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THE SOLUTION OF THE COSMOLOGICAL CONSTANT PROBLEM AND THE FORMATION OF THE SPACE-TIME CONTINUUM

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ABSTRACT. The application of the microscopic theory of superconductivity to describe the early Universe makes it possible to solve the problem of dark energy. In the cosmological models with superconductivity (CMS) this problem is solved in a natural way: dark energy is the result of pairing of primary fermions with the Planck mass, and its calculated density is equal to $6 \cdot 10^{-30} \text{ g/cm}^3$ and is in good agreement with data of PLANK collaboration. At the same time the birth of space-time domains can also be described in the proposed model. Characteristic parameters of interaction of primary fermions determine the changes of the scale and values of different, but conjugate with each other, phase transitions – for the dark energy, the observed evolving Universe and other component of the condensate of primary fermions.

Keywords: gravity, superfluid gas, fermions, evolution of the universe, dark energy, vacuum energy.

1. Introduction

It was provided a number of approaches for solve the problem of the cosmological constant and dark energy (Weinberg, 1989). Previously we described the process of formation of dark energy as a condensate of primary fermions, by analogy with the theory of superconductivity by Bardeen-Cooper-Schrieffer (BCS) (Bardeen, Cooper & Schrieffer, 1957). The vacuum was regarded as an analogue of a crystal at Planck distances (Fomin, 1990). However, we can consider the more general problem of the formation of Bose condensate from Fermi gas, which gives better understanding of the dark energy nature and a new approach to solving the problem of the cosmological constant and dark energy. At the present time the dark energy manifests itself as anti-gravity, not only on a cosmological scale, but on the scale of galaxy groups (Karachentsev et al., 2009; Bisnovatyi-Kogan & Chernin, 2012; Chernin et al., 2013; Eingorn & Zhuk, 2012; Bri-lenkov, Eingorn & Zhuk, 2015; Bukalov, 2015).

2. The energy spectrum of the superfluid gas and density of dark energy

Let us consider the degenerate almost ideal Fermi gas with attraction between the particles, which are the primary fermions with a mass close to the Planck mass: $M \approx M_p$. It is well known, that even in the presence of an arbitrarily

weak attraction between the particles, the ground state of the system is unstable respect to the restructuring, changing whole system and lowering its energy (Bardeen, Cooper & Schrieffer, 1957; Pitaevskii & Lifshitz, 1980). This instability arises from the Cooper effect, i.e. aspiration to the formation of bound states of fermions pairs that are in the p -space near the Fermi surface and have momenta equal in direction and antiparallel spins. For consideration of this problem, following to (Pitaevskii & Lifshitz, 1980), we introduce the Bogolyubov transformation of the operators, which bring together the operators of the particles with opposite momenta and spins:

$$\hat{b}_{p^-} = u_p \hat{a}_{p^-} + v_p \hat{a}_{-p,+}^+, \quad \hat{b}_{p^+} = u_p \hat{a}_{p^+} - v_p \hat{a}_{-p,-}^+ \quad (1)$$

The indexes + and – refer to the two values of the spin projection. With gas isotropy the coefficients u_p , v_p can depend only on the absolute value of the momentum p . The operators comply with the creation and annihilation of quasiparticles on condition:

$$\hat{b}_{p\alpha}^+ \hat{b}_{p\alpha}^+ + \hat{b}_{p\alpha}^+ \hat{b}_{p\alpha}^- = 1, \quad (2)$$

where the index α numbers the two values of the spin projection. Other pairs of operators are anticommutative. Therefore, the transform coefficients are imposed a condition:

$$u_p^2 + v_p^2 = 1. \quad (3)$$

The transformation inverse to (1) takes the form

$$\hat{a}_{p^-} = u_p \hat{b}_{p^+} + v_p \hat{b}_{-p,-}^+, \quad \hat{a}_{p^+} = u_p \hat{b}_{p^-} - v_p \hat{b}_{-p,+}^+ \quad (4)$$

Due to the primary role of the interaction between pairs of particles with opposite momenta and spins we write only the Hamiltonian with the members, in which $p_1 = -p_2 \equiv p$, $p'_1 = -p'_2 \equiv p'$. It is following:

$$\hat{H} = \sum_{p\alpha} \frac{p^2}{2m} \hat{a}_{p\alpha}^+ \hat{a}_{p\alpha} - \frac{g}{V} \sum_{pp'} \hat{a}_{p^+}^+ \hat{a}_{-p'}^+ \hat{a}_{-p,-}^- \hat{a}_{p^+}^-, \quad (5)$$

where $g = 4\pi\hbar^2 |b|/m$ is a “coupling constant”, $b < 0$ is the scattering length.

For the account of constancy of the number of particles in the system a new Hamiltonian is introduced as a difference $\hat{H}' = \hat{H} - \mu \hat{N}$, where $\hat{N} = \sum_{p\alpha} \hat{a}_{p\alpha}^+ \hat{a}_{p\alpha}$ is a particle

number operator. In this case the chemical potential is determined by the condition that the average value \bar{N} is equal to a given number of particles in the system (Pitaevskii & Lifshitz, 1980).

Introducing $\eta_p = p^2 / 2m - \mu$ and $\mu \approx p_F^2 / 2m$, we get near the Fermi surface $\eta_p = v_F(p - p_F)$, where $v_F = p_F / m$. Subtract $\mu \hat{N}$ from the expression (5). Then

$$\hat{H}' = \sum_{p\alpha} \eta_p \hat{a}_{p\alpha}^+ \hat{a}_{p\alpha} - \frac{g}{V} \sum_{pp'} \hat{a}_{p'+}^+ \hat{a}_{-p'}^+ \hat{a}_{-p,-} \hat{a}_{p+}. \quad (6)$$

Making the transformation (4) with (2) and (3) and replacing p by $-p$, we get

$$\begin{aligned} \hat{H}' = & 2 \sum_p \eta_p v_p^2 + \sum_p \eta_p |u_p^2 - v_p^2| |\hat{b}_{p+}^+ \hat{b}_{p+} + \hat{b}_{p-}^+ \hat{b}_{p-}| + \\ & + 2 \sum_p \eta_p u_p v_p |\hat{b}_{p+}^+ \hat{b}_{-p,-}^+ + \hat{b}_{-p,+}^+ \hat{b}_{p+}| - \frac{g}{V} \sum_{pp'} \eta_p \hat{B}_p^+ \hat{B}_{p'}^+, \end{aligned} \quad (7)$$

$$\hat{B}_p = u_p^2 \hat{b}_{-p,-}^+ \hat{b}_{p+}^+ - v_p^2 \hat{b}_{p+}^+ \hat{b}_{-p,-}^+ + v_p u_p |\hat{b}_{-p,-}^+ \hat{b}_{-p,-}^+ - \hat{b}_{p+}^+ \hat{b}_{p+}^+|.$$

The choice of the coefficients u_p, v_p can be carried out from the condition of minimum energy E of the system at a given entropy. The entropy is determined by the combinatorial expression (Pitaevskii & Lifshitz, 1980):

$$S = - \sum_{p\alpha} [n_{p\alpha} \ln n_{p\alpha} + (1 - n_{p\alpha}) \ln(1 - n_{p\alpha})].$$

In the Hamiltonian (7) diagonal matrix elements have only members, containing products $\hat{b}_{p\alpha}^+ \hat{b}_{p\alpha} = n_{p\alpha}$, $\hat{b}_{p\alpha}^+ \hat{b}_{p\alpha}^+ 1 - n_{p\alpha}$. Therefore, we find

$$\begin{aligned} E = & 2 \sum_p \eta_p v_p^2 + \sum_p \eta_p |u_p^2 - v_p^2| |n_{p+} + n_{p-}| - \\ & - \frac{g}{V} \left[\sum_p u_p v_p (1 - n_{p+} - n_{p-}) \right]^2. \end{aligned} \quad (8)$$

Varying this expression on the parameters u_p (taking into account the relation (3)), we obtain as the condition for the minimum

$$\begin{aligned} \frac{\delta E}{\delta u_p} = & - \frac{2}{v_p} (1 - n_{p+} - n_{p-}) \left[2 \eta_p u_p v_p - \right. \\ & \left. - \frac{g}{V} (u_p^2 - v_p^2) \sum_{p'} u_{p'} v_{p'} (1 - n_{p'+} - n_{p'-}) \right] = 0. \end{aligned}$$

Hence, we find the equation

$$2 \eta_p u_p v_p = \Delta (u_p^2 - v_p^2), \quad (9)$$

where Δ is the sum of:

$$\Delta = \frac{g}{V} \sum_p u_p v_p (1 - n_{p+} - n_{p-}). \quad (10)$$

From (9) and (3) we express u_p, v_p via η_p and Δ :

$$\left. \begin{aligned} u_p^2 \\ v_p^2 \end{aligned} \right\} = \frac{1}{2} \left(1 \pm \frac{\eta_p}{\sqrt{\Delta^2 + \eta_p^2}} \right). \quad (11)$$

Substituting these values in the (10), we obtain an equation that determines Δ :

$$\frac{g}{2V} \sum_p \frac{1 - n_{p+} - n_{p-}}{\sqrt{\Delta^2 + \eta_p^2}} = 1.$$

In equilibrium, the occupation numbers of quasiparticles does not depend on the spin direction and are given by the Fermi distribution with zero chemical potential:

$n_{p+} = n_{p-} \equiv n_p = [e^{\varepsilon/T} + 1]^{-1}$. Going also from summation to integration over p -space, we can write this equation in the form

$$\frac{g}{2} \int \frac{1 - 2n_p}{\sqrt{\Delta^2 + \eta_p^2}} \frac{d^3 p}{(2\pi\hbar)^3} = 1. \quad (12)$$

When $T = 0$ quasiparticles are absent, $\Delta = \Delta_0$, so that $n_p = 0$ and the equation (12) takes the form

$$\frac{g}{2(2\pi\hbar)^3} \int \frac{4\pi p^2 dp}{\sqrt{\Delta_0^2 + \eta_p^2}} = 1. \quad (13)$$

The main contribution to the integral in (13) comes from momentum range, in which $\Delta_0 \ll v_F |p_F - p| \ll v_F p_F \sim \mu$ and the integral is logarithmic (Δ_0 is small in comparison with μ , that confirmed by the result). After cutting the logarithmic integral for $\eta = \tilde{\varepsilon} \sim \mu$ we obtain

$$\int \frac{p^2 dp}{[\Delta_0^2 + v_F^2 (p_F - p)^2]^{1/2}} \approx \frac{p_F^2}{v_F} \int \frac{d\eta}{(\Delta_0^2 + \eta^2)^{1/2}} \approx \frac{2p_F^2}{v_F} \ln \frac{\tilde{\varepsilon}}{\Delta_0}.$$

Here $g m p_F \ln(\tilde{\varepsilon} / \Delta_0) / 2\pi^2 \hbar^3 = 1$, and

$\Delta_0 = \tilde{\varepsilon} \exp[-2\pi^2 \hbar^3 / g m p_F] = \tilde{\varepsilon} \exp[-\pi \hbar / 2 p_F |b|]$ or $\Delta_0 = \tilde{\varepsilon} \exp(-2 / g \chi_F)$, where $\chi_F = m p_F / \pi^2 \hbar^3$ is the energy density of the particle states on the Fermi surface ($\chi d\varepsilon$ is the number of states in the interval $d\varepsilon$).

Let us consider the shape of the energy spectrum of the system. The energy of the elementary excitations is $\varepsilon_{p+} = \varepsilon_{p-} \equiv \varepsilon(p)$. It is the change in energy of the entire system when changing the quasiparticle occupation numbers: $\varepsilon = \delta E / \delta n_{p\alpha} |_{u_p v_p}$. The calculating of $\varepsilon(p)$ gives

(Pitaevskii & Lifshitz, 1980): $\varepsilon(p) = \sqrt{\Delta^2 + \eta_p^2}$. Thus, the energy of quasiparticles can not be less than Δ . For $p = p_F$ $\varepsilon(p) = \Delta$. Therefore, the excited states of the system are separated from the main energy gap, as well as the quasi-particles must appear in pairs, it is possible to write down the value of this gap as 2Δ . From $\varepsilon(p) \neq 0$ it follows that the Fermi gas has superfluidity. Thus from quasiparticles with energies $\varepsilon(p)$ a gas appears, which moves translationally as a single unit relative to the fluid with velocity v . Such gas from quasiparticle corresponds to the normal component of the superfluid. The rest of the liquid will behave like a superfluid component. The density of such superfluid liquid is equal to the sum of the normal and superfluid components: $\rho = \rho_n + \rho_s$. An important property of superfluid motion is its potentiality: $\text{rot } v_s = 0$. The energy 2Δ is the energy of the Cooper pairs. It must be expended to break a pair. The value of the distance between the particles with correlated momenta, or the coherence length, is

$\xi_0 = \pi v_F / \Delta_0 = \hbar e^{2p_F |b|} / p_F$.

From thermodynamics of superfluid Fermi gas it follows (Pitaevskii & Lifshitz, 1980) that $\Delta = 0$, when

$$T_c = \gamma \Delta_0 / \pi \approx 0.57 \Delta_0$$

$$\Delta = T_c \sqrt{\frac{8\pi^2}{7\zeta(3)} \left(1 - \frac{T}{T_c}\right)} = 3.063 T_c \sqrt{1 - \frac{T}{T_c}}. \quad (14)$$

We calculate the heat capacity of the gas. At low temperatures, we start from the formula

$$\delta E = \sum_p \varepsilon (\delta n_{p+} + \delta n_{p-}) = 2 \sum_p \varepsilon \delta n_p$$

to change the total energy by varying the quasiparticle occupation numbers. Divided into δT and go from summation to integration, we obtain the heat capacity:

$$C = V \frac{mp_F}{\pi^2 \hbar^3} \int_{-\infty}^{\infty} \varepsilon \frac{\partial n}{\partial T} d\eta, \text{ where } V \text{ is a volume.}$$

When $T \ll \Delta$ quasi-particle distribution function is $n \approx e^{-\varepsilon/T}$, and their energy is $\varepsilon \approx \Delta_0 + \eta^2 / 2\Delta_0$. We integrate and get:

$$C = V \frac{\sqrt{2} mp_F \Delta_0^{5/2}}{\pi^{3/2} \hbar^3 T^{3/2}} e^{-\Delta_0/T}. \quad (15)$$

Thus, for $T \rightarrow 0$ the heat capacity decreases exponentially. This is a direct consequence of the presence of the gap in the energy spectrum. The difference between the basic levels of the superfluid and normal systems is (Pitaevskii L.P., Lifshitz E.M., 1980):

$$E_s - E_n = -V \frac{mp_F}{4\pi^2 \hbar^3} \Delta_0^2. \quad (16)$$

The sign “-” in (16) is the instability of the “normal” ground state in the case of attraction between gas particles. On one particle it falls $\sim \Delta^2 / \mu$. We apply the theory outlined above to description the dark energy of the Universe and calculation of its density. We transform (16) into the expression for the density:

$$-\Delta \rho = \frac{E_s - E_n}{V} = -\frac{mp_F}{4\pi^2 \hbar^3} \Delta_0^2. \quad (17)$$

The observed density of dark energy can be regarded as the binding energy of the fermions. Therefore, considering it as the difference between the densities of the energies of the levels of the superfluid and normal systems, it is necessary to attribute this difference as negative, indicating instability of the normal ground state for an arbitrarily small attraction between fermions, according to (17).

With $\Delta \rho = \rho_{DE} = \frac{1}{8\pi G_N} \Lambda = \frac{mp_F}{4\pi^2 \hbar^3} \Delta_0^2$ we choose, for example, $v_F = \pi c / 8$, in order to the fermion velocity on the Fermi surface was lower than the speed of light. Then

$$\Delta_0 = \tilde{\varepsilon} e^{-\frac{\pi \hbar}{2p_F |\lambda|}} = M_p e^{-\frac{\pi \hbar}{2p_F |\lambda|}} / 4\pi = M_p e^{-\frac{\pi \lambda_i}{2|\lambda|}} /$$

$$/4\pi = M_p e^{-\frac{1}{\lambda_i}} / 4\pi, \text{ where } M_p \text{ is the Planck mass.}$$

At $\Lambda = \Delta_0^2 / 4 = \tilde{\varepsilon}^2 e^{-2\frac{\pi \hbar}{2p_F |\lambda|}} = \tilde{\varepsilon}^2 e^{-2/\lambda_i}$, where λ_i is the constant of fermions interaction. We estimate the value of λ_i . Since $\Lambda^{1/2} = \tilde{\varepsilon} / e^{1/\lambda_i} = M_p / e^{1/\lambda_i} C$, then assuming a natural cutoff parameter of maximum energy equal to $\tilde{\varepsilon} = M_p$, when $\lambda_i \cong \alpha_{em} = (137.0599)^{-1}$ and $C = 8\pi$, we obtain:

$$\rho_{DE} = \frac{1}{4\pi G_N} \frac{1}{8\pi t_p e^{1/\lambda_i}} = \frac{1}{256\pi^3 G_N^2} \frac{c^5}{\hbar e^{2\alpha_{em}^{-1}}}, \quad (18)$$

$\rho_{DE} = 6.09 \cdot 10^{-27} \text{ kg/m}^3$ in excellent agreement with the PLANK data (Planck Collaboration, 2013).

Thus, at the present time, at $z = 0$, the observed density of dark energy interaction parameter of primary fermions is very close to the electromagnetic fine structure constant α_{em} or equal to it. There are two possibilities: either the interaction of fermions has electromagnetic nature, or the equality $\lambda_i \cong \alpha_{em}^{-1}$ points to the existence of “shadow” or “mirror” long-range interactions, like the electromagnetic ones, for the charges of the shadow sector of matter, manifests itself as a primary condensate of fermions, or dark energy.

The density of the normal and superfluid systems of energy close to the Planck density, and dark energy is a small contribution to this density, making $10^{-120} \rho_p$ at the present time, but in the grand unification epoch it was $10^{-12} \rho_p$. Therefore formally the quantum field theory correctly estimates the true vacuum energy density as the Planck density, but it is the density of the superfluid system, without making a direct contribution to the observed forms of energy, and therefore, into gravity. This contribution is made by only the energy of the system of fermions. If λ coincides with the α_{em} or changed synchronously as a constant of shadow interaction, then it is possible to evaluate the dynamics of changes of $\lambda = \alpha_{em}$ depending on the energy density in the early Universe. Let us consider the process of formation of modern values of dark energy in the hot early Universe. As we know from quantum electrodynamics, the value of the electromagnetic fine structure constant is a function of the four-momentum Q^2 :

$$\alpha_i^{-1} = \alpha_{em} - \frac{\beta}{3\pi} \ln \left(\frac{Q}{2m_e} \right)^2, \quad (19)$$

where m_e is the electron mass, $\alpha_{em} = e^2 / \hbar c$ is the fine structure constant. Then, the effective density of dark energy is:

$$\rho_{DE} = \frac{\Lambda}{8\pi G_N} = \frac{c^5}{256\pi^3 G_N^2 \hbar e^{2\left(\alpha_{em}^{-1} - \frac{\beta}{3\pi} \ln \left(\frac{Q}{2m_e} \right)^2\right)}} = \frac{c^5}{256\pi^3 G_N^2 \hbar e^{2\alpha_{em}^{-1}}} \left(\frac{Q}{2m_e} \right)^{\frac{4\beta}{3\pi}}, \quad (20)$$

where $Q = kT / c$ is momentum of radiation quanta and matter in the early Universe:

Thus, the density of dark energy as the binding energy of fermions is controlled by the density of radiation energy and substance. At the same time ρ_{DE} reaches a minimum and becomes constant at $Qc = 2m_e c^2 = 1.022 \text{ MeV}$. For energy GUT $\mu_{GUT} \approx 10^{15} \text{ eV}$ we have

$$\alpha^{-1}(\mu_{GUT}) = \alpha^{-1}(\mu_0) - \frac{\beta_i}{2\pi} \ln \left(\frac{\mu_{GUT}}{\mu_0} \right). \quad (21)$$

The law of variation of λ_i determines the dynamics of change in Λ . Before the start of transition to the superconducting state, with $\lambda_i^{-1} \rightarrow 0$, $\rho_s = 0$ and

$$\rho_n = \rho_p = \frac{1}{4\pi G_N (8\pi t_p)^2}. \quad (22)$$

In general $\rho_s = \rho_p |1 - e^{-2\lambda_i^{-1}}|$.

$$\rho_{DE} = \frac{|\hbar\omega_p|^4}{256\pi^3 e^{2\alpha^{-1}(\mu_0)}} \left(\frac{\mu_{GUT}}{\mu_0} \right)^{\frac{2\beta_i}{\pi}}. \quad (23)$$

To determine the law of variation of α_i^{-1} we will look at some aspects of the Universe formation. If the Universe started from the Planck density $\rho_p \sim M_p^4$, then up to the moment of the phase transition it expanded in vacuum-like state. The scale factor a as the radius of the Universe at the moment of transition from the vacuum-like in a hot state was $a = R_\Lambda = R_H T_{CMBR} / T_{GUT} = \Lambda_i^{-1/2} = 8\pi t_p e^{\alpha_i^{-1}}$, where $R_H = c/H$ is a modern value of the Hubble radius, T_{CMBR} is the CMB temperature. At the same time τ_p is a parameter for the phase transition. At the moment of the phase transition we can estimate the radius of the Universe R_U , and accordingly the value α_i^{-1} depending on the value of the energy gap Δ_{DE} . For example, when $\rho_{DE} = \langle \phi \rangle^4 = (246.3 \text{ GeV})^4$, $\alpha_i^{-1} = 73.1$ and $k_B T_{GUT} = 1.35 \cdot 10^{15} \text{ GeV}$, then $a = 2.292 \text{ sm}$.

Because dark energy is the only one of the components of the observable Universe, but it is comparable to other, so it is rightful to consider the energy density of the entire observable Universe as evolving dynamically changing difference of density of normal and superfluid fermion systems, i.e. being in a state of a phase transition with changing energy density. Then the density $\Delta\rho$ can be identified with the critical density of the Universe. When

$$\Delta\rho = \rho_c = \frac{3}{8\pi G_N} H_0^2 = \frac{mp_F}{4\pi^2 \hbar^3} \Delta_0^2 \quad \text{and} \quad m = M_p, \quad \text{choose}$$

$p_F = \pi M_p c / 4$, in order to the fermion velocity on the Fermi surface will be lower than the speed of light. Then the square of the dynamically changing energy gap determines the Hubble radius: $\Delta_0^2 = 6H_0^2$. That means that the time parameter t_H is a function of the occurring phase transition of type II, corresponding to the Universe evolution and the variable λ_j :

$$\Delta_j = \frac{\tilde{\xi}}{e^{2p_i |b|}} = \frac{M_p}{4\pi e^{2p_i |b|}} = \frac{M_p}{4\pi e^{\frac{\pi\lambda_i}{2|b|}}} = \frac{M_p}{4\pi e^{\frac{1}{\lambda_j}}}. \quad (24)$$

From $t_H = H_0^{-1} = 1.4 \cdot 10^{10} \text{ years}$, $t_H = 8\pi t_p e^{\lambda_j^{-1}} = 8\pi t_p e^{\frac{\pi\lambda_i}{2|b|}}$, $\lambda_j^{-1} = \pi\lambda_{F_j} / 2|b| \approx 137 \cong \alpha_{em}^{-1}$ at $z = 0$, where α_{em} is the fine structure constant.

The critical density corresponds to the Hubble parameter with a value $H_0 = 69.76 \text{ km/s/Mpc}$ is

$$\begin{aligned} \rho_c &= \frac{3}{8\pi G_N} H_0^2 = \frac{3}{8\pi G_N} \left(\frac{1}{8\pi t_p e^{\lambda_j^{-1}}} \right)^2 \\ &= \frac{3}{8\pi G_N} \left(\frac{1}{8\pi t_p} e^{\frac{\pi\lambda_i}{2|b|}} \right)^2 \approx 9.14 \cdot 10^{-30} \text{ g/sm}^3. \end{aligned} \quad (25)$$

This value ρ_c is in good agreement with the PLANK results (Planck Collaboration, 2013). It should be noted that the proximity of the dark energy density value of the critical density of matter and generally can be explained by the proximity or the equality of the interaction parameters at the present time. Such equality can be explained by the approximation of the various parameters λ_i to a single value, similar to the parameters behavior in the grand unification epoch: $\lambda_i \cong \lambda_j \cong \lambda_z \cong \lambda_{em}$. Thus, the observed dark energy and matter can be regarded as a set of quasi-particles with energy of communication of primary fermions. Therefore, the observed world can be seen as the difference between two energy levels of a fermion system, which density is close to the Planck density: $\rho_n = \rho_p \approx 3M_p^4 / 8\pi$, $\rho_s = \rho_n - \rho_c = 3(M_p^4 - M_p^2 \Delta_j^2) / 8\pi$

Thus, we can describe the observed critical density of the Universe as the difference between the densities of the superfluid and normal fermion systems, and this process is dynamic, providing the energy difference, which coincides with the energy of the observable Universe. Therefore, in the beginning we can start from the Planck density, when $\rho_s = 0$, to $\rho_{p_n} - \rho_{s(t)} = \rho_{GUT}$ and then to

$\rho_s \rightarrow \rho_p |1 - e^{2/\lambda_j}|$. Thus, the energy density of the superfluid fermion systems can be increased from zero to a density close to the Planck density.

References

- Bardeen J., Cooper L., Schrieffer J.R.: 1957, *Phys. Rev.*, **108**, 1175–1204.
- Bisnovaty-Kogan G.S., Chernin A.D.: 2012, *Astrophys. Space Sci.*, **338**, 337.
- Brilenkov R., Eingorn M., Zhuk A.: 2015, arXiv: 1507.07234.
- Bukalov A.V.: 2015, *Odessa Astron. Publ.*, **28/2**, 114.
- Chernin A.D. et al.: 2013, *Astron. Astrophys.*, **553**, 101.
- Eingorn M., Zhuk A.: 2012, arXiv:1205.2384.
- Fomin P.I.: 1990, *Probl. phys. kinetics and physics of solid body*, 387–398.
- Karachentsev I.D. et al.: 2009, *MNRAS*, **393**, 1265.
- Planck Collaboration: arXiv:1303.5062 [astro-ph.CO].
- Pitaevskii L.P., Lifshitz E.M.: 1980, *Statistical Physics Part 2*.
- Weinberg S.: 1989, *Reviews of Modern Physics*, **61**, 1–23.