PLENARY SESSION

HOLOGRAPHIC DYNAMICS

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ABSTRACT. This paper is a brief introduction to the new physical conception – holographic dynamics. Formulated in the early 90's of the last century, the holographic principle helped to overcome some fundamental challenges facing the traditional cosmology.

Key words: holographic principle, entropy force, cosmological constant, dark energy, Friedmann equation, black hole, ageographic model.

1. Introduction

The mankind has created two Great axiomatic theories. It is Geometry and Thermodynamics. The first of them provides description of the space and time and forms the basis of the General theory of relativity created by Einstein, Hilbert and many other scientists. The second describes giant number of various processes in the World surrounding us. Actually, only this physical theory contains the postulate specifing the direction of time flow. As Nobel winner I.Prigozhin figuratively noted - «arrow of time». Apparently, time is the only thing that connects these two Great theories. Attempts to get into the darkness surrounding origin of our World demands understanding of how Space-time emerge. Most of theories start much later, postulating existence of space-time and discussing processes proceeding in it. Extremely attractive is the approach presenting attempt to develop the concept in which the space-time itself is somehow emerging structure. At the heart of the new approach lays the holographic principle, dynamics which is based on this principle has received the name of holographic dynamics. This work represents short introduction to this subject.

2. Holographic principle

Let's begin with the formulation of a holographic principle. The traditional point of view assumed that dominating part of degrees of freedom in our World are composed by fields that fill the space. However it became then clear that such estimate complicates construction of quantum gravity: it is necessary to introduce small distance cutoffs for all integrals in the theory in order to make it sensible. As a consequence, our world should be described on a three-dimensional discrete lattice with period of order of Planck length. In the latest time some physicists share even more radical point of view: instead of the three-dimensional lattice, complete description of Nature requires only two-dimensional one, situated on the space boundary of our World. Such approach bases on the so-called «holographic principle» ['t Hooft : 1993, Susskind : 1995, Bousso : 2002]. Therefore, central place in the holographic principle is occupied by the assumption that all the information about the Universe can be coded on some two-dimensional surface - the holographic screen.

All that is in it, we will write on it. Presence of such record means occurrence of space in it. The most important in this postulate - the space is understood not as primary object, but as consequence of the inventory of everything that is "inside"sphere. In other words, the space is macroscopical object. Certainly, possibility and record precision are limited by the size of a surface. For taking this restriction into concideration, the axiom postulating quantity of information which can be written down on this surface is introduced. So the quantity of the bits which have been written on a surface, having area A, cannot exceed one bit per plank square L_{Pl}^2

$$N = \frac{A}{L_{Pl}^2} = \frac{Ac^3}{G\hbar} \qquad (1)$$

The data record on the two-dimensional screen is closely connected with fundamental physical quantity - entropy. Actually, information $I = -\Delta S$ is understood as change of entropy ΔS. The screen on which the record is carried out, is called the holographic screen. Thus, certain entropy can be assigned to the holographic screen. Naturally, penetration of objects in the screen would lead to renewal of record on the holographic screen. So, if the particle of mass m comes closer to the holographic screen record, entropy of the holographic screen changes on:

$$\Delta S = 2\pi k_B \frac{mc}{\hbar} \Delta x \tag{2}$$

Presence of mass in the coefficient can be justified by additivity of entropy, and proportionality to Δx is natural even as result of small-parameter expansion. In other words, this postulate looks quite naturally. Now we use the first law of thermodynamics. Equating work of force to entropy change on the screen, we will receive

$$F\Delta x = T\Delta S$$

In other words, entropy change leads to some force occurrence

$$F = T\frac{\Delta S}{\Delta x} \tag{3}$$

It is possible to call it entropy force. This force has purely statistical nature. The value of this force is defined by screen temperature. Thus it is necessary to introduce a postulate defining temperature of the holographic screen. As such postulate, a principle of equal distribution of energy on freedom degrees is used.

$$E = \frac{1}{2} k_B T N \tag{4}$$

Thus, the temperature of the holographic screen is defined by energy E contained in the screen and number of bits N on the screen or number of degrees of freedom of the screen. The formulated postulates lead to the holographic dynamics pretendind on the description of occurrence of everything. This theory operates the forces having the statistical nature or entropy forces. Within the scope of such holographic theory becomes clear even the nature of the fundamental Newton's second law [Verlinde : 2010].

3. Entropic force. The law of Gravity

Let's show within the limits of the holographic theory occurrence of the gravitational force, acting on a particle of mass m located near some holographic screen. Let the mass M be placed behind the spherical screen, then full energy according to Einstein's formula is defined as:

$$E = Mc^2 \tag{5}$$

Using parities (1), (2) and (4), we will receive for the temperature:

$$k_B T = \frac{2MG}{cA}$$

Having substituted this temperature in the parity (3) for entropy force and considering that screen area is $A = 4\pi r^2$, we will receive

$$F = G\frac{mM}{r^2} \quad , \tag{6}$$

It is well-known famous law of universal gravitation. Now, it is possible to state that the factor of proportionality G coincides with a gravitational constant. Certainly, the most important in such way of deducion of this law is the change of interpretation of gravitational force. From this point of view gravitational force gains sense of entropic statistical force. It is caused by aspiration to the most chaotic state. Such point of view at gravitational force is well compatible with the basic observable facts. We will concern only one of them. The main property of gravitation is its universality. Gravitation equally acts on everything, that possesses energy, irrespective of presence of huge distinctions between different structures of matter. Within the scope of statistical or entropy nature of gravitational force this property looks obvious.

Certainly, the deduction provided here concerns only the elementary nonrelativistic and stationary case. The holographic theory is extended on much more general situation. By means of holographic principles equations of Einsteins General Theory of Relativity are deduced (see for example [Барьяхтар, Болотин, Тур, Яновский : 2010]). It should be noted that one of the first attempts to explain gravitation from thermodynamic considerations can be found in [Jacobson: 1995]. In this work, Einstein's equations were derived from the fundamental relation $dQ = TdS$ and proportional to the entropy area horizon.

4. Holographic Universe

In this section we use holographic principle to describe the dynamics of the Universe. We will demonstrate that this approach not only allows to reproduce the achievements of traditional description, but also to resolve a number of problems which were encountered in it. Using Hubble sphere of radius $R = H^{-1}$ as a holographic screen, we will reproduce the Friedmann equations without resorting to either Einstein equations, or Newtonian dynamics. Holographic screen has an area of $A = 4\pi R^2$ and carries information (maximum) of $N = 4\pi R^2/L_{Pl}^2$ bits. The change in information dN during the period of time dt due to the expansion of the Universe $R \to R + dR$ is

$$dN = \frac{dA}{L_{Pl}^2} = \frac{8\pi R}{L_{Pl}^2} dR \tag{7}$$

Here we use $c = k_B = 1$. The change in Hubble radius leads to the change of Hawking temperature $\left(T = \frac{\hbar}{2\pi R}\right)$, which is

$$dT = -\frac{\hbar}{2\pi R^2} dR \tag{8}$$

Using the equidistribution of energy we can write

$$dE = \frac{1}{2}NdT + \frac{1}{2}TdN = \frac{\hbar}{L_{Pl}^2}dR = \frac{dR}{G} \qquad (9)$$

$\left(L_{Pl}^2 = \frac{\hbar G}{c^3}\right)$ and dR can be expressed in the form

$$dR = -H\dot{H}R^3 dt \qquad (10)$$

On the other hand, we can calculate the energy flux through the Hubble sphere given the energy-momentum tensor for the substance which fills the Universe. Treating this substance as an ideal fluid and using $T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}$, we obtain

$$dE = A\,(\rho + p)\,dt \qquad (11)$$

Equating (9) and (11) using (10), we obtain

$$\dot{H} = -4\pi G\,(\rho + p) \qquad (12)$$

As is well known, the system

$$\begin{array}{l} \dot{H} = -4\pi G\,(\rho + p)\,; \\ \dot{\rho} + 3H\,(\rho + p) = 0 \end{array} \qquad (13)$$

is equivalent to the standard Friedmann equations

$$\begin{array}{l} H^2 = \frac{8\pi G}{3}\rho; \\ \frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) \end{array} \qquad (14)$$

Thus, we can say that the goal is achieved.

5. Holographic Dark energy

The derivation of the Friedmann equations from the holographic principle is an important result, but by itself it is merely a recreation of what is known. Can we somehow use this principle to construct a new approach to the description of the dynamics of the universe? If so, can we use this approach to overcome the fundamental problems of the traditional approach? Holographic dynamics gives a positive answer to this question. As an example, let's present a solution of the cosmological constant problem.

In the Standard cosmological model, the density of dark energy is about 70% of the critical density

$$\rho_\Lambda \approx 0.7\frac{3H_0^2}{8\pi G} \approx 10^{-47}GeV \qquad (15)$$

If dark energy in the form of the cosmological constant is really zero-point energy oscillations of vacuum, then

$$\rho_{vac} \approx \frac{k_{\max}^4}{16\pi^2}$$

where $k_{\max}$- is a certain cutoff scale. The natural choice for this role is the Planck mass $k_{\max} = M_{Pl} \simeq 1.22 \times 10^{19}GeV$, since this value is typically synonymous with the limits of use of GR. The result of this choice is

$$\rho_{vac} \approx 10^{74}GeV$$

This value is larger than the observed value by 120 orders of magnitude. This contradiction is the so-called cosmological constant problem. Physicists have never before faced such a gigantic numerical contradiction.

The holographic principle allows us to replace rough dimensional estimate with a more strict evaluation. In any effective quantum field theory, defined in the spatial region with the characteristic size L and that uses the ultraviolet cutoff (UV-cutoff) Λ, entropy $S \propto \Lambda^3 L^3$. According to the holographic principle, this value must be in agreement with the inequality

$$L^3\Lambda^3 \leq S_{BH} = \frac{A}{4L_{Pl}^2} = \pi L^2 M_{Pl}^2 \qquad (16)$$

Here, S_{BH} is the entropy of a black hole with the gravitational radius$L$. We have therefore obtained an important result[Cohen : 1998]: in the bounds of holographic dynamics, the value of the IR-cutoff (L) is strictly linked to the value of the UV-cutoff (Λ) . In other words, physics on small UV-scales depends on the parameters of physics on small IR-scales. Specifically, in the case of the saturation of the inequality (16)

$$L \sim \Lambda^{-3}M_{Pl}^2$$

In the cosmological aspect that interests us, the correlation between small and large scales can be obtained from a natural demand: the total energy bound in an area of the size L must not be larger than the mass of a black hole of the same size:

$$L^3\rho_{de} \leq M_{BH} \sim LM_{Pl}^2 \qquad (17)$$

Here, ρ_{de} is the density of so-called «holographic dark energy» [Li : 2004]. If this inequality was not fulfilled, the universe would be composed entirely of black holes. If we in the context of cosmology take L as the size of the current universe, for instance the Hubble scale H^{-1} , then the dark energy density will be close to the observed data (15)

$$\rho_{de} \approx 10^{-46}GeV$$

Thus, the problem of the cosmological constant in the holographic dynamics absent.

Even though the value of the density of dark energy is correct, problems arise with the equation of state due to our choice of the Hubble radius as the IR-scale [Hsu : 2004] This is easily seen from the following. Let's analyze a Universe composed of holographic dark energy, defined by the relation (17) and matter. In this case, $\rho = \rho_{de} + \rho_M$. From the first Friedmann equation it follows that$\rho \propto H^2$. If $\rho_{de} \propto H^2$, then the dynamic behavior of holographic dark energy and dark matter is identical. In this case equation of state for holographic dark energy$p_{de} = w\rho_{de} = 0$, meaning$w = 0$. Obviously such value of the parameter w violates the condition of the cosmic accelerated expansion $w < -1/3$.

To make holographic dark energy cause accelerated expansion of the Universe, the IR-cutoff must be a scale different from the Hubble radius. Specifically, the problem can be solved if we use the event horizon as the IR-cutoff. However, for this model problems arise with the causality principle, since according to the definition of the event horizon, the current dynamics of dark energy will depend on the future evolution of the scale factor. One of the possible ways of resolving this issue is to equalize the scale of the IR-cutoff with a length scaled defined by the age of the universe T . In this case

$$\rho_\Lambda \propto M_{Pl}^2 T^{-2} \tag{18}$$

The holographic model of dark energy, in which the scale of the IR-cutoff is the age of the universe (Ageogaphic model [Cai : 2007]) allows us to 1) obtain the observed value of density of dark energy; 2) provide accelerated expansion at the latter phases of the Universe's evolution; 3) resolve contradictions related to the causality principle.

6. Emergence of Space

To understand why cosmic space emerges Padmanabhan [Padmanabhan 1,2 :2012] proposed a specific version of holographic principle. At first consider a pure de Sitter universe with a Hubble constant H. Let us assume that such a universe obeys the holographic principle in the form

$$N_{sur} = N_{bulk} \tag{19}$$

Here the N_{sur} is the number of degrees of freedom on the spherical surface of Hubble radius H^{-1} given by

$$N_{sur} = \frac{4\pi}{L_{Pl}^2 H^2} \tag{20}$$

The $N_{bulk} = |E| \,/\, (1/2)\, k_B T$ is the effective number of degrees of freedom which are in equipartition at the horizon temperature $T = H/2\pi k_B$. If we take |E|to be the energy $|\rho + 3p|\, V$ contained inside the Hubble volume $V = \frac{4\pi}{3H^3}$ we obtain

$$N_{bulk} = \frac{16\pi^2\, |\rho + 3p|}{H^4} \tag{21}$$

For de Sitter universe $p = -\rho$, then Eq. (1) reduces to the standard Friedmann equation

$$H^2 = \frac{8\pi L_{Pl}^2}{3}\rho \tag{22}$$

Our universe is only asymptotically de Sitter. This would suggest that the expansion of the universe, which conceptually equivalent to the emergence of space, is being driven towards holographic equipartition. Then the basic law governing the emergence of space must relate the emergence of space to the difference $N_{sur} - N_{bulk}$. The most natural and simplest form of such a law will be

$$\frac{dV}{dt} = L_{pl}^2\, (N_{sur} - N_{bulk}) \tag{23}$$

where V is the Hubble volume in Planck units and t is the cosmic time in Planck units. Using (13) and (14) we obtain the standard dynamical Friedmann equation. By this means Friedmann equations can be reinterpreted as an evolution to holographic equipartition. In other words, "if the holographic principle is not correct, it is very difficult to understand why Friedmann equations hold in our universe"[Padmanabhan 1 :2012].

6. Summary

First successful applications of holographic principle brought forth hopes to build on its base adequate description of Universe dynamics, lacking number of problems, innate in traditional approach. Conversely, that success from our point of view became the source of unreasonable optimism. One has to remember the words of Churchill «Success is the ability to go from one failure to another with no loss of enthusiasm». Holographic dynamics is one of the most perspective branches of theoretical physics. There were too few failures to expect ultimate success.

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