THE MATHEMATICAL MODEL OF THE PHOTOMETRIC VARIABILITY AND CLASSIFICATION OF SEMIREGULAR PULSATING ASYMPTOTIC GIANTS BRANCH STARS

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ABSTRACT. The modern review of the stars which are located at the position of asymptotic giant branch at the HR-diagram is presented. The most interesting problems connected to these objects are noted, as well as attention is paid to classification and to the evolutionary status. We provided mathematical modeling of the mean light curve of the semiregular supergiant S Per. It is shown, that by means of the periodogram analysis, it is possible to determine the period of the main variability and to provide further detailed classification of semiregular pulsating stars, approximating their mean light curves with a trigonometric polynomial. It is offered to use the photometric period for estimates of physical parameters of pulsing stars.

Key words: variable stars, pulsations, data analysis

After in the center of the star will burn out helium, having formed a carbon-oxygen core, reactions will move to a layer around of a core where helium was still kept, and in higher layers burning hydrogen proceeds. At this stage, a star again turns to red giants, forming asymptotic giant branch (AGB) at the HR-diagram.

We carried out research of semiregular AGB stars using methods of mathematical modeling and their making subsequent classification. The algorithms and programs developed by I.L.Andronov were used for the analysis of photometric signals. They were described by Andronov (1994, 1997, 2003).

All AGB stars are pulsating.

Pulsating instability arises at certain stages of stellar evolution, therefore classification of pulsating variable stars on duration of the period, the shape of the light curve, to the spectral type and other observational parameters reflects their evolutionary status, that is characteristic to group of stars with certain ranges of mass, age and chemical composition.

The basic problems solved now, concerning AGB stars, is in computation of dynamic models of atmospheres, definition of a pulsation mode, studying of the mechanism of the mass loss and the further evolution of stars.

One of the main questions of the theory of stellar pulsations is the mode, in which AGB stars pulsate.

For the solution of the problem about the mode of stellar pulsation, it is necessary to classify all observations and types of stelar activity.

Any ways of classification of stars on variability types lean on a general view of a light curve and a spectral class. However, such approach is not always successful, if it is a question of semiregular variables (SR). For them often it is impossible to consider the general light curve as it contains parts, characteristic for stars of various types. This is probably because SR-stars, in the majority, first, multiperiodic and all components of this multiperiodicity prove very actively, thus having similar amplitude with the main variability. And, secondly, the period of the basic variation also changes (Kudashkina, 2003).

Subclasses of SR-stars are strongly mixed. Especially it concerns stars of subtypes SRb. Kerschbaum and Hron (1992) introduced their division into 'red' and 'blue', based on statistical researches of the periods, amplitudes, temperatures, mass loss rates, presence the dust shell and features of spectra. Properties of stars in visual and infra-red areas of a spectrum have been used in their work. The same authors specify, that SRa-stars are intermediate objects between long period Mira-type stars and SRb-stars.

For example, the star AF Cyg may be quite a prototype of a separate class of stars (as, for example, RV Tau). This object shows consecutive pulsations periods 'switching' between two main values (Andronov, Chernysheva, 1989).

It is necessary to notice, that the SRc-class, in fact, marks only stars which are supergiants, sometimes with variability of type SRa, but more often the type of variability is not certain in any way. As a representative of the SRc-class, we shall consider the star S Per.

S Per is the supergiant, belongs to stars with harmonious variability. For the periodogram analysis, the database of the French Association of Variable Stars Observers (AFOEV) and the methods of the analysis of multiperiodic fluctuations, described by Andronov (1994, 1997), were used.

The period value is P=809.^{d}91 (Kudashkina & Andronov, 2000). Actually, at the periodograms, there is not a single high peak, but there are two. The first corresponds to a period of P=16173^d±158^d.with which average brightness changes (we shall notice, that the interval of observations exceeds 20000 days), and the second peak is dual P_1= 809.d^6±0.^{d}22 and P_2=768.^{d}8±0.^{d}31.

Extended researches of the period show the following. Having divided all existing light curve about for hundred years into six intervals, for each interval we applied the periodogram analysis. The results are presented in Table 1. Each following value of the period for a given interval is computed for the residuals of observationsfrom a best sine fit (the 'prewhitening'), S (f) – height of peak at the periodogram, which is a square of the correlation coefficient between the observations and the sine fit.

In last time interval, the light curve of S Per has a regular form with the stable period of 816.8 days (Fig. 1). We consider, that the star pulsates in a fundamental mode.

Using the dependence 'the period - absolute bolometric magnitude', published by Feast (1989) for supergiants, we shall estimate M_{bol} for S Per.

$$M_{bol} = -7.20 \cdot \lg(P) + 12.8$$

We obtain M_{bol}≈-8.17.

Abramyan (1984) has determined the following parameters for S Per, using infrared-observation and dependences between luminosity, mass, effective temperature and the period: M_V=-6.1, M_{bol}=-8.7, $M/M_\odot$=26.3, T_{ef}=2950 K. Spectral class M4Ia-M4.5Iab.

Let's take advantage of these values of mass and effective temperature for estimates of radius of a star.

We use classical relations,

$$\lg(L/L_o) = -0.4(M_{bol} - 4.7)$$

where $L_\odot$ – luminosity of the Sun.

We have from here L≈140605$L_\odot$.

For absolutely black body

$$L = 4\pi R^2 \sigma T_{ef}^4 ,$$

we receive R=1.0·10^{14} cm or, about, 1400$R_\odot$.

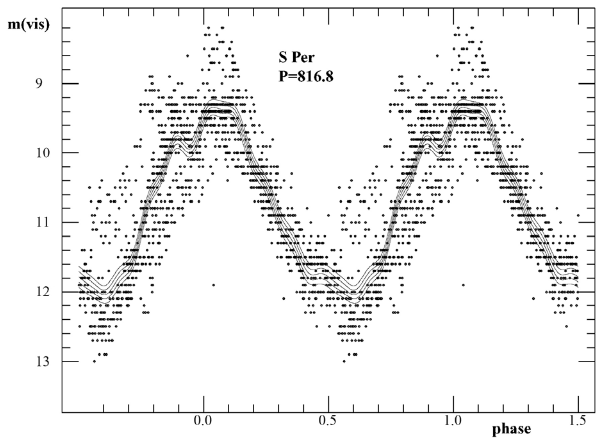

Figure 1: Phase light curve of the star S Per on the observational data-base AFOEV and its approximation by a trigonometric polynomial of statistically optimal degree s=8 (Andronov, 1994; Andronov & Baklanov, 2004).

Table 1: The results of the periodogram analysis for S Per.

Interval (JD+24…)	Period values	S(f)
23500-25000 (1500^d)	991.^{d}46±18.^{d}64 431.43± 9.84 643.99±45.50	0.94 0.48 0.40
25000-29000 (4000^d)	834.15± 3.26 489.88± 2.08 1631.76±25.04	0.44 0.36 0.33
29000-32000 (3000^d)	768.91± 7.16 544.26± 4.04 998.83±15.20	0.75 0.53 0.39
39400-43200 (3800^d)	909.40± 6.52 787.89± 8.45 1090.17±15.31	0.83 0.55 0.45
43200-47000 (3800^d)	759.33± 5.48 599.31± 6.07 298.55± 1.65	0.42 0.35 0.33
47000-51000 (4000^d)	816.82± 1.55 2463.06±40.01 1106.98±8.90	0.77 0.44 0.38

As to T_{ef}=2900 K estimated by Alvarez and Mennessier (1997), this corresponds to a spectral class of about M8. However, at them it is a question of stars-giants of the Mira-type. Nevertheless, the radius of S Per seems to be overestimated. In this connection, it is interesting to substitute the obtained value in the known formula from the theory of stellar pulsations (Cox, 1980)

$$P_0 = Q\sqrt{\frac{(R/R_O)^3}{(M/M_O)^2}} ,$$

where R_0, M_0 – are solar radius and mass, respectively.

For our value of period of P_0=816.^{d}8, Q≈0.077, that, generally speaking, should agree with theoretical values (Q from 0.06 till 0.08) for semiregular variables.

Conclusions. Thus, the extensive and non-uniform class of semiregular variables requires the close approach and audit which quite with advantage can be lead, using modern mathematical methods and already existing observation material.

In the present work it is shown, that stable light curve and a period of pulsations can be used to estimate of physical parameters of a star.

All variations of the photometric parameters inherent to considered stars, for example, the form of a light curve in visual area, undoubtedly, are connected to physical properties of a star. Therefore it is possible to make classification of semiregular variable stars on these parameters. Similar work has been done by Chinarova and Andronov (2001). Average values of parameters of light curves for 173 SR-stars of various subtypes were received as a result. This result can serve as the basis for mathematical modeling of the complicated processes taking place in a star and its envelope, influencing on the shape of the light curve.

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