (binary pulsars), it is quite natural to inquire about their possible observational or experimental windows. From what we already know [2-4], among these windows are:

possible deviations from the Newton [18] and Coulomb laws, or new interactions,

possible variations of the effective gravitational constant with a time rate smaller than the Hubble one,

possible existence of monopole modes in gravitational waves,

different behavior of strong field objects, such as multidimensional black holes, wormholes and p -branes,

standard cosmological tests,

possible non-conservation of energy in strong field objects and accelerators, if brane-world ideas about gravity in the bulk turn out to be true etc.

Our realized program [2-4] in multidimensional cosmology includes exact solutions with sourses as:

Λ, $(\Lambda+\varphi)$ (nonsingular, dynamically compactified, inflationary);

PF, (PF+φ) (nonsingular, inflationary);

viscous fluid (nonsingular, generation of mass, entropy); explaining quintessence and coincidence in 2-component model;

giving stochastic behaviour near the singularity, billiards in Lobachevsky space, when D=11 is critical and φ destroys the billiards (94);

Ricci-flat solutions above for any n, also with curvature in one factor-space and with curvatures in 2 factor-spaces only for N=10,11;

with scalar fields , dilatons, forms of arbitrary rank (1998, inflationary, bouncing, Λ generation);

first billiads for p-branes (1999);

dilatonic fields with potentials, billiards for these systems;

quantum variants (WDW-equation) for above cases and

solutions with present acceleration and variation of G [20,21]. In particular, we considered cosmological models in diverse dimensions leading to present acceleration and a relatively small time variation of the effective gravitational constant G now. In case of two factor spaces with non-zero curvatures without matter, we have suggested a mechanism for predicting small $\dot{G}$ /G. When the 3-space has a negative curvature and the internal space has a positive curvature, we got an accelerating expansion of our 3-dimensional space and a small value of $\dot{G}$ /G.

Integrable models with extra arbitrary dimensions were obtained and studied also in spherical symmetry cases, see [2-4].

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