# ON THE THEORETICAL BASING THE DIRECTION DEPENDENCE COSMOLOGICAL DECELERATION PARAMETER

L.M.Chechin
Astrophysical institute named after Fessenkov V.G., Almaty, Kazakhstan
chechin-lm@mail.ru

ABSTRACT. For theoretical describing the asymmetry of Hubble's diagrams and calculating the anisotropy of deceleration parameter phenomenon the concepts of Universe rotation and its two-component model were involved. Our result $\left(\frac{\Delta q}{q_0}\right)_{\max} \leq 0.48$ is in good correlation with the value $\left(\frac{\Delta q}{q_0}\right)_{\max} = 0.76^{-0.46}_{+0.41}$ in (Cai & Tuo, 2011).

**Key words:** anisotropy of the deceleration parameter, Universe principal axis

## 1. Introduction

One of the novel cosmological effects that are intensively searched at last time is the space asymmetry of Hubble's diagram (Hudson et al., 2004). In this context it's necessary to mark the original article (Schwarz & Weinhorst, 2007) where the asymmetry of Hubble's diagrams for the North and the South sky hemispheres was examined accurately. This asymmetry, according to authors, cannot be explains by peculiar motion of the observer, but most apparently due to the any bulk flow along the direction ((l, b) = ($300^0$, $10^0$)) in the Universe existence that earlier was argued in article (Hudson et al., 2004). Recently R.-G.Cai and Z.-L.Tuo (2011) determined more precisely this direction ((l, b) $= 314^{0-13^0}_{+20^0}$, $28^{0-33^0}_{+11^0}$) and found the maximum anisotropy of the deceleration parameter $\frac{\Delta q}{q} = 0.76^{-0.46}_{+0.41}$.

Their results are possible to summaries as follows – our Universe is anisotropic in realty and possesses by any principal space axis. That is why the cosmological deceleration parameter will be anisotropic, also and must be depend on the principal space direction in definite way. These statements require theoretical basing the direction dependence of the cosmological deceleration parameter phenomenon.

## 2. Basic cosmological equations

Our searching we start from the well-known results. The uniform isotropic metric of the space-flat Universe ($k = 0$) have the standard form

$$ds^2 = dt^2 - a^2(t)\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta d\varphi^2\right)\right]. \quad (1)$$

Einstein's equations for the scale factor $a(t)$ are

$$\ddot{a} = -\frac{4\pi G}{3}(\rho + 3p)a, \quad (2)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = -\frac{8\pi G}{3}\rho, \quad (3)$$

$$\dot{\rho}a + 3(\rho + p)\dot{a} = 0. \quad (4)$$

These equations is possible to deduce and from the Newtonian mechanics in the following way. Let's consider the spherical volume of radius $r$ where concentrates any substance with the density $\rho$ and with the Hubble velocity distribution

$$\vec{v} = H\vec{r}. \quad (5)$$

In the motionless frame of reference the equation of motion of a probe particle that locates on the surface of this sphere, have the usual form

$$\frac{d\vec{v}}{dt} = -\frac{GM}{r^3}\vec{r} = -\frac{4\pi G}{3}\rho\vec{r}. \quad (6)$$

Making the well-known Tolman transformation $\rho \to \rho + 3p$, that allows taking into account the pressure influence on equation of motion, and putting it into (6) we get equation (2). Next, multiplying left and right sides of (6) by $\vec{v} = \frac{d\vec{r}}{dt}$ we get equation (3), that is connected with (6) by the law of energy conservation (4) (Zel'dovich & Novikov, 1983).

In article (Chechin, 2010) it was shown that cosmic vacuum produces not only the Universe expansion but its rotation, also. Here the main results of this article are reproducing briefly.

Let's start from searching the rotational movement of galaxies caused by the antigravitational vacuum force, only. As the model of examining type of galaxy the elliptical galaxy was chosen. For this shape of galaxy its equations of rotational motion are

$$\begin{aligned} \frac{d\theta}{dt} &= \frac{1}{C\omega\sin\theta}\cdot\frac{\partial U}{\partial\psi}, \\ \frac{d\psi}{dt} &= -\frac{1}{C\omega\sin\theta}\cdot\frac{\partial U}{\partial\theta} \end{aligned} . \quad (7)$$

In (7) $\omega$ is the first integral of the rotational motion, i.e. $\omega = const$. It describes the component of angular velocity with respect to the specific momentum - $C$. Next, at deducing (7) it was put forward condition that galaxy angular velocity is very small. This allowed neglect the squared angular velocity components and the corresponding angular accelerations. And at last, it was assumed that arbitrary potential in (7) equals to the vacuum potential $U = U_V$, where

$$U_V = -4G \times I_j S_j \times \left( \begin{array}{c} \cos\psi \int \frac{e_1^2}{R_0^3} dR_0 + \sin\psi\cos\theta \int \frac{e_1 e_2}{R_0^3} dR_0 - \sin\psi\cos\theta \int \frac{e_1 e_2}{R_0^3} dR_0 + \\ + \cos\psi\cos\theta \int \frac{e_2^2}{R_0^3} dR_0 + \sin\psi\sin\theta \int \frac{e_1 e_3}{R_0^3} dR_0 - \cos\psi\sin\theta \int \frac{e_2 e_3}{R_0^3} dR_0 + \\ + \sin\theta \int \frac{e_2 e_3}{R_0^3} dR_0 + \cos\theta \int \frac{e_3^2}{R_0^3} dR_0 \end{array} \right) \quad (8)$$

Analysis of equation (7) shown that solution for the precession angle evolving is $\psi(t) = 8\frac{GI_j S_J}{C\omega R_0^2}\cdot t$. Basing on this result it is easy to calculate the angular velocity of the elliptical galaxy around $OZ$ axis. As for this case the following condition $\omega = -\dot{\psi}$ takes place, than its module equals

$$\omega = \omega_V = \sqrt{8\frac{GI_j S_j}{CR_0^2}} = const \cdot \quad (9)$$

This expression describes the angular velocity that galaxy acquires due to the vacuum antigravitational force.

Admitting $I_j S_j \sim 3IS \sim \rho_V l^4$ and putting that $C \sim l^2$, we find $\omega_V \propto \sqrt{8G\rho_V}\cdot\frac{l}{R_0} \propto \omega_0 \cdot \frac{l}{R_0}$. So, its maximal magnitude will be under the condition $l \propto R_0$. Then expression for the vacuum angular velocity simplifies and takes on the form

## 3. Universe rotation axis

$$\omega_V = \omega_0 \propto \sqrt{G\rho_V} \, . \quad (10)$$

This expression interprets as the minimal angular velocity in the Universe that possesses an arbitrary object due to the vacuum presence. Its present numerical value is $10^{-19}\sec^{-1}$. Hence, the vacuum creates the identical initial angular velocity for all of cosmic objects, including the Universe itself.

At the earliest stages of the Universe evolution, for instance at the baryonic asymmetry epoch when vacuum density was $10^{-15} g/cm^3$ of order, the angular velocity occurs equal $\omega_V \propto 10^{-11}\sec^{-1}$. For the very early Universe when vacuum density was - $10^{90}\, g/cm^3$, the Universe angular velocity is $\omega_V \propto 10^{42}\sec^{-1}$. This magnitude practically equals to the result of article [13], which was done in the framework of general relativity theory ($\omega \propto 10^{43}\sec^{-1}$).

Henceforth, from these investigations we get the following conclusion - the Universe rotation leads to picking out the principal direction in the space, it may be named as the Universe rotation axis. (Mark, that measurement along this axis only gives the Hubble parameter for the uniform Universe, because in the perpendicular directions the Carioles and centrifugal forces act, also.)

## 4. Basing the direction dependence of the cosmological deceleration parameter

For enriching our target, which was formulated in section 1, put that distance

$$r = r_0 + \delta r \, , \quad (11)$$

where $r_0$ is the distance in uniform space, while $\delta r$ - small addition (perturb term) for describing the possible space anisotropy. Putting (11) into the Newtonian equation (6) we get the equation

$$\frac{d^2(r_0 + \delta r)}{dt^2} = -\frac{4\pi G}{3}\rho(r_0 + \delta r), \quad (12)$$

that may be decomposed on two parts, easily: main part

$$\frac{d^2 r_0}{dt^2} = \frac{dv_0}{dt} = -\frac{4\pi G}{3}\rho r_0 \quad (13)$$

and perturb one

$$\frac{d^2 \delta r}{dt^2} = -\frac{4\pi G}{3}\rho\delta r \, . \quad (14)$$

Later on these equations will be considered as are independent each other.

Performing the above mentioned Tolman transformation $\rho \to \rho + 3p$ and substituting it into (13) we find equation

$$\frac{d^2 r_0}{dt} = -\frac{4\pi G}{3}(\rho + 3p) r_0 . \tag{15}$$

For the case of vacuum ($\rho = \rho_v$, $p = -\rho_v$) the inflationary regime of the Universe expanding follows from (15) immediately –

$$r_0 = \mathrm{R} \cdot \exp\left(\sqrt{\frac{8\pi G}{3}\rho_v} \cdot t\right) = R \cdot \exp(H_0 \cdot t) . \tag{16}$$

It leads to the Hubble expansion law

$$v_0 = \frac{dr_0}{dt} = \dot{r}_0 = \sqrt{\frac{8\pi G}{3}\rho_v} \cdot r_0 = H_0 r_0 \tag{17}$$

and to the corresponding acceleration

$$\ddot{r}_0 = H_0{}^2 r_0 . \tag{18}$$

Now consider the equation (14). Suppose that in this equation $\rho = \rho_b$, where $\rho_b$ is the baryonic substance density. The baryonic substance pressure $p_b$ let equals zero, for simplicity. Last requirement means considering the presence of two-component substance – cosmic vacuum and baryonic dust – in the Universe, that are not interact each other in the main approximation.

By introducing the designation $\Omega^2 = \frac{4\pi G}{3}\rho_b$, from (14) it follows

$$\frac{d^2 \delta r}{dt^2} + \Omega^2 \delta r = 0 . \tag{19}$$

This oscillatory-type equation possesses by two roots

$$\delta r_{\pm} = \pm \delta R \cdot \exp(i\Omega \cdot t) = \pm \delta R \cdot \cos(\Omega \cdot t) . \tag{20}$$

They lead to the presence of two perturb (with respect to (17)) velocities

$$\begin{aligned} \delta v_+ &= \frac{d\delta r_+}{dt} = +\delta R \cdot i\Omega \exp(i\Omega \cdot t) = -\delta R \cdot \Omega \cdot \sin \Omega t \\ \delta v_- &= \frac{d\delta r_-}{dt} = -\delta R \cdot i\Omega \exp(i\Omega \cdot t) = +\delta R \cdot \Omega \cdot \sin \Omega t \end{aligned} \tag{21}$$

and two corresponding accelerations

$$\begin{aligned} \frac{d^2 \delta r_+}{dt^2} &= \frac{d\delta v_+}{dt} = -\Omega^2 \cdot \delta r_+ \\ \frac{d^2 \delta r_-}{dt^2} &= \frac{d\delta v_-}{dt} = -\Omega^2 \cdot \delta r_- \end{aligned} . \tag{22}$$

From physical viewpoint expressions (20) – (22) mean that presence of baryonic dust matter creates two space-opposite fluxes that are propagate on the background of expanding and accelerating “Hubble vacuum flux” along the Universe rotation axis (see division 3). That is why it possible writes down the expressions for total distance, velocity and acceleration of any probe particle (galaxy)

$$\begin{aligned} r &= r_0\left(1 \pm \frac{\delta r}{r_0}\right), \\ \dot{r} &= \dot{r}_0\left(1 + \frac{\delta \dot{r}}{\dot{r}_0}\right), \\ \ddot{r} &= \ddot{r}_0\left(1 + \frac{\delta \ddot{r}}{\ddot{r}_0}\right). \end{aligned} \tag{23}$$

Thus the cosmological deceleration parameter $q$ with the accuracy no higher than $\frac{\delta r}{r_0} \sim \frac{\delta \dot{r}}{\dot{r}_0} \sim \frac{\delta \ddot{r}}{\ddot{r}_0} < 1$ is

$$q_{\pm} \approx -1 \pm 3\frac{\delta R \cdot \cos \Omega t}{r_0} \mp 2\frac{\delta R \cdot \Omega \cdot \sin \Omega t}{H_0 r_0} \pm \frac{\delta R \cdot \Omega^2 \cdot \cos \Omega t}{H_0^2 r_0} . \tag{24}$$

Basing on the definitions of $H_0$ and $\Omega$ we introduce the new coefficient $\kappa = \frac{\Omega}{H_0} = \sqrt{\frac{\rho_b}{2\rho_v}}$. As in unit of the critical density $\rho_b \sim 0.04$ and vacuum density $\rho_v \sim 0.7$, coefficient $\kappa \approx 0.17$, henceforth.

From (24) it is possible find the relative acceleration difference between two baryonic fluxes with respect to the “Hubble vacuum flux” –

$$\frac{\Delta q}{q_0} = 2\frac{\delta R}{R}\left[(3 + \kappa)\frac{\cos \kappa H_0 t}{\exp H_0 t} - 2\kappa \frac{\sin \kappa H_0 t}{\exp H_0 t}\right]. \tag{25}$$

Assuming that for modern epoch $H_0 t \sim 1$ we approximately get $\frac{3+\kappa}{e^1} \sim 1.2$. Hence, the first term in right side of (25) tends to 1.2, while the second term tends to zero. So,

$$\frac{\Delta q}{q_0} \approx 2.4 \frac{\delta R}{R} . \tag{26}$$

Basing on our assumption $\frac{\delta r}{r_0} < 1$, that was argued earlier, we may put that it will satisfy if the ratio

$\left(\frac{\delta R}{R}\right)_{\max} \le 0.2$. This leads to the estimation $\left(\frac{\Delta q}{q_0}\right)_{\max} \le 0.48$ that is in good correlation (case of the upper magnitude index) with the value $\left(\frac{\Delta q}{q_0}\right)_{\max} = 0.76^{-0.46}_{+0.41}$ in (Cai & Tuo, 2011).

**5. Conclusions**

From observational data it was established the asymmetry of Hubble's diagrams for the North and the South sky hemispheres (Hudson et al., 2004). Moreover it was estimated the space anisotropy of the deceleration parameter phenomenon, that was done by R.-G.Cai and Z.-L.Tuo (2011). These facts require the adequate theoretical basing, hence.

For doing this the concepts of Universe vacuum rotation and its two independent component model (cosmic vacuum and baryonic dust) were attracted. Our result on the phenomenon of anisotropy the deceleration parameter calculation - $\left(\frac{\Delta q}{q_0}\right)_{\max} \le 0.48$ - is in good correlation (case of the upper magnitude index) with the value $\left(\frac{\Delta q}{q_0}\right)_{\max} = 0.76^{-0.46}_{+0.41}$, which was evaluated in (Cai & Tuo, 2011).

**References**

Cai R.-G., Tuo Zh.-L.: 2011, *arXiv:1109.0941v4 [astro-ph.CO]*.
Schwarz D., Weinhorst B.: 2007, *astro-ph 0706.0165v2.*
Hudson M.J, Smith R.J., Lucey J.R., Branchini E.: 2004, *arXiv:astro-ph/0404386v1*
Zel'dovich Ya.B., Novikov I.D.: 1983, *Relativistic Astrophysics, 2. The structure and evolution of the Universe*. University of Chicago Press, 751 p.
Chechin L.M.: 2010, *Astronomy Reports,* **54,** 719.