SIGNIFICANCE OF TENSION FOR KALUZA-KLEIN MODELS: CRITICAL REMARKS

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ABSTRACT. In this very brief review the crucial problematic aspects of gravitational interaction in the weak field limit of Kaluza-Klein models are clarified. We explain why some models meet the classical gravitational tests, while the others do not. In the case of toroidal compactification of extra spatial dimensions we show how the presence of tension (both with and without effects of nonlinearity with respect to the scalar curvature R) of a single gravitating source results in agreement with the observations. It takes place for so-called latent solitons, in particular, black strings and black branes.

In the case of spherical compactification there is the additional (with respect to the Newtonian one) Yukawa interaction for models with the stabilized internal space. For large Yukawa masses the effect of this interaction is negligibly small, and such models satisfy the gravitational tests at the same level of accuracy as general relativity. However, gravitating masses acquire effective relativistic pressure in the external space. Obviously, such pressure contradicts the observations. We demonstrate that tension is the only possibility to preserve the dust-like equation of state in the external space. Therefore, tension plays a crucial role for multidimensional models.

Key words: Kaluza-Klein models, extra dimensions, black strings, black branes, tension, gravitational tests.

The idea of space-time multidimensionality is one of the most intriguing and breath-taking hypotheses of the past and present centuries. Now there is a large variety of multidimensional models supposing existence of extra spatial dimensions. It is absolutely obvious that the search for them is most effective if we know which of these models are viable. In other words, we need to know which of these models do not contradict the experimental data. For example, the well known gravitational experiments (the perihelion shift, the deflection of light and the time delay of radar echoes) in the Solar system turn out to be very good filters for screening out non-physical theories. It is well known that the weak field approximation is enough for deriving the corresponding formulas for these experiments (Landau & Lifshitz, 2000). For example, in the framework of general relativity these formulas demonstrate excellent agreement with the experimental data of astronomical observations.

In our recent papers we investigated in detail popular Kaluza-Klein models with toroidal compactification of the internal spaces. As we have explicitly shown, in order to be at the same level of agreement with the gravitational tests as general relativity, the gravitating masses should have tension (or, in other words, negative relativistic pressure) in the internal spaces. This statement is true for both linear (Eingorn & Zhuk, 2011; Eingorn et al., 2011) and nonlinear $f(R)$ (Eingorn & Zhuk, 2011a; Eingorn & Zhuk, 2012) models. Unfortunately, the physically reasonable sources of the gravitational field with the dust-like equation of state $p = 0$ in all spaces contradict the observations (Eingorn & Zhuk, 2010). For the proper value of tension (which takes place for the latent solitons as well as black strings and black branes as their particular cases) the contradiction is eliminated (see also Eingorn & Zhuk, 2012a).

However, at the given moment we are not aware of the physical meaning of tension for the ordinary astrophysical objects similar to our Sun, which is not relativistic. Consequently, in order to describe motion of a test body in the vicinity of the Sun, there is no need to take into account the relativistic properties of black holes (for example, the presence of the horizon). On the contrary, it is quite sufficient to use the corresponding black strings or black branes metrics in the weak field approximation (see Eingorn & Zhuk, 2011; Eingorn et al., 2011).

Therefore, we continued to search for multidimensional models that satisfy the gravitational tests, being free from such physically unclear property of gravitating masses as tension. For this purpose, we have considered Kaluza-Klein models with spherical compactification of the internal space, being a 2-sphere (Chopovsky et al., 2011; Chopovsky et al., 2012) and a d-sphere (Eingorn et al.,

2012), where $d \geq 2$ is absolutely arbitrary. Here we have shown that the Yukawa-type admixture to the standard metric coefficients is generated. The characteristic range of the Yukawa interaction for these models is proportional to the scale factor of the internal spherical space (or, in other words, to the radius of the internal sphere): $\lambda \sim a$.

It should be noted that the sizes of the extra dimensions in Kaluza-Klein models are bounded by the recent collider experiments, and at the same time there are quite strong restrictions imposed on the Yukawa parameter λ from the inverse square law tests (see, for example, Kapner et al., 2007; Adelberger et al., 2003). According to both these limitations, λ is in many orders of magnitude smaller than the radius of the Sun.

Consequently, with very high accuracy we can drop the admixture of the Yukawa terms to the metric coefficients at distances greater than the radius of the Sun, and as a result we achieve good agreement with the gravitational tests for the models with spherical compactification. In these models the gravitating mass may have the dust-like equation of state in all spatial dimensions, and its tension may be absent, at least, at first glance.

Nevertheless, the considered Kaluza-Klein models with spherical compactification of an arbitrary number of extra dimensions and dust-like sources of the gravitational field turn out to be also inconsistent with the observations, but now from the thermodynamic point of view.

Indeed, we have shown that any gravitating body acquires the effective relativistic pressure in the external space. It means that any system of nonrelativistic particles may have the relativistic momentum crossing any spatial area (see Klimontovich, 1986). Of course, it contradicts the laboratory observations. Such predicted, but unobserved relativistic pressure disappears only in the case of the bare tension in the internal space. Therefore, in order to be in agreement with the experimental data, we again should include tension of bare gravitating masses in the internal space, as it takes place in the case of toroidal compactification.

As a conclusion, we want to stress that Kaluza-Klein models with different types of compactification (toroidal, spherical) face severe problems for physically reasonable dust-like sources of the gravitational field, while non-dust-like sources with tension can not be considered as an adequate way out. This critical situation indicates that one should either focus on alternative multidimensional models such as brane world models, or repudiate the idea of space-time multidimensionality at all. If the latter choice is made, the given investigation can be considered as a part of a direct experimental proof that our world is really spatially three-dimensional, as it was accepted for ages.

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