MULTIDIMENSIONAL SOLITONS WITH SPHERICAL COMPACTIFICATION

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ABSTRACT. Multidimensional static spherically symmetric (with respect to the external 3-dimensional space) vacuum solutions of the Einstein equation were well-known and prevalent in literature for a long time. These solutions are called solitons. Among them, there is a class of particular solutions, called latent solitons, which are indistinguishable from General Relativity concerning the gravitational tests. Black strings and black branes belong to this class. We construct an exact soliton solution, when the internal space represents a sphere of some finite radius.

Key words: Kaluza–Klein models: black branes.

The Kaluza–Klein (KK) model with spherical topology of two additional spatial dimensions is investigated. We consider the metrics of the form:

$$ds^2 = \tilde{A}(\tilde{r}_3)c^2dt^2 + \tilde{B}(\tilde{r}_3)d\tilde{r}_3^2 + \tilde{C}(\tilde{r}_3^2)\left(d\theta^2 + \sin^2\theta\right) + \tilde{E}(\tilde{r}_3^2)\left(d\xi^2 + \sin^2\eta\right),$$

where tilde denotes Schwarzschild-like parametrization for the metrics and the radial coordinate into the external (non-compact) space. The energy-momentum tensor (EMT) of the corresponding background matter may be presented in the form of a perfect fluid with the vacuumlike equation of state in the external space and an arbitrary (parameterized with ω_1) equation of state in the internal (compact) space:

$$T_{ik} = \begin{cases} \varepsilon(\tilde{r}_3)g_{ik}, & \text{for} \quad i,k = 0,\dots,3; \\ -\omega_1\varepsilon(\tilde{r}_3)g_{ik}, & \text{for} \quad i,k = 4,5. \end{cases}$$

We require that the internal space is exactly the 2-sphere: $\tilde{E} \equiv -a^2$, where a is the radius of the internal space. Then, from multidimensional Einstein equations

$$R_{ik} = \kappa_6\left(T_{ik} - \frac{1}{4}Tg_{ik} - \frac{1}{2}\Lambda_6 g_{ik}\right).$$

we get the following equality between the background value of the energy density and the internal space radius: $\varepsilon = \Lambda_6/\omega_1 = \left[(1+\omega_1)\kappa_6 a^2\right]^{-1}$. Choosing different values of the parameter ω_1 we can simulate different types of the background matter.

To get the external spacetime in the form of the Schwarzschild metrics, we have to introduce a compact gravitating object which is spherically symmetric in the external space and uniformly smeared over the internal space. Let the EMT of this source have the following nonzero covariant components:

$$T_{00} = \hat{\varepsilon}g_{00}, \quad T_{ii} = -\hat{p}_1 g_{ii},$$

for $i = 4,5$. In case of a pointlike gravitating mass in the weak-field limit we have $\hat{\varepsilon} \approx m\delta(\mathbf{\tilde{r}}_3)/(4\pi a^2)$. Noting that we want to get the Schwarzschild solution in the external space, it can be easily realized that Einstein equations are compatible only if the equation of state $\hat{p}_1 = -\hat{\varepsilon}/2$ holds. The corresponding exact solution (black brane with spherical compactification) is:

$$ds^2 = \left(1 - \frac{r_g}{\tilde{r}_3}\right)c^2dt^2 - \left(1 - \frac{r_g}{\tilde{r}_3}\right)^{-1}d\tilde{r}_3^2 - \tilde{r}_3^2 d\Omega_2^2 - a^2(d\xi^2 + \sin^2\xi d\eta^2).$$

Here $r_g = 2G_N m/c^2$. Using the relation between the Schwarzschild-like radial coordinate $\tilde{r}_3$ and the isotropic radial coordinate $r_3 = \sqrt{x^2+y^2+z^2}$, in the weak-field limit (up to the terms $1/c^2$) we can find :

$$ds^2 \approx \left(1 - \frac{r_g}{r_3}\right)c^2dt^2 - \left(1 + \frac{r_g}{r_3}\right)(dx^2 + dy^2 + dx^2) - a^2(d\xi^2 + \sin^2\xi d\eta^2).$$

This metrics shows that the PPN parameters $\beta = \gamma = 1$, similar to general relativity. Hence, our black brane satisfies the gravitational experiments at the same level of accuracy as general relativity.

References

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