ONE-DIMENSIONAL SIMULATIONS OF INHOMOGENEITY GROWTH IN PRESSURELESS GRAVITATING MATTER

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ABSTRACT. We study a model of the inhomogeneity growth in the Universe filled with a pressureless matter. The standard hydrodynamical equations for the cosmological perturbations in the comoving frame are treated taking into account all nonlinear terms. We consider perturbations of the uniform isotropic cosmological background but with plane periodic initial conditions for the perturbations. The problem is reduced to ordinary differential equations for an infinite chain of Fourier coefficients for hydrodynamical variables. We perform a numerical integration of these equations (with proper truncation) for an ensemble of random initial conditions. The power spectrum of the density contrast in obtained by means of an averaging procedure.

Key words: large scale structure, cosmology, hydrodynamics.

1. Introduction

It is now well known that the (non-baryonic) dark matter (DM) constitutes a considerable fraction of the cosmological density. However, little is known about the DM structure. This presents the serious challenge to computational astrophysics because different DM models can manifest themselves in different scenarios of structure formation on the galactic and extragalactic level. The related problems involve numerical simulations of the galaxy formation process, working out predictions concerning the galactic environment (number of dwarf satellite galaxies) and structure of their central regions (the cusp-core problem) either with cold dark matter (DM) or within warm DM models (Navarro *et.al.* 1996, Navarro *et. al.* 1997, Avila *et. al.* 2001, Bode *et. al.* 2001, Moore *et. al.* 2006, Schneider *et. al.* 2012). Interesting possibility to obtain bounds on masses of DM particles stems from observations of Ly-α forest (see, e.g., Boyarsky *et. al.* 2009) and references therein); comparison of theory with observations requires accurate calculations of the power spectrum of cosmological inhomogeneity on kiloparsec scales. In this field the computational methods are well known that use the N-body simulations combined with the smoothed particle hydrodynamics (Springel 2005, Brandbyge *et. al.* 2008, Schneider *et. al.* 2012). However, it is important to have independent numerical schemes.

On the other hand, some analytical and semi-analytical methods to study a weak cosmological inhomogeneity were proposed (Bernard *et. al.* 2002, Taruya *et. al.* 2002, Wong 2008), which work with the Fourier-transformed hydrodynamical variables. These schemes either use some low order perturbative schemes or deal with the correlation functions under some additional *a*priori assumptions in order to close the infinite chain of unknown functions (Pietroni 2008, Lesgourgues *et. al.* 2009). A comparison of different approaches can be found in (Carlson *et. al.* 2009).

In this paper we use an alternative method with a direct integration of hydrodynamical equations in the Fourier space. The hydrodynamical variables are considered within the spatial "cell of periodicity", so we can reduce the problem to an infinite chain of interacting equations for the Fourier coefficients. Therefore, we also must use some truncation for these coefficients, which, however, can be controlled by usual numerical means. Here our aim is to demonstrate a workability of the numerical method.

In our previous work (Sliusar & Zhdanov 2012) we treated analogous fully one-dimensional problem using the same method. The problem treated in the present paper is in fact a special case of *three-dimensional* problem though with the plane initial conditions that depend upon only one spatial variable x. The main distinctive feature of this paper is due to different relations for the uniform background. In (Sliusar & Zhdanov 2012) we succeeded to find an analytical (though implicit) solution that made it possible a comparison with the numerical solution; in the present paper we do not have such a possibility.

In section 2 we write down the equations for the Fourier coefficients and present some results for the solutions with a special initial conditions in order to see how initial perturbation proliferates from low wavenumbers to larger ones. The we consider a set of solutions with randomly distributed initial conditions to obtain a power spectrum after a statistical averaging.

2. Numerical simulations

Further we deal with the homogenous cosmological background; R stands for the cosmological scale factor and τ is the "conformal time", x is the comoving spatial coordinate. For the epoch after the recombination one can neglect the contribution of the cosmological constant up to the redshifts $z \sim 2 \div 3$ so it is quite reasonable to assume $R \sim \tau^2$ typical for purely cold dark matter model. Hereafter $\delta(x, \tau)$ is the density contrast, $\theta(x, \tau) = \partial v/\partial x$, v is the peculiar velocity. We assume L to be a spatial periodicity scale so we can deal with the (one-dimensional) Fourier coefficients ($n = 0, \pm 1, \pm 2, ...$)

$$\theta(x, \tau) = \sum_n a_n(\tau) \exp(ik_n x),$$

$$\delta(x, \tau) = \sum_n b_n(\tau) \exp(ik_n x),$$

where $k_n = 2\pi n/L$ and choice of L depends on a concrete problem, its scale accuracy etc. For simplicity we confine ourselves to a symmetric case: $\theta(x, \tau) = \theta(-x, \tau)$ and $\delta(x, \tau) = \delta(-x, \tau)$ are even functions yielding

$$a_n(\tau) = a_{-n}(\tau), \quad b_n(\tau) = b_{-n}(\tau). \qquad (1)$$

Also, we shall exclude terms $a_0 \equiv 0, \quad b_0 \equiv 0$. One can show that this assumption (as well as the previous one about symmetry) can be obtained as a consequence of appropriate initial conditions. These simplifications are not essential in principle, and in a concrete situation they can be compensated by some choice of L. Therefore, below we deal only with natural values of n.

Standard considerations (see, e.g., (Wong 2008, Pietroni 2008, Lesgourgues *et. al.* 2009, Sliusar & Zhdanov 2012) of hydrodynamical equations, which also take into account Poisson equation for the gravitational potential, lead to the equations for the Fourier coefficients. We rewrite these equations taking into account symmetry relations (1):

$$\frac{da_n}{d\tau} = -\frac{2a_n}{\tau} - \frac{6b_n}{\tau^2} - n\sum_{p=1}^{\infty} \frac{a_p}{p}\left(a_{|n-p|} - a_{n+p}\right), \qquad (2)$$

$$\frac{db_n}{d\tau} = -a_n - n\sum_{p=1}^{\infty} \frac{a_p}{p}\left(b_{|n-p|} - b_{n+p}\right) = 0, \qquad (3)$$

$n = 1, 2, ...$

The numerical solution of the equations (2), (3) was performed using the 4-th order Runge-Kutta method after a truncation of the infinite chain of coefficients a_n, b_n. Calculations were carried out by the specially written GPGPU code using OpenCL SDK by AMD. The time, required to calculate a_n and b_n, in case of 256 values of n (values of wavenumbers k) is 10 seconds for single realization of initial conditions. It is important to note that it is easy to extend the corresponding algorithms to the 3-D case.

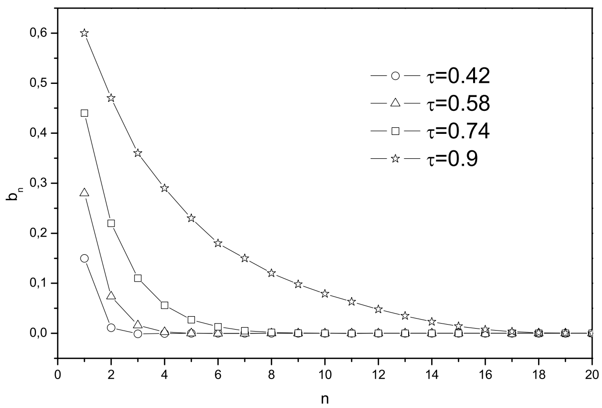

Figure 1: Symmetric problem: Fourier coefficients for the density contrast at conformal times $\tau = 0.42, 0.58, 0.74, 0.9$. Initial conditions at $\tau = 0.1$: all $a_n = 0$, $b_n = 0$ except $b_1 = 0.01$

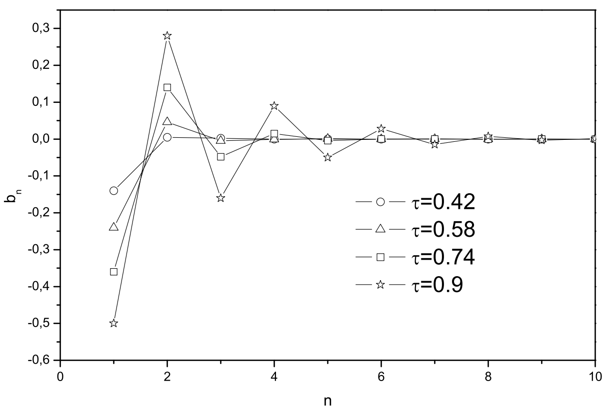

Figure 2: The same as on Fig.1 with initial conditions at $\tau = 0.1$: $b_1 = -0.01$, the other coefficients = 0

We calculated coefficients a_n, b_n for several values of τ with the same initial conditions given at $\tau = 0.1$. All coefficients a_n, b_n at $\tau = 0.1$ were assumed zero except b_1 which has been chosen in different ways (see figure captions). This means that at the initial time only a mode with the largest scale was present and the fluid velocity is exactly that of the homogeneous expansion. On Figs.1,2 we present the coefficients b_n of the density contrast for several values of τ. We observe a successive growth of $|b_n|$. For example, on Fig. 1 we see almost monotonous growth of b_n with time which shows a proliferation of perturbation from smaller to larger values of n (in fact there is also a very small precursor wave of negative amplitude which is indistinguishable on this figure). An analogous proliferation from larger to smaller wavenumbers is well known in the hydrodynamical turbulence theory. The solution of Eqs.(2,3) cannot be extended for all τ that is typical for ordinary differential equations with a right-hand side containing a nonlinear (here quadratic) terms. In our case the solution described on Fig. 1 diverges after $\tau \sim 0.9$. This infinite growth corresponds to collapse due to the absence of peculiar velocity at the initial moment and absence of pressure that might resist the gravitational attraction.

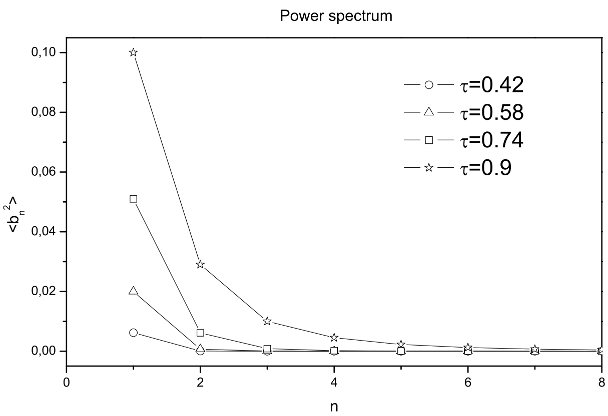

Figure 3: Averages $< b_n^2(\tau) >$) over 100 realizations for $\tau = 0.42, 0.58, 0.74, 0.9$ with random initial data $b_1(0.1)$ that have been distributed uniformly on $[-0.01, 0.01]$

Then we calculated a set of solutions with random initial data for b_1 with subsequent averaging to obtain $< b_n^2(\tau) >$, which is essentially a power spectrum of the inhomogeneity (here $< ... >$ stands for the statistical average). The result is shown on Fig. 3.

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