# DARK SIDE OF QUARK BAG MODEL

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ABSTRACT. We calculate the present expansion of our Universe endowed with relict colored objects – quarks and gluons – that survived hadronization either as isolated islands of quark-gluon nuggets or spread uniformly in the Universe. The QNs can play the role of dark matter.
**Key words**: dark matter theory

## 1. Introduction

We consider the possibility that a small fraction of colored objects – quarks and gluons – escaped hadronization. They may survive as islands of colored particles, called quark-gluon nuggets (for brevity sometimes also quark nuggets (QNs)). This possibility was first considered by E. Witten [1] and scrutinized further in [2-4]. In his paper [1], E. Witten discusses the possibility that QNs can survive even at zero temperature and pressure. If so, the "hot" quark-gluon phase in the form of QNs may affect the present expansion of the Universe. Indeed, our investigation shows that nuggets can contribute to dark matter provided that their interaction with ordinary matter is weak.

## 2. Equations of state in the quark-gluon bag model

We first briefly remind the quark bag equation of state, a simple model of quark confinement. For vanishing chemical potential, $\mu = 0$, it is a system of two equations:

$$p_q(T) = A_q T^4 - B, \qquad (2.1)$$

$$p_h(T) = A_h T^4. \qquad (2.2)$$

The first line corresponds to the "hot" phase of deconfined quarks and gluons, and the second one relates to confined particles, i.e. hadrons. A system of strongly interacting particles, made of free quarks and gluons, is cooling down and meets the "cold" phase transforming into colorless hadrons. The coefficients are defined by the degrees of freedom and are equal to: $A_q \approx 1.75$, $A_h \approx 0.33$, $B = (A_q - A_h)T_c^4$ and $T_c \approx 200\ MeV$. Knowing the pressure, $P(T)$, for $\mu = 0$, one can easily calculate the remaining thermodynamical quantities, e.g., for the energy density we have

$$\varepsilon(T) = T\frac{dp}{dT} - p. \qquad (2.3)$$

The above equations of state (EoS) is not unique. Here we will use one, which was considered by C. Källman [5], who introduced a temperature-dependent bag "constant", namely, by replacing in the first line of the EoS, eq. (2.1), $B \to B(T) = \bar{B}T$, where $\bar{B} = (A_q - A_h)T_c^3$. This modification has immediate consequences, namely, by producing a minimum in the "hot" line of the EoS, corresponding to metastable deeply supercooled states of the deconfined strongly interacting matter. Also, it drives inflation of the Universe, as shown in [6, 7].

Since our idea is that a small fraction of deconfined quarks and gluons survives to present days, we are interested in the "hot" branch of the bag EoS. As we mentioned above, there is a number of different modifications of eq. (2.1). For our present purposes, however, two simple representatives will be sufficient. They are the Källman modified model (which we call Model I):

$$p_q(T) = A_q T^4 - \bar{B}T \equiv \bar{A}_1 T + \bar{A}_4 T^4, \qquad (2.4)$$

and the original model (Model II) described by eq. (2.1):

$$p_q(T) = A_q T^4 - B \equiv \bar{A}_0 + \bar{A}_4 T^4. \qquad (2.5)$$

## 3. Quark nuggets

Now, we want to investigate cosmological consequences of this assumption. Obviously, for considered models, a cosmological scenario strongly depends on thermodynamical properties of quark-gluon plasma (QGP). We focus on two possible eqs. (2.4) and (2.5). With the help of standard thermodynamical eq. (2.3) we get the expressions for the energy density:

$$\varepsilon = 3\bar{A}_4 T^4 \tag{3.1}$$

and

$$\varepsilon = -\bar{A}_0 + 3\bar{A}_4 T^4 \tag{3.2}$$

for Model I and Model II, respectively. Eqs. (2.4), (2.5), (3.1) and (3.2) describe the pressure and energy density inside of the nuggets. The total pressure and energy density of all nuggets in the Universe can be calculated as follows. Let us take, e.g., Model I with eq. (2.4). Then, for total pressure of nuggets we get

$$P = \frac{\sum_i p_{qi} v_i}{V} = \frac{A_1 T + A_4 T^4}{a^3}, \tag{3.3}$$

where $p_{qi}$ is the pressure of the i-th nugget with the volume $v_i$ and $V \propto a^3$ is the total volume of the Universe ($a$ is the scale factor of the Friedmann-Robertson-Walker metric). We consider the case where all nuggets have the same pressure (2.4) and their volumes are either constant or only slightly varying with time. The total volume of nuggets $\sum_i v_i$ is included in the coefficients $A_1$ and $A_4$ (i.e. $A_{1,4}$ have dimension $\bar{A}_{1,4} \times cm^3$). Therefore,

$$\frac{A_1}{A_4} = \frac{\bar{A}_1}{\bar{A}_4} = -0.8114 T_c^3. \tag{3.4}$$

Similarly, from eq. (3.1), for the energy density of all nuggets we get

$$\mathrm{E} = \frac{3A_4 T^4}{a^3}. \tag{3.5}$$

The same procedure holds for the Model II. Let us consider two models separately.

**Model I**

Here, the pressure and energy density of all nuggets are given by the above formulae (3.3) and (3.5), respectively. In these formulae, temperature is a function of the scale factor $a$: $T = T(a)$. Let us specify this dependence. From the energy conservation equation

$$d(\mathrm{E}a^3) + P d(a^3) = 0 \tag{3.6}$$

we can easily get

$$T = \left( \frac{(C/a)^{3/4} - A_1}{A_4} \right)^{1/3}. \tag{3.7}$$

We consider the model where the coefficients $A_1 < 0$ and $A_4 > 0$. In eq. (3.7), $C \geq 0$ is the constant of integration which is defined by the temperature $T_0$ and scale factor $a_0$ at the present time:

$$C = (A_1 + A_4 T_0^3)^{3/4} a_0 = A_4^{4/3}(-0.8114 T_c^3 + T_0^3)^{4/3} a_0. \tag{3.8}$$

The temperature $T$ tends to the constant value when the scale factor approaches infinity:

$$T \to T_\infty = \left( \frac{-A_1}{A_4} \right)^{1/3} = 0.9327 T_c \text{ for } a \to \infty, \tag{3.9}$$

and the pressure goes asymptotically to zero: $P \to 0$. On the other hand, for $C \equiv 0$, the temperature is constant all the time $T \equiv T_\infty$, and nuggets behave as a matter with zero pressure $P = 0$. It is worth noting that in this model the temperature of the QNs at present time is not arbitrary low, rather it is close to the critical temperature $T_c$ of the phase transition.

We consider our Universe starting from the moment when we can drop the radiation. It is well known that the radiation dominated (RD) stage is much shorter than the matter dominated (MD) stage. Hence, the neglect of the RD stage does not affect much the estimate of the lifetime of the Universe. Starting from the MD stage, the first Friedmann equation for our model reads

$$3\frac{\mathrm{H}^2 + K}{a^2} = \kappa \mathrm{E} + \kappa \varepsilon_0^{mat} \left( \frac{a_0}{a} \right)^3 + \Lambda. \tag{3.10}$$

where we take into account the cosmological constant $\Lambda$ and the (usual + dark) matter with the present value of the energy density $\varepsilon_0^{mat}$. In (3.10), $\mathrm{H} = a'/a = (da/d\eta)/a$, $\kappa = 8\pi G_N / c^4$, $G_N$ is the gravitational constant and $K = \pm 1,\ 0$ is the spatial curvature. The conformal time $\eta$ is connected with the synchronous time $t$: $a d\eta = c dt$. Taking into account eqs. (3.5) and (3.7), we get for the Hubble parameter $H = (1/a)da/dt = (c/a^2)da/d\eta$ the following expression:

$$H^2 = H_0^2 \{ \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \left( \frac{a_0}{a} \right)^{\frac{9}{4}} \right]^{\frac{4}{3}} + \Omega_M \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda + \Omega_K \left( \frac{a_0}{a} \right)^2 \}, \tag{3.11}$$

where the cosmological parameters are

$$\Omega_M = \frac{c^2}{3H_0^2} \kappa \varepsilon_0^{mat}, \ \Omega_\Lambda = \frac{c^2}{3H_0^2} \Lambda, \quad \Omega_K = -K \left( \frac{c}{a_0 H_0} \right)^2 \tag{3.12}$$

and we introduce the dimensionless parameters

$$\beta = \left( \frac{C}{a_0} \right)^{3/4} \frac{1}{A_4^{1/4}} \left( \frac{\kappa c^2}{a_0^3 H_0^2} \right)^{3/4}, \ \gamma = -\frac{A_1}{A_4^{1/4}} \left( \frac{\kappa c^2}{a_0^3 H_0^2} \right)^{3/4}. \tag{3.13}$$

From the second Friedmann equation

$$\frac{2\mathrm{H}' + \mathrm{H}^2 + K}{a^2} = -\kappa P + \Lambda \tag{3.14}$$

after some obvious algebra we obtain the deceleration parameter

$$-q = \left( \frac{H_0}{H} \right)^2 \{ \frac{\gamma}{2} \left[ \beta \left( \frac{a_0}{a} \right)^{39/4} + \gamma \left( \frac{a_0}{a} \right)^9 \right]^{1/3} - \left[ \beta \left( \frac{a_0}{a} \right)^3 + \gamma \left( \frac{a_0}{a} \right)^{9/4} \right]^{4/3} - \frac{\Omega_M}{2} \left( \frac{a_0}{a} \right)^3 + \Omega_\Lambda \}. \tag{3.15}$$

At the present time $t_0$, eqs. (3.11) and (3.15) read

$$1=(\beta+\gamma)^{4/3}+\Omega_M+\Omega_\Lambda+\Omega_K, \quad (3.16)$$

$$-q_0=\frac{\gamma}{2}(\beta+\gamma)^{1/3}-(\beta+\gamma)^{4/3}-\frac{\Omega_M}{2}+\Omega_\Lambda. \quad (3.17)$$

Additionally, we obtain from (3.11) the differential equation

$$d\tilde{t}=\frac{\tilde{a}d\tilde{a}}{\sqrt{\left(\beta+\gamma\tilde{a}^{3/4}\right)^{4/3}+\Omega_M\tilde{a}+\Omega_\Lambda\tilde{a}^4+\Omega_K\tilde{a}^2}}, \quad (3.18)$$

where we introduce the dimensionless quantities

$$\tilde{a}=\frac{a}{a_0}, \ \tilde{t}=H_0t. \quad (3.19)$$

Therefore, the age of the Universe $\tilde{t}_0$ is defined by the equality

$$-\tilde{t}_0=\int_1^0\frac{\tilde{a}d\tilde{a}}{\sqrt{\left(\beta+\gamma\tilde{a}^{3/4}\right)^{4/3}+\Omega_M\tilde{a}+\Omega_\Lambda\tilde{a}^4+\Omega_K\tilde{a}^2}} \quad (3.20)$$

Now, we consider the case of the flat space $K=0\rightarrow\Omega_K=0$ . Then, eq. (3.16) reads

$$1=(\beta+\gamma)^{4/3}+\Omega_M+\Omega_\Lambda. \quad (3.21)$$

Eq. (3.17) demonstrates that accelerated expansion of the Universe at the present time (i.e. $-q_0>0$ ) can be ensured by the first and the last terms on the right side of this equation. It is tempting to explain the acceleration only at the expense of the first term, i.e. due to the presence of QNs when the cosmological constant is absent. However, simple analysis of eqs. (3.17) and (3.21) in the case $\Omega_\Lambda=0$ shows that the acceleration $-q_0>0$ is achieved only for $\beta<0$ that contradicts our model. The inclusion of the negative curvature $\Omega_K>0$ does not affect this conclusion due to the smallness of $\Omega_K$ .

As we have mentioned above, nuggets behave as matter either asymptotically when $a\rightarrow\infty$ or for all time in the case $C=0\rightarrow\beta=0$ . In the latter case we can exactly restore the ΛCDM model so long as eqs. (3.17) and (3.21) take the usual form for this model:

$$1=\Omega_{M,total}+\Omega_\Lambda \quad (3.22)$$

and

$$-q_0=-\frac{1}{2}\Omega_{M,total}+\Omega_\Lambda \ \Rightarrow \ \Omega_\Lambda=\frac{1}{3}-\frac{2}{3}q_0, \quad (3.23)$$

where $\Omega_{M,total}\equiv\gamma^{4/3}+\Omega_M$ . Let $\Omega_M$ correspond to just the visible matter. According to observations, $\Omega_M\approx 0.04$ . Then, we can easily restore the parameters of the ΛCDM model. For example, taking the deceleration parameter $q_0\approx -0.595$ , as in the ΛCDM model, we get $\Omega_\Lambda\approx 0.73$ and $\gamma\approx 0.33\rightarrow\gamma^{4/3}\approx 0.23$ . Therefore, $\Omega_{M,total}\approx 0.27$ . For the Universe age we get from (3.18) (where we should put $\beta=0, \quad \Omega_K=0$ ) $\tilde{t}_0\approx 1\Rightarrow t_0\approx H_0^{-1}\sim 13.7\times 10^9\,yr$ . Hence, weekly interacting QNs may be candidates for dark matter.

A similar analysis of Model 2 leads to the same physical results

## 4. Conclusion

We introduced the expansion of the present Universe, using the "hot", i.e. the quark-gluon branch of the bag EoS. Although we made reference to the role of this type of the EoS during the early universe, namely its inflation phase, here we postponed possible speculations about the continuous evolution of the universe, within the present formalism, from it early, quark-gluon stage to the present days, admitting only the possible continuity in the existence in the present Universe of a small fraction of colored objects – quarks and gluons – which escaped hadronization. Our scenario showed that the colored objects survived in the form of isolated islands, called quark-gluon nuggets. This cosmological scenario is defined by EoS of QGP. We focus on two possible eqs. (2.4) and (2.5) dubbed Model I and Model II, respectively. We have shown that there are no fundamental differences in the obtained conclusions for these models, and that weekly interacting (with visible matter) QNs can play the role of dark matter for both of the models.Therefore, the considered scenario provide new possible ways of solving the problems of dark matter.

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