HOT DENSE MAGNETIZED SPINOR MATTER IN PARTICLE AND ASTROPARICLE PHYSICS: THE ROLE OF BOUNDARIES

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ABSTRACT. We study the influence of boundaries on chiral effects in hot dense relativistic spinor matter in a strong magnetic field which is orthogonal to the boundaries. The most general set of boundary conditions ensuring the confinement of matter within the boundaries is employed. We find that the chiral magnetic effect disappears, whereas the chiral separation effect stays on, becoming dependent on temperature and on a choice of boundary conditions. As temperature increases from zero to large values, a stepped-shape behaviour of the chiral separation effect as a function of chemical potential is changed to a smooth one. A choice of the boundary condition can facilitate either amplification or diminution of the chiral separation effect; in particular, the effect can persist even at zero chemical potential, if temperature is finite. This points at a significant role of boundaries for physical systems with hot dense magnetized spinor matter, i.e. compact astrophysical objects (neutron stars and magnetars), relativistic heavy-ion collisions, novel materials known as the Dirac and Weyl semimetals.

Keywords: hot dense matter, strong magnetic field, chiral effects.

1. Introduction

Properties of hot dense spinor matter in a strong magnetic field are intensively studied during the last decade. An interest to this subject is driven by diverse areas of contemporary physics, ranging from particle and astroparticle physics to cosmology and even condensed-matter physics. Relativistic heavy-ion collisions (Kharzeev D.E.: 2014), compact astrophysical objects (neutron stars and magnetars) (Ferrario L., Melatos A., and Zrake J.: 2015), the early universe (Tashiro H., Vachaspati T., and Vilenkin A.: 2012), novel materials known as the Dirac and Weyl semimetals (Vafek O. and Vishwanath A.: 2014) are the main physical systems where such studies are relevant. In the case of temperature, chemical potential, and the inverse magnetic length exceeding considerably the mass of a relativistic quantized spinor matter field, it has

been shown in theory (Metlitski M.A. and Zhitnitsky A.R.: 2005; Fukushima K., Kharzeev D.E., and Warringa H.J.: 2008) that persistent and nondissipative currents emerge in thermal equilibrium, resulting in a variety of chiral effects in hot dense magnetized matter; see review in Miransky V.A. and Shovkovy I.A.: 2015, and references therein.

So far chiral effects were mostly considered in unbounded (infinite) matter, which may be relevant for cosmological applications, perhaps. For all other applications (to particle, astroparticle, and condensed matter physics), an account has to be taken of the finiteness of physical systems, and the role of boundaries in chiral effects in bounded matter has to be clearly exposed. The concept of quantized matter fields which are confined to bounded spatial regions is quite familiar in the context of condensed matter physics: collective excitations (e.g., spin waves and phonons) exist only inside material samples and do not spread outside. Nevertheless, a quest for boundary conditions ensuring the confinement of quantized matter was initiated in particle physics, in the context of a model description of hadrons as bags containing quarks (Bogolioubov P.N.: 1968; Chodos A., Jaffe R.L., Johnson K., Thorn C. B., and Weisskopf V.: 1974). Motivations for a concrete form of the boundary condition may differ in detail, but the key point is that the boundary condition has to forbid any flow of quark matter across the boundary, see Johnson K.: 1975. However, from this point of view, the bag boundary conditions proposed in Bogolioubov P.N.: 1968, and in Chodos A., Jaffe R.L., Johnson K., Thorn C. B., and Weisskopf V.: 1974 are not the most general ones. It has been rather recently realized that the most general boundary condition ensuring the confinement of relativistic quantized spinor matter within a simply connected boundary involves four arbitrary parameters (Akhmerov A.R. and Beenakker C.W.J.: 2008; Al-Hashimi M.H. and Wiese U.-J.: 2012), and the explicit form of such a condition has been given (Sitenko Yu.A.: 2015; Sitenko Yu.A. and Yushchenko S.A.: 2015; Sitenko Yu.A.: 2016a). To study an impact of the background magnetic field on confined matter, one has to choose the magnetic field configuration with respect to the boundary sur-

face. The primary interest is to understand the effect of a boundary which is transverse to the magnetic field strength lines. Then the simplest geometry is that of a slab in the uniform magnetic field directed perpendicular. It should be noted that such a geometry can be realized in condensed matter physics by putting slices of Dirac or Weyl semimetals in an external transverse magnetic field. Note also that the slab geometry is conventional in a setup for the Casimir effect (Casimir H.B.G.: 1948); see review in Bordag M., G. L. Klimchitskaya G. L., Mohideen U., and Mostepanenko V. M.: 2009.

As a first step toward the full theory of chiral effects in bounded matter, the chiral effects in dense magnetized ultrarelativistic spinor matter at zero temperature in a slab were considered in publication Gorbar E.V., Miransky V.A., Shovkovy I.A., and Sukhachov P.O.: 2015, with the use of the bag boundary condition of Bogolioubov P.N.: 1968. Our aim is to extend the consideration to the case of nonzero temperature and the most general boundary condition.

2. Thermal equilibrium for chiral spinor matter

We start with the operator of the second-quantized spinor field in a static background,

$$\hat{\Psi}(\mathbf{r}, t) = \sum_{E_\lambda > 0} e^{-iE_\lambda t} \langle \mathbf{r}|\lambda\rangle \, \hat{a}_\lambda + \sum_{E_\lambda < 0} e^{-iE_\lambda t} \langle \mathbf{r}|\lambda\rangle \, \hat{b}_\lambda^\dagger, \tag{1}$$

where $\hat{a}_\lambda^\dagger$ and $\hat{a}_\lambda$ ($\hat{b}_\lambda^\dagger$ and $\hat{b}_\lambda$) are the spinor particle (antiparticle) creation and destruction operators satisfying anticommutation relations,

$$\left[\hat{a}_\lambda, \hat{a}_{\lambda'}^\dagger\right]_+ = \left[\hat{b}_\lambda, \hat{b}_{\lambda'}^\dagger\right]_+ = \langle\lambda|\lambda'\rangle, \tag{2}$$

and $\langle\mathbf{r}|\lambda\rangle$ is the solution to the stationary Dirac equation,

$$H\langle\mathbf{r}|\lambda\rangle = E_\lambda \langle\mathbf{r}|\lambda\rangle, \tag{3}$$

H is the Dirac Hamiltonian, λ is the set of parameters (quantum numbers) specifying a one-particle state, E_λ is the energy of the state; wave functions $\langle\mathbf{r}|\lambda\rangle$ satisfy the requirement of orthonormality

$$\int_\Omega \mathrm{d}^3 r \, \langle\lambda|\mathbf{r}\rangle \langle\mathbf{r}|\lambda'\rangle = \langle\lambda|\lambda'\rangle \tag{4}$$

and completeness

$$\sum \langle\mathbf{r}|\lambda\rangle \langle\lambda|\mathbf{r}'\rangle = I\delta(\mathbf{r} - \mathbf{r}'); \tag{5}$$

summation is over the whole set of states, and Ω is the quantization volume.

Conventionally, the operators of dynamical variables (physical observables) in second-quantized theory are defined as bilinears of the fermion field operator (1). One can define the fermion number operator,

$$\hat{N} = \frac{1}{2} \int_\Omega \mathrm{d}^3 r (\hat{\Psi}^\dagger \hat{\Psi} - \hat{\Psi}^T \hat{\Psi}^{\dagger T})$$

$$= \sum \left[\hat{a}_\lambda^\dagger \hat{a}_\lambda - \hat{b}_\lambda^\dagger \hat{b}_\lambda - \frac{1}{2} \mathrm{sgn}(E_\lambda) \right], \tag{6}$$

and the energy (temporal component of the energy-momentum vector) operator,

$$\hat{P}^0 = \frac{1}{2} \int_\Omega \mathrm{d}^3 r (\hat{\Psi}^\dagger H \hat{\Psi} - \hat{\Psi}^T H^T \hat{\Psi}^{\dagger T})$$

$$= \sum |E_\lambda| \left(\hat{a}_\lambda^\dagger \hat{a}_\lambda + \hat{b}_\lambda^\dagger \hat{b}_\lambda - \frac{1}{2} \right), \tag{7}$$

where superscript T denotes a transposition and $\mathrm{sgn}(u)$ is the sign function [$\mathrm{sgn}(\pm u) = \pm 1$ at $u > 0$]. The average of operator $\hat{U}$ over the grand canonical ensemble is defined as (see, e.g., Das A.: 1997)

$$\left\langle \hat{U} \right\rangle_{T,\mu} = \frac{\mathrm{Sp}\,\hat{U}\exp\left[-(\hat{P}^0 - \mu\hat{N})/T\right]}{\mathrm{Sp}\,\exp\left[-(\hat{P}^0 - \mu\hat{N})/T\right]}, \tag{8}$$

where equilibrium temperature T is defined in the units of the Boltzmann constant, chemical potential is denoted by μ, and Sp denotes the trace or the sum over the expectation values in the Fock state basis created by operators in (2). Let us take operator $\hat{U}$ in the form

$$\hat{U} = \frac{1}{2} \left(\hat{\Psi}^\dagger \Upsilon \hat{\Psi} - \hat{\Psi}^T \Upsilon^T \hat{\Psi}^{\dagger T} \right), \tag{9}$$

where Υ is an element of the Dirac-Clifford algebra. The explicit form of $\hat{U}$, $\hat{P}^0$ and $\hat{N}$ in terms of the creation and destruction operators is inserted in (9); then one obtains

$$\left\langle \hat{U} \right\rangle_{T,\mu} = -\frac{1}{2}\mathrm{tr}\,\langle\mathbf{r}|\Upsilon \tanh[(H - \mu I)(2T)^{-1}]|\mathbf{r}\rangle, \tag{10}$$

where tr denotes the trace over spinor indices. We are considering the quantized charged spinor field in the background of a static uniform magnetic field with strength $\mathbf{B} = \boldsymbol{\partial} \times \mathbf{A}$, where $\mathbf{A}$ is the vector potential of the magnetic field. Assuming that the magnetic field is strong (supercritical) and ultrarelativistic spinor matter is at high temperature and high density,

$$|eB| >> m^2, \quad T >> m, \quad |\mu| >> m, \tag{11}$$

we shall neglect the mass of the spinor matter field, putting $m = 0$ in the following. Thus the Dirac Hamiltonian takes form

$$H = -i\gamma^0\boldsymbol{\gamma} \cdot (\boldsymbol{\partial} - ie\mathbf{A}), \tag{12}$$

where e is the charge of the matter field and natural units $\hbar = c = 1$ are used. Owing to the presence of chiral symmetry,

$$[H, \gamma^5]_- = 0, \qquad (13)$$

where $\gamma^5 = -\mathrm{i}\gamma^0\gamma^1\gamma^2\gamma^3$ (γ^0, γ^1, γ^2, and γ^3 are the generating elements of the Dirac-Clifford algebra, and γ^5 is defined according to Okun L.B.: 1982), one can introduce also the following average:

$$\left\langle \hat{U} \right\rangle_{T,\mu_5} = \frac{\mathrm{Sp}\,\hat{U}\exp\left[-(\hat{P}^0 - \mu_5\hat{N}^5)/T\right]}{\mathrm{Sp}\exp\left[-(\hat{P}^0 - \mu_5\hat{N}^5)/T\right]}, \qquad (14)$$

where

$$\hat{N}^5 = \frac{1}{2}\int\limits_\Omega \mathrm{d}^3r(\hat{\Psi}^\dagger\gamma^5\hat{\Psi} - \hat{\Psi}^T\gamma^{5T}\hat{\Psi}^{\dagger T}) \qquad (15)$$

is the axial charge and μ_5 is the chiral chemical potential. For operator $\hat{U}$ in the form of (9), one obtains

$$\left\langle \hat{U} \right\rangle_{T,\mu_5} = -\frac{1}{2}\mathrm{tr}\left\langle \mathbf{r}|\Upsilon\tanh[(H - \mu_5\gamma^5)(2T)^{-1}]|\mathbf{r}\right\rangle, \qquad (16)$$

Thus, there are two types of thermal averaging, when chiral symmetry is present. For instance, one can define the vector current density as either an average over the standard grand canonical ensemble,

$$\mathbf{J} = \left\langle \hat{U} \right\rangle_{T,\mu}\bigg|_{\Upsilon = \gamma^0\boldsymbol{\gamma}}, \qquad (17)$$

or an average over the grand canonical ensemble with the chiral chemical potential,

$$\mathbf{J} = \left\langle \hat{U} \right\rangle_{T,\mu_5}\bigg|_{\Upsilon = \gamma^0\boldsymbol{\gamma}}. \qquad (18)$$

As we shall see, there is no contradiction between these two different definitions of the same physical quantity, since at least one of them results in zero.

3. Chiral effects in the unnbounded space

A solution to the Dirac equation in the background of a static uniform magnetic field is well described in the literature, see, e.g., Akhiezer A.I. and Berestetskij V.B.: 1965. The one-particle energy spectrum in the case of the massless spinor field is

$$E_{nk} = \pm\omega_{nk}, \quad \omega_{nk} = \sqrt{2n|eB| + k^2},$$
$$-\infty < k < \infty, \quad n = 0, 1, 2, ..., \qquad (19)$$

k is the value of the wave number vector along the magnetic field, and n numerates the Landau levels. A straightforward calculation of (18) with the use of the explicit form of spinor wave functions of definite chiralities immediately yields

$$\mathbf{J} \equiv \left\langle \hat{U} \right\rangle_{T,\mu_5}\bigg|_{\Upsilon = \gamma^0\boldsymbol{\gamma}} = -\frac{e\mathbf{B}}{2\pi^2}\mu_5, \qquad (20)$$

whereas (17) turns out to be zero. Similarly, one can calculate the axial charge density,

$$J^{05} \equiv \left\langle \hat{U} \right\rangle_{T,\mu,\mu_5}\bigg|_{\Upsilon = \gamma^5} = \frac{|eB|}{2\pi^2}\Bigg\{\mu_5$$
$$+ \sum_{n=1}^\infty \int_0^\infty \mathrm{d}k\,\frac{\sinh[(\mu_5 + \mu)/T]}{\cosh[(\mu_5 + \mu)/T] + \cosh(\sqrt{2n|eB| + k^2}/T)}$$
$$+ \sum_{n=1}^\infty \int_0^\infty \mathrm{d}k\,\frac{\sinh[(\mu_5 - \mu)/T]}{\cosh[(\mu_5 - \mu)/T] + \cosh(\sqrt{2n|eB| + k^2}/T)}\Bigg\}, \qquad (21)$$

and the axial current density,

$$\mathbf{J}^5 \equiv \left\langle \hat{U} \right\rangle_{T,\mu}\bigg|_{\Upsilon = \gamma^0\boldsymbol{\gamma}\gamma^5} = -\frac{e\mathbf{B}}{2\pi^2}\mu. \qquad (22)$$

Note that $\mathbf{J}$ (20) and $\mathbf{J}^5$ (22) are known as the chiral magnetic (Fukushima K., Kharzeev D.E., and Warringa H.J.: 2008) and chiral separation (Metlitski M.A. and Zhitnitsky A.R.: 2005) effects, respectively. Only the lowest Landau level ($n = 0$) contributes to (20) and (22), and both effects are of topological origin, being related to a quantum anomaly, see Alekseev A.Y., Cheianov V.V., and Frohlich J.: 1998; Giovannini M. and Shaposhnikov M.: 1998; Son D.T. and Zhitnitsky A.R.: 2004; Kharzeev D.E., McLerran L.D., and Warringa H.J.: 2008; Basar G. and Dunne G.V.: 2013. Defining the left and right current densities,

$$\mathbf{J}_L = \frac{1}{2}\left(\mathbf{J} + \mathbf{J}^5\right) \qquad (23)$$

and

$$\mathbf{J}_R = \frac{1}{2}\left(\mathbf{J} - \mathbf{J}^5\right), \qquad (24)$$

as well as the left and right chemical potentials,

$$\mu_L = \mu + \mu_5 \qquad (25)$$

and

$$\mu_R = \mu - \mu_5, \qquad (26)$$

one can rewright (20) and (22) as

$$\mathbf{J}_L = -\frac{e\mathbf{B}}{4\pi^2}\mu_L \qquad (27)$$

and

$$\mathbf{J}_R = \frac{e\mathbf{B}}{4\pi^2}\mu_R. \qquad (28)$$

Actually, the chiral magnetic effect was first discovered in the form of (27) in Vilenkin A.: 1980 (with a missing extra factor of 1/2).

4. Chiral effects in a slab

To study an influence of a background magnetic field on the properties of hot dense spinor matter, one has to account for the fact that the realistic physical systems are bounded. Our interest is in an effect of the static magnetic field with strength lines which are orthogonal to a boundary. Then, as was already noted, the simplest geometry of a material sample is that of a straight slab in the uniform magnetic field directed perpendicular. The one-particle energy spectrum in this case is [cf. (19)]

$$E_{nl} = \pm\omega_{nl}, \quad \omega_{nl} = \sqrt{2n|eB| + k_l^2}, \quad n = 0, 1, 2, \ldots,$$
$$(29)$$

where the values of the wave number vector along the magnetic field, k_l, are to be determined by the boundary condition.

The most general boundary condition ensuring the confinement of relativistic spinor matter within a simply connected boundary is (see Sitenko Yu.A. and Yushchenko S.A.: 2015; Sitenko Yu.A.: 2016a)

$$\left\{ I - \gamma^0 \left[e^{i\varphi\gamma^5} \cos\theta \right. \right.$$
$$\left. \left. + (\gamma^1 \cos\varsigma + \gamma^2 \sin\varsigma) \sin\theta \right] e^{i\tilde{\varphi}\gamma^0(\boldsymbol{\gamma}\cdot\mathbf{n})} \right\}$$
$$\times \chi(\mathbf{r}) \mid_{\mathbf{r}\in\partial\Omega} = 0, \qquad (30)$$

where $\mathbf{n}$ is the unit normal to surface $\partial\Omega$ bounding spatial region Ω and $\chi(\mathbf{r})$ is the confined spinor matter wave function, $\mathbf{r} \in \Omega$; matrices γ^1 and γ^2 in (30) are chosen to obey condition

$$[\gamma^1, \boldsymbol{\gamma}\cdot\mathbf{n}]_+ = [\gamma^2, \boldsymbol{\gamma}\cdot\mathbf{n}]_+ = [\gamma^1, \gamma^2]_+ = 0, \quad (31)$$

and the boundary parameters in (30) are chosen to vary as

$$-\frac{\pi}{2} < \varphi \le \frac{\pi}{2}, \quad -\frac{\pi}{2} \le \tilde{\varphi} < \frac{\pi}{2},$$
$$0 \le \theta < \pi, \quad 0 \le \varsigma < 2\pi. \qquad (32)$$

The MIT bag boundary condition (Johnson K.: 1975),

$$(I + i\boldsymbol{\gamma}\cdot\mathbf{n})\chi(\mathbf{r}) \mid_{\mathbf{r}\in\partial\Omega} = 0, \qquad (33)$$

is obtained from (30) at $\varphi = \theta = 0$, $\tilde{\varphi} = -\pi/2$.

The boundary parameters in (30) can be interpreted as the self-adjoint extension parameters. The self-adjointness of the one-particle energy (Dirac Hamiltonian in the case of relativistic spinor matter) operator in first-quantized theory is required by general principles of comprehensibility and mathematical consistency; see, e.g., Bonneau G., Faraut J., and Valent G.: 2001. To put it simply, a multiple action is well defined for a self-adjoint operator only, allowing for the

construction of functions of the operator, such as resolvent, evolution, heat kernel and zeta-function operators, with further implications upon second quantization.

In the case of a disconnected boundary consisting of two simply connected components, $\partial\Omega = \partial\Omega^{(+)} \bigcup \partial\Omega^{(-)}$, there are in general eight boundary parameters: φ_+, $\tilde{\varphi}_+$, θ_+, and ς_+ corresponding to $\partial\Omega^{(+)}$; and φ_-, $\tilde{\varphi}_-$, θ_-, and ς_- corresponding to $\partial\Omega^{(-)}$. If spatial region Ω has the form of a slab bounded by parallel planes, $\partial\Omega^{(+)}$ and $\partial\Omega^{(-)}$, separated by distance a, then the boundary condition takes form

$$\left(I - K^{(\pm)} \right) \chi(\mathbf{r}) \mid_{z=\pm a/2} = 0, \qquad (34)$$

where

$$K^{(\pm)} = \gamma^0 \left[e^{i\varphi_\pm\gamma^5} \cos\theta_\pm \right.$$
$$\left. + (\gamma^1 \cos\varsigma_\pm + \gamma^2 \sin\varsigma_\pm) \sin\theta_\pm \right] e^{\pm i\tilde{\varphi}_\pm\gamma^0\gamma^z}, \qquad (35)$$

coordinates $\mathbf{r} = (x, y, z)$ are chosen in such a way that x and y are tangential to the boundary, while z is normal to it, and the position of $\partial\Omega^{(\pm)}$ is identified with $z = \pm a/2$. The confinement of matter inside the slab means that the vector bilinear, $\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\chi(\mathbf{r})$, vanishes at the slab boundaries,

$$\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\chi(\mathbf{r}) \mid_{z=\pm a/2} = 0, \qquad (36)$$

and this is ensured by condition (34). As to the axial bilinear, $\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\gamma^5\chi(\mathbf{r})$, it vanishes at the slab boundaries,

$$\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\gamma^5\chi(\mathbf{r}) \mid_{z=\pm a/2} = 0, \qquad (37)$$

in the case of $\theta_+ = \theta_- = \pi/2$ only, that is due to relation

$$[K^{(\pm)} \mid_{\theta_\pm=\pi/2}, \gamma^5]_- = 0. \qquad (38)$$

However, note that, as a massless spinor particle is reflected from an impenetrable boundary, its helicity is flipped. Since the chirality equals plus or minus the helicity, chiral symmetry has to be necessarily broken by the confining boundary condition. Thus the case of $\theta_+ = \theta_- = \pi/2$ is not acceptable on the physical grounds. Moreover, there is a symmetry with respect to rotations around a normal to the slab, and the cases differing by values of ς_+ and ς_- are physically indistinguishable, since they are related by such a rotation. The only way to avoid the unphysical degeneracy of boundary conditions with different values of ς_+ and ς_- is to fix $\theta_+ = \theta_- = 0$. Then $\chi^\dagger(\mathbf{r})\gamma^0\gamma^z\gamma^5\chi(\mathbf{r})$ is nonvanishing at the slab boundaries, and the boundary condition takes form

$$\left\{ I - \gamma^0 \exp\left[i\left(\varphi_\pm\gamma^5 \pm \tilde{\varphi}_\pm\gamma^0\gamma^z \right) \right] \right\} \chi(\mathbf{r}) \mid_{z=\pm a/2} = 0.$$
$$(39)$$

Condition (39) determines the spectrum of the wave number vector in the z direction, k_l. The requirement that this spectrum be real and unambiguous yields constraint (see Sitenko Yu.A. and Yushchenko S.A.: 2015; Sitenko Yu.A.: 2016a)

$$\varphi_+ = \varphi_- = \varphi, \quad \tilde{\varphi}_+ = \tilde{\varphi}_- = \tilde{\varphi}; \qquad (40)$$

then the k_l spectrum is determined implicitly from relation

$$k_l \sin\tilde{\varphi} \cos(k_l a) + (E_{...l} \cos\tilde{\varphi} - m\cos\varphi)\sin(k_l a) = 0, \qquad (41)$$

where m is the mass of the spinor matter field and $E_{...l}$ is the energy of the one-particle state. In the case of the massless spinor matter field, $m = 0$, and the background uniform magnetic field perpendicular to the slab, $\mathbf{B} = (0,0,B)$, $E_{...l}$ takes the form of E_{nl} (29), and relation (41) is reduced to

$$k_l \sin\tilde{\varphi}\cos(k_l a) + E_{nl}\cos\tilde{\varphi}\sin(k_l a) = 0, \qquad (42)$$

depending on one parameter only, although the boundary condition depends on two parameters,

$$\left\{ I - \gamma^0 \exp\left[\mathrm{i}\left(\varphi\gamma^5 \pm \tilde{\varphi}\gamma^0\gamma^z \right) \right] \right\} \chi(\mathbf{r})|_{z=\pm a/2} = 0. \qquad (43)$$

Using the explicit form of standing waves with the k_l spectrum determined by (42), one can compute the averages of type (8). As to the averages of type (14), they cannot be defined, because the confining boundary condition necessarily breaks chiral symmetry: standing waves inside a slab are formed from counterpropagating waves of opposite chiralities. It is straightforward to check the validity of relations

$$\mathbf{J} = J^{05} = 0. \qquad (44)$$

Similarly, the components of the axial current density, which are orthogonal to the direction of the magnetic field, vanish as well,

$$J^{x5} = J^{y5} = 0. \qquad (45)$$

As to the component of the axial current density, which is along the magnetic field, only the lowest Landau level ($n = 0$) contributes to it, and hence the z component of the axial current density is

$$J^{z5} = -\frac{eB}{2\pi a}\left[\sum_{k_l^{(+)}>0} f_+(k_l^{(+)}) - \sum_{k_l^{(-)}>0} f_-(k_l^{(-)}) \right.$$
$$\left. -\frac{1}{2}\sum_{k_l^{(+)}>0} 1 + \frac{1}{2}\sum_{k_l^{(-)}>0} 1 \right], \quad (46)$$

where

$$f_\pm(k) = \left[e^{(k\mp\mu)/T} + 1 \right]^{-1}, \qquad (47)$$

and the two last sums which are independent of temperature and chemical potential correspond to the contribution of the vacuum fluctuations. The calculation of the sums over l yields (see Sitenko Yu.A.: 2016b, 2016c)

$$J^{z5} = -\frac{eB}{2\pi a}$$
$$\times\left\{ \mathrm{sgn}(\mu)F\left(|\mu|a + \mathrm{sgn}(\mu)\left[\tilde{\varphi} - \mathrm{sgn}(\tilde{\varphi})\pi/2\right]; Ta \right) \right.$$
$$\left. -\frac{1}{\pi}\left[\tilde{\varphi} - \mathrm{sgn}(\tilde{\varphi})\pi/2\right] \right\}, \quad (48)$$

where

$$F(s;t) = \frac{s}{\pi}$$
$$-\frac{1}{\pi}\int_0^\infty \mathrm{d}v\, \frac{\sin(2s)\sinh(\pi/t)}{[\cos(2s)+\cosh(2v)][\cosh(\pi/t)+\cos(v/t)]}$$
$$+\frac{\sinh\{[\arctan(\tan s)]/t\}}{\cosh[\pi/(2t)]+\cosh\{[\arctan(\tan s)]/t\}}. \quad (49)$$

In view of relation

$$\lim_{a\to\infty}\frac{1}{a}F(|\mu|a; Ta) = |\mu|/\pi, \qquad (50)$$

the case of a magnetic field filling the whole (infinite) space (Metlitski M.A. and Zhitnitsky A.R.: 2005) is obtained from (48) as a limiting case [cf. (22)],

$$\lim_{a\to\infty} J^{z5} = -\frac{eB}{2\pi^2}\mu. \qquad (51)$$

Unlike this unrealistic case, the realistic case of a magnetic field confined to a slab of finite width is temperature dependent, see (48) and (49). In particular, we get

$$\lim_{T\to 0} J^{z5} = -\frac{eB}{2\pi a}$$
$$\times\left[\mathrm{sgn}(\mu)\left[\!\left[\frac{|\mu|a + \mathrm{sgn}(\mu)\tilde{\varphi}}{\pi} + \Theta(-\mu\tilde{\varphi}) \right]\!\right] \right.$$
$$\left. -\frac{\tilde{\varphi}}{\pi} + \frac{1}{2}\mathrm{sgn}(\tilde{\varphi}) \right] \qquad (52)$$

and

$$\lim_{T\to\infty} J^{z5} = -\frac{eB}{2\pi^2}\mu; \qquad (53)$$

here $[\![u]\!]$ denotes the integer part of quantity u (i.e. the integer which is less than or equal to u), and $\Theta(u) = \frac{1}{2}[1 + \mathrm{sgn}(u)]$ is the step function. As follows from (48), the boundary condition that is parametrized by $\tilde{\varphi}$ can serve as a source which is additional to the spinor matter density: the contribution of the boundary to the axial current effectively enhances or diminishes the

contribution of the chemical potential. Because of the boundary condition, the chiral separation effect can be nonvanishing even at zero chemical potential,

$$J^{z5}|_{\mu=0} = -\frac{eB}{2\pi a}\left\{ F(\tilde{\varphi} - \mathrm{sgn}(\tilde{\varphi})\pi/2; Ta) \right.$$
$$\left. -\frac{1}{\pi}[\tilde{\varphi} - \mathrm{sgn}(\tilde{\varphi})\pi/2] \right\}; \qquad (54)$$

the latter vanishes in the limit of infinite temperature,

$$\lim_{T\to\infty} J^{z5}|_{\mu=0} = 0. \qquad (55)$$

The trivial boundary condition, $\tilde{\varphi} = -\pi/2$, yields spectrum $k_l = (l+\frac{1}{2})\frac{\pi}{a}$ $(l = 0, 1, 2, \ldots)$, which is the same in the setups of both bag models (Bogolioubov P.N.: 1968) and (Chodos A., Jaffe R.L., Johnson K., Thorn C. B., and Weisskopf V.: 1974). The axial current density at zero temperature for this case was obtained in Gorbar E.V., Miransky V.A., Shovkovy I.A., and Sukhachov P.O.: 2015,

$$J^{z5}|_{T=0,\ \tilde{\varphi}=-\pi/2} = -\frac{eB}{2\pi a}\mathrm{sgn}(\mu)\left[\!\left[\frac{|\mu|a}{\pi} + \frac{1}{2}\right]\!\right]. \quad (56)$$

The "bosonic-type" spectrum, $k_l = l\frac{\pi}{a}$ $(l = 0, 1, 2, \ldots)$, is yielded by $\tilde{\varphi} = 0$, and, due to the contribution of the vacuum fluctuations, the axial current density is continuous at this point,

$$\lim_{\tilde{\varphi}\to 0_+} J^{z5} = \lim_{\tilde{\varphi}\to 0_-} J^{z5}. \qquad (57)$$

5. Summary and discussion

We have considered the influence of boundaries on chiral effects in hot dense magnetized relativistic spinor matter. An issue of the confining boundary condition plays the key role in this study. In the absence of boundaries there exist the chiral magnetic effect which is exhibited by the nondissipative vector current along the magnetic field, see (20), and the chiral separation effect which is exhibited by the nondissipative axial current in the same direction, see (22); both currents are temperature independent. As boundaries are introduced and the matter volume is shrinked to a slab which is transverse to the magnetic field, the fate of these currents is different. The axial current stays on, becoming dependent on temperature and on a choice of boundary conditions, see (48) and (49); as temperature increases from zero to large values, a stepped-shape behaviour of the axial current density as a function of chemical potential is changed to a smooth one, see (52) and (53). The vector current, as well as the axial charge, is extinct, see (44), because of boundary condition (36). Thus, the chiral magnetic effect in a slab is eliminated by the confining boundary condition.

Magnetic fields of the order of the QCD energy scale squared can be produced in the quark-gluon plasma created in relativistic heavy-ion collisions (as a result of electric currents from the colliding charged ions); see Kharzeev D.E.: 2014. Magnetic fields up to 10^{15} Gauss may exist in some compact stars (magnetars); see Glendenning N.K.: 2000; Turola R, Zane S., and Watts A.L.: 2015. Even stronger fields are generated in progenitor magnetars during the core collapse after the supernovae explosion (Ferrario L., Melatos A., and Zrake J.: 2015). It has been claimed that the chiral magnetic effect (20) plays a significant role in these processes; see Boyarsky A., Ruchayskiy O., and Shaposhnikov M.: 2012; Sigl G. and Leite N.: 2016. Another claim is that the chiral magnetic effect is responsible for a solution of the problem of large kicks of pulsars (Charbonneau J. and Zhitnitsky A.: 2010; Charbonneau J., Hoffman, and Heyl J.: 2010). The latter problem is that pulsars exhibit rapid proper motion with velocities ranging from 100 to 1600 km/s, and about 15% of pulsars has velocities above 1000 km/s (Arzoumanian Z., Chernoff D.F., and Cordes J.M.: 2002). Large velocities are unambiguously confirmed with the model independent measurement of pulsar B1505+55 moving at 1083^{+103}_{-90} km/s (Chatterjee S., Vlemmings W.H.T., Brisken W.F., et al: 2005). Persistent topological current (20) has been proposed in the capacity of an engine which pushes a fast pulsar like a rocket (Charbonneau J. and Zhitnitsky A.: 2010).

However, if finiteness of a physical system in the direction of the magnetic field is taken into account, then the topological vector current, as well as the axial charge, disappears, that is due to the boundary condition confining spinor (electron, quark, or nucleon) matter inside the magnetar. This circumstance changes essentially the whole picture of the large kick mechanism. The only necessary ingredients are a strong magnetic field and the high density of the relativistic charged spinor matter in the core of a compact star (magnetar or protomagnetar). Such an environment develops axial currents with the specific temperature dependence which is almost stepped-shape in the case of degenerate matter $(T \ll |\mu|)$ and a somewhat smooth otherwise, see (48), (49), (52), and (53). The pertinent carriers of these currents are electrons in the case of the nuclear matter, or electrons and quarks in the case of the deconfined quark matter (in supposed quark stars). Since the currents are persistent, they are capable of delivering the asymmetry of neutrinos created by weak interaction from the interior to the surface without dissipation, thus producing a proper motion of the star as a whole. The details of the mechanism are yet to be elaborated.

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