ASTROPHYSICS

COMPARATIVE ANALYSIS OF NUMERICAL METHODS OF DETERMINATION OF PARAMETERS OF BINARY STARS. CASE OF SPHERICAL COMPONENTS

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ABSTRACT. We discuss methods for modeling eclipsing binary stars. There are few realizations of the Wilson-Devinney (1971) code and its improvements, e.g. "Binary Maker", "Phoebe". A parameter search using the Monte-Carlo method was realized by Zola et al. (2010), which is efficient in expense of too many evaluations of the test function. We compare existing algorithms of minimization of multi-parametric functions. To study methods, we adopt a simplified model of an eclipsing binary at a circular orbit assuming spherical components with an uniform brightness distribution. This model resembles more advanced models in a sense of correlated parameter estimates due to a similar topology of the test function. Such a model may be applied to detached Algol-type systems, where the tidal distortion of components is negligible.

Key words: variable stars, eclipsing binaries, algols, data analysis, time series analysis, parameter determination.

Introduction

Determination of the model parameters of various astrophysical objects, comparison with observations and, if needed, further improvement of the model, is one of the main directions of science, particularly, of the study of variable stars.

There is some software which allows to compute theoretical light curves of binary stars taking into account physical processes taking place there. Methods of modeling light curves of binary stars have been described by many authors (Kopal 1959, Tsessevich 1971, Shul'berg 1971, Rucinski 2010, Kallrath and Milone 2009).

Since the paper of Wilson and Devinney (1971) and its further improvements (Wilson 1979, 1994), there are some realizations of the corresponding method. The well known programs are "Binary Maker" (Bradstreet, 2005, *http://www.binarymaker.com/*), PHOEBE (Prsa et al. 2011, *http://sourceforge.net/projects/phoebe/*).

To determine the statistically best sets of the parameters, there are some methods for optimization of the test function which is dependent on these parameters (cf. Cherepashchuk 1992).

As for the majority of binary stars the observations are not sufficient to determine all parameters, for smoothing the light curves may be used "phenomenological fits". Often were used trigonometric polynomials (="restricted Fourier series"), following a pioneer work of Pickering (1881) and other authors, see Parenago and Kukarkin (1936) for a detailed historical review. Andronov (2012) proposed a method of phenomenological modeling of eclipsing variables (most effective for algols, but also applicable for EB and EW – type stars).

Model

The simplest model is based on the following main assumptions: the stars are spherically symmetric (this is physically reliable for detached stars with components being deeply inside their Roche lobes); the surface brightness distribution is uniform. This challenges the limb darkening law, but is often used for teaching students because of simplicity of the mathematical expressions, cf. Andronov (1991). Similar simplified model of an eclipsing binary star is presented by Dan Bruton (*http://www.physics.sfasu.edu/astro/ebstar/ebstar.html*).

The scheme is shown in Fig.1. The parameters are L_1, L_2 (proportional to luminosities), radii R_1, R_2, distance R between the projections of centers to the celestial sphere.

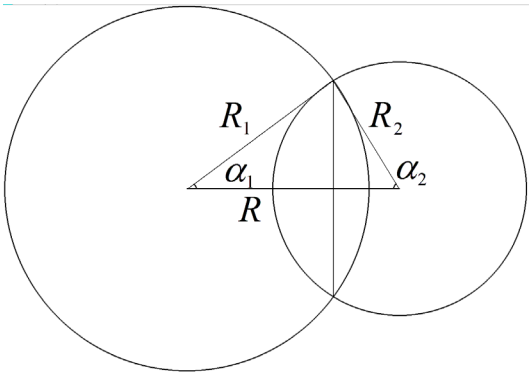

Fig. 1. Scheme of eclipsing binary system with spherical components.

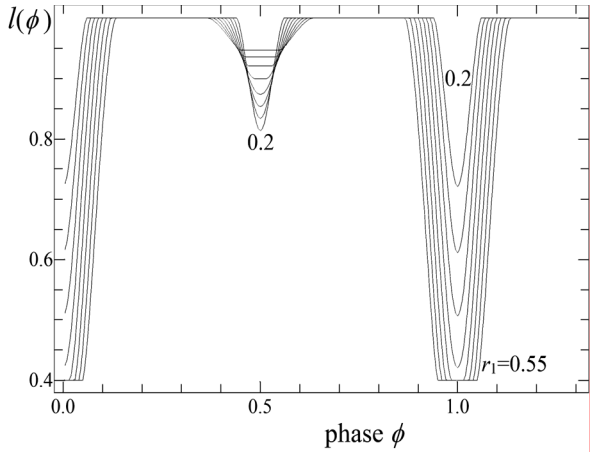

Fig. 2. A set of light curves generated for R_1 in a range from 0.2 to 0.55 with a step of 0.05 for fixed values of other parameters listed in the text.

The square of the eclipsed segment is S=S_1+S_2,

$$S_1=R_1^2(\alpha_1\text{-sin }\alpha_1\cos\alpha_1) \qquad (1)$$
$$S_2=R_2^2(\alpha_2\text{-sin }\alpha_2\cos\alpha_2)$$

where the angles α_1, α_2 may be determined from the cosine theorem:

$$\cos\alpha_1=\frac{R^2+R_1^2-R_2^2}{2R_1R}=\frac{R^2+\eta}{2R_1R},$$
$$\cos\alpha_2=\frac{R^2+R_2^2-R_1^2}{2R_2R}=\frac{R^2-\eta}{2R_2R}, \qquad (2)$$

where obviously $\eta=R_1^2-R_2^2$. The total flux is L=L_1+L_2, if $R \geq R_1$+R_2 (i.e. both stars are visible, S=0). For $R \leq R_1$+R_2, S=πR_2^2 (assuming that R_2≤R_1). Generally, $L = L_1$+$L_2 - S/\pi R_j^2$, where j is the number of star which is behind another, i.e. j=1, if $\cos 2\pi\phi \leq 0$, and j=2, if $\cos 2\pi\phi \geq 0$. Here ϕ is phase (ϕ=0 corresponds to a full eclipse, independently on which star has larger brightness). For scaling purposes, a dimensionless variable $l(\phi)$=$L(\phi)/(L_1$+$L_2)$ is usually introduced.

For tests, we used a light curve generated for the following parameters: R_1=0.3, R_2=0.2, L_1= 0.4, L_2=0.6 and i=80°. The phases were computed with a step of 0.02. This light curve as well as other generated for a set of values of R_1 is shown in Fig.2.

As a test function we have used:

$$F=\sum_{i=1}^{n}\frac{(x_i-\alpha x_c(\phi_i))^2}{\sigma_i^2} \qquad (3)$$

where x_i (or l_i) are values of the signal at phases ϕ_i with a corresponding accuracy estimate σ_i, and x_c are theoretical values computed for a given trial set of m parameters.

For normally distributed errors and absence of systematic differences between the observations and theoretical values, the parameter F is a random variable with a χ^2_{n-m} statistical distribution (Anderson, 1958, Cherepashchuk 1992). For the analysis carried out in this work, we used a simplified model with $\sigma_i=1$. This assumption doesn't challenge the basic properties of the test function.

The scaling parameter is sometimes determined as $x(0.75)/x_c(0.75)$, i.e. at a phase where both components are visible, and the flux (intensity) has its theoretical maximum (in the "no spots" model). To improve statistical accuracy, it may be recommended to use a scaling parameter computed for all real observations:

$$\alpha=\frac{\sum_{i=1}^{n}\frac{x_i}{\sigma_i^2}}{\sum_{i=1}^{n}\frac{x_c(\phi_i)}{\sigma_i^2}} \qquad (4)$$

Even in our simplified model, the number of parameters is still large (4). At Figure 3, the lines of equal levels of F are shown. One may see that the zones of small values are elongated and inclined showing a high correlation between estimates of 2 parameters. In fact this correlation is present for other pairs of parameters. This means that there may be relatively large regions in the multi–parameter space which produce theoretical light curves of nearly equal coincidence with observations.

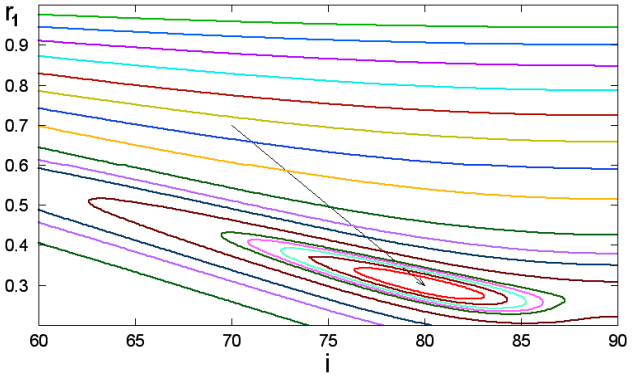

Fig. 3. Lines of equal values of the test function F for fixed values of other parameters. The arrow shows position of the "true" parameters used to generate the signal.

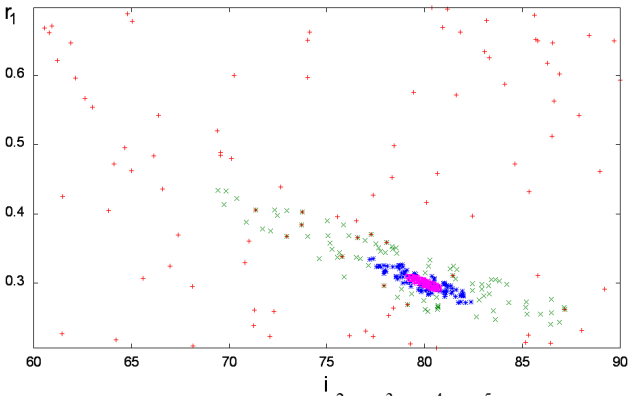

Fig. 4. Best 100 points after 10^2, 10^3, 10^4, 10^5 trial computations, respectively.

In the software by Zola et al. (2010), the Monte-Carlo method is used, and at each trial computation of the light curve, the random parameters are used in a corresponding range:

$$C_k = C_{k,min} + (C_{k,min} - C_{k,min}) \cdot \mathrm{rand}, \quad (5)$$

where rand is an uniformly distributed random value.

Then one may plot "parameter – parameter" diagrams for "best" points after a number of N trial computations. The "best" means sorting of sets of the parameters according to the values of the test function F.

Initially, the points are distributed uniformly. With an increasing N, "better" (with smaller F) point concentrate to a minimum. There may be some local minima, if the number of parameters will be larger (e.g. spot(s) present in the atmosphere(s) of component(s)).

Andronov and Tkachenko (2013) had made computations for an artificial function of m (=1,2,3) variables. The minimal value δ (as a true value was set to zero), which was obtained using N trial computations in the Monte-Carlo method is statistically proportional to

$$\delta \sim N^{2/m}, \quad (6)$$

i.e. the number of computations $N \sim \delta^{-m/2}$ drastically increases with both an increasing accuracy and number of parameters.

For our simplified model, the numerical experiments statistically support this relation. Also, the distance between the "successful computations" (when the test function becomes smaller than all previous ones) $\Delta N \sim N$. Obviously, it is not realistic to make computations of the test function for billions times to get a set of statistically optimal parameters.

In the "brute force" method, the test functions are computed using a grid in the m – dimensional space, so the interval of each parameter is divided by n_i points. The number of computations is $N=n_1 n_2 \ldots n_m$ should be still large. Either the Monte–Carlo method, or the "brute force" one allow to determine positions of the possible local extrema in an addition to the global one.

However, if the preliminary position is determined, one should use faster methods to reach the minimum. Classically, there may be used the method of the "steepest descent", where the new set of parameters may be determined as

$$C_{k+1,i} = C_{k,i} - \lambda h_{k,i}, \quad (7)$$

where $C_{k,i}$ is the estimated value of the coefficient C_i at k-th iteration, $h_{k,i}$ – proposed vector of direction for the coefficient C_i, and λ is a parameter. Typically one may use one of the methods for one–dimensional minimization (cf. Press et al. 2007, Korn and Korn, 1968), determine a next set of the parameters $C_{k,i}$, recompute a new vector $h_{k,i}$ and again minimize λ. In the method of the steepest descent, one may use a gradient $h_{k,i} = \partial F / \partial C_i$ as a simplest approximation to this vector. Another approach (Newton-Raphson) is to redefine a function $F(\lambda)=F(C_i, i=1..m)$, compute the root of equation $\partial F / \partial \lambda = 0$, and then to use a parabolic approximation to this function. Thus

$$\lambda = (\partial F / \partial \lambda) / (\partial^2 F / \partial \lambda^2). \quad (8)$$

There may be some modifications of the method based on a decrease of λ, which may be recommended, if the shape of the function significantly differs from a parabola.

In the method of "conjugated gradients", the function is approximated by a second-order polynomial. Finally it is usually recommended to use the Marquardt (1963) algorithm. We tested this algorithm with a combination of the "steepest descent" (when the determinant of the Hessian matrix is negative) and "conjugated gradients" (if positive), which both are efficient for a complex behavior of the test function.

We developed the software realizing various methods for study of variable stars. The results of this study will be used in the frame of the projects "Ukrainian Virtual Observatory" (UkrVO) (Vavilova et al., 2012) and "Inter-Longitude Astronomy' (Andronov et al., 2010).

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