MAGNETOROTATIONAL SUPERNOVAE AND MAGNETO-DIFFERENTIAL-ROTATIONAL INSTABILITY

S.G. Moiseenko[1], G.S. Bisnovatyi-Kogan[1,2]

[1] Space Research Institute, Russian Academy of Sciences, Moscow, Russia, *moiseenko@iki.rssi.ru, gkogan@iki.rssi.ru*

[2] National Research Nuclear University MEPhI, Moscow, Russia

ABSTRACT. Results of simulations of magnetorotational(MR) core-collapse supernova explosion mechanism are presented. For the simulations we have used specially developed Lagrangian completely conservative operator-difference scheme on triangular grid of variable structure. 2D numerical simulations show that the Magneto-Differential-Rotational Instability (MDRI) (Tayler-type instability) develops which leads to the exponential growth of all components of magnetic field. The instability reduces time evolution of the MR explosion. The explosion energy found in MR explosion corresponds to observations. We made simulations of the MR mechanism supernova explosion with different equations of state and different procedures of neutrino transport. We found that the results of the MR explosion does not significantly change.

Key words: Stars: Supernovae; Magnetohydrodynamics; Numerical simulations.

1. Introduction

Supernova explosions are ones of the brightest and most energetic events in the Universe. The explanation of the physical mechanism of core-collapsed supernova explosions is one of important problems of modern astrophysics. The mechanism of core-collapse supernova explosion connected with magnetic field and rotation is called magnetorotational(MR). The idea of MR mechanism was suggested by G.S.Bisnovatyi-Kogan in 1970 (Bisnovatyi-Kogan, 1970). Due to the collapse of the iron core significant part of the gravitational energy of the core is emitted in the form of neutrino. If the iron core has initial rotation before collapse then due to the nonuniform contraction the rotation in the core will be differential. If the core had initial poloidal magnetic field the differential rotation leads to the appearing and amplification of the toroidal component of the magnetic field. When the magnetic pressure becomes comparable with gas pressure the compression wave start to move outwards the core. Running over steeply decreasing density profile it transforms into the fast MHD shock wave. The magnetic field plays role of the MHD piston which supports the shock.

The first simulations of the MR processes in stars have been done by LeBlanc & Wilson, 1970, after which MR processes in the stars in relation to the core collapse supernova explosion had been simulated by Bisnovatyi-Kogan et al., 1976, Ardelyan et al., 1979, Müller & Hillebrandt, 1979, Ohnishi,1983, Symbalisty, 1984.

In last several decades a number of papers were devoted to the numerical simulations of MR mechanism (Yamada and Sawai,2004, Takiwaki et al., 2004, Kotake et al., 2004, Kotake et al., 2006, Ardeljan, et al. 2005, Sawai et al., 2005, Moiseenko et al., 2006, Obergaulinger et al., 2006, Burrows et al., 2007, Dessartet al., 2007, Scheidegger et al., 2008, Bisnovatyi-Kogan et al., 2008, Mikami et al. 2008, Takiwaki et al., 2009, Takiwaki and Kotake, 2011, Endeve et al., 2012, Sawai et al, 2013).

2. Formulation of the problem

2.1. Basic equations

Consider a set of magnetohydrodynamical equations with selfgravitation and infinite conductivity:

$$\begin{gathered}\frac{\mathrm{d}\mathbf{x}}{\mathrm{d}t} = \mathbf{v},\\ \frac{\mathrm{d}\rho}{\mathrm{d}t} + \rho\nabla\cdot\mathbf{v} = 0,\\ \rho\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -\mathrm{grad}\left(P + \frac{\mathbf{H}\cdot\mathbf{H}}{8\pi}\right) + \frac{\nabla\cdot(\mathbf{H}\otimes\mathbf{H})}{4\pi} - \rho\nabla\Phi,\\ \rho\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{\mathbf{H}}{\rho}\right) = \mathbf{H}\cdot\nabla\mathbf{v},\ \Delta\Phi = 4\pi G\rho, \quad (1)\\ \rho\frac{\mathrm{d}\varepsilon}{\mathrm{d}t} + P\nabla\cdot\mathbf{v} + \rho F(\rho,T) = 0,\\ P = P(\rho,T),\ \varepsilon = \varepsilon(\rho,T),\end{gathered}$$

where $\frac{\mathrm{d}}{\mathrm{d}t} = \frac{\partial}{\partial t} + \mathbf{v}\cdot\nabla$ is the total time derivative, $\mathbf{x} = (r,\varphi,z)$, $\mathbf{v} = (v_r, v_\varphi, v_z)$ is the velocity vector, ρ is the density, P is the pressure, $\mathbf{H} = (H_r, H_\varphi, H_z)$ is the magnetic field vector, Φ is the gravitational potential, ε is the internal energy, G is gravitational constant, $\mathbf{H} \otimes \mathbf{H}$ is the tensor of rank 2, and $F(\rho, T)$ is the rate of neutrino losses.

r, φ, and z are spatial Lagrangian coordinates, i.e. $r = r(r_0, \varphi_0,$ and $z_0, t)$, $\varphi = \varphi(r_0, \varphi_0, z_0, t)$, and $z = z(r_0, \varphi_0, z_0, t)$, where r_0, φ_0, z_0 are the initial coordinates of material points of the matter.

Taking into account symmetry assumptions ($\frac{\partial}{\partial\varphi} = 0$), the divergency of the tensor $\mathbf{H}\otimes\mathbf{H}$ can be presented in the following form:

$$\nabla\cdot(\mathbf{H}\otimes\mathbf{H}) = \begin{pmatrix} \frac{1}{r}\frac{\partial(rH_rH_r)}{\partial r} + \frac{\partial(H_zH_r)}{\partial z} - \frac{1}{r}H_\varphi H_\varphi \\ \frac{1}{r}\frac{\partial(rH_rH_\varphi)}{\partial r} + \frac{\partial(H_zH_\varphi)}{\partial z} + \frac{1}{r}H_\varphi H_r \\ \frac{1}{r}\frac{\partial(rH_rH_z)}{\partial r} + \frac{\partial(H_zH_z)}{\partial z} \end{pmatrix}.$$

Axial symmetry ($\frac{\partial}{\partial\varphi} = 0$) and symmetry to the equatorial plane are assumed. The problem is solved in the restricted domain. At $t = 0$ the domain is restricted by the rotational axis $r \geq 0$, equatorial plane $z \geq 0$, and outer boundary of the star where the density of the matter is zero, while poloidal components of the magnetic field H_r, and H_z can be non-zero.

At the rotational axis ($r = 0$) the following boundary conditions are defined: $(\nabla\Phi)_r = 0$, $v_r = 0$. At the equatorial plane ($z = 0$) the boundary conditions are: $(\nabla\Phi)_z = 0$, $v_z = 0$. At the outer boundary (boundary with vacuum) the following condition is defined: $P_{\text{outer boundary}} = 0$.

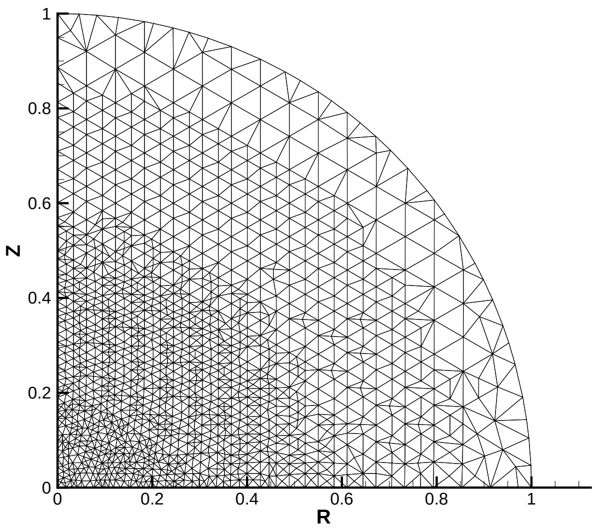

Figure 1: Example of the triangular Lagrangian grid.

2.2. Numerical method

The numerical method used in present simulations is based on the implicit operator-difference completely conservative scheme on the Lagrangian triangular grid (Fig.1) of variable structure. The implicitness of the applied numerical scheme allows to make large time steps. It is important to use implicit scheme in such kind of problems due to the presence of two strongly different timescales. The small timescale is defined by the huge sound velocity in the central parts of the star. The big time scale is defined by the characteristic timescale of the evolution of the magnetic field. Conservation properties of the numerical scheme are important for the exact fulfillment of the energy balance and divergence-free property of the magnetic field.

During the simulation of the MRE the time step for the implicit scheme was $\sim 10 \div 300$ times larger than time step for the explicit scheme (CFL stability condition). It means that the total number of time steps is $10 \div 300$ times less than it would have been done for explicit scheme, what allows us to decrease time approximation error. We did not make direct comparison of CPU time per time step for our implicit scheme and an explicit scheme. Our estimations show that the total number of arithmetic operations for the implicit scheme is ≈ 20 times larger than it is required for explicit scheme. Considerable decrease of the required number of time steps leads to corresponding reduce of the time approximation error.

Grid reconstruction procedure applied here for the reconstruction of the triangular lagrangian grid is used both for the correction of the "quality" of the grid and for the dynamical adaptation of the grid.

The method applied here was developed, and its stability was investigated in the papers by Ardeljan, Kosmachevskii, 1995, Ardeljan N.V, Kosmachevskii K.V., Chernigovskii S.V., 1987 and references therein. It was tested thoroughly with different tests (Ardeljan N.V., Bisnovatyi-Kogan G.S., Moiseenko S.G., 2000).

3. Results

3.1. Core collapse simulation

As the initial step for the simulation of MR supernova explosion we made simulation of the collapse of non magnetized rotating iron core (Ardeljan et al 2004). The ratios between the initial rotational and gravitational energies and between the internal and gravitational energies of the star are the following:

$$\frac{E_{rot}}{E_{grav}} = 0.0057, \ \frac{E_{int}}{E_{grav}} = 0.727.$$

At $t = 0.1424$s the density in the center of the star reaches its maximum value $\rho_{c,max} = 5.655 \cdot 10^{14}$g/cm^3. At the final stage of the core collapse ($t = 0.261$s.) we obtain a differentially rotating configuration. The central proto-neutron star with a radius ~ 12.8km rotates almost rigidly with the rotational period 0.00152s. The angular velocity rapidly decreases with the increase in

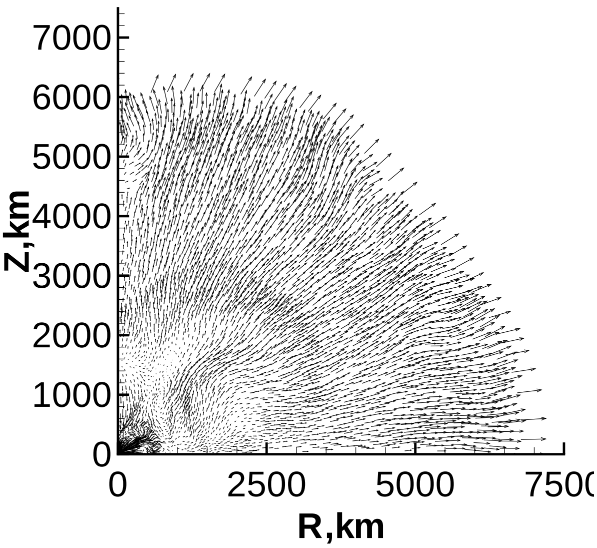

Figure 2: The MR explosion for *quadrupole-like* initial magnetic field. The velocity field at $t = 0.30s$ after 'switching on' the magnetic field.

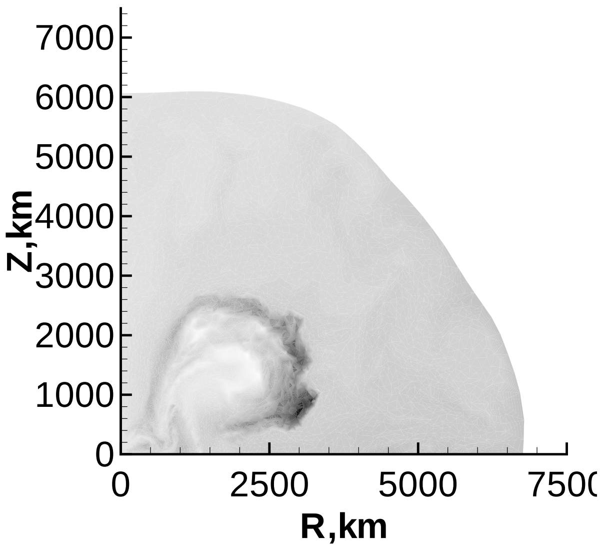

Figure 3: The MR explosion for *quadrupole-like* initial magnetic field. The specific angular momentum spacial distribution at $t = 0.30s$ after 'switching on' the magnetic field.

the distance from the star center. The particles of the matter situated at the outer boundary in the equatorial plain rotate with the period $\sim$ 35s due to strong expansion of the envelope after the collapse of the core.

3.2. Magnetorotational explosion

After core collapse and formation of the proto-neutron star initial poloidal quadrupole-like magnetic field was 'switched on' (Ardeljan, Bisnovatyi-Kogan, Moiseenko, 2005). Due to the differential rotation the toroidal component of the magnetic field H_φ appears and amplifies with the time. At the beginning of the simulations the toroidal component of the magnetic field grows linearly with the time at the periphery of the proto-neutron star. At the developed stage of the H_φ evolution the toroidal magnetic energy begins to grow much faster due to developing magnetorotational instability leading also to a rapid growth of the poloidal components.

Due to the quadrupole-like type of the symmetry of the initial magnetic field the MHD shock is stronger, and it moves faster near the equatorial plane $z = 0$. The matter of the envelope of the star is ejected preferably near the equatorial plane (Figs.2,3).

The shape of the MR supernova explosion qualitatively depends on the initial configuration of the magnetic field. In the paper by Moiseenko, Bisnovatyi-Kogan, Ardeljan, 2005, we made simulations of the MR supernova explosion with the initial dipole-like poloidal magnetic field.

In the case of the dipole-like magnetic significant part of the ejected matter obtains a velocity along the rotational axis (Figs.4,5).

The explosion energy of the MR supernova was in the 'quadrupole' and the 'dipole' cases approximately the same $0.5 \sim 0.6 \times 10^{51}$erg.

At the initial stage of the evolution of the toroidal component of the magnetic field it grows linearly with the time, after about 100 rotational periods of the central core, its linear growth changes to the exponential one, and the poloidal components also start to grow exponentially. The reason for that exponential growth is an onset of the variant of MRI, which was investigated by Tayler, 1973. It is directly connected with the differential rotation, and is induced by it. We may call it Magneto-Differential-Rotational-Instability(MDRI).

We simulated an MR supernova explosion for various initial core masses and rotational energies (Bisnovatyi-Kogan, Moiseenko, Ardelyan, 2008). The initial core mass was varied from $1.2M_\odot$ to $1.7M_\odot$. The specific rotational energy at the time when the magnetic field was switched on, E_{rot}/M_{core}, was varied from $0.19 \cdot 10^{19}$ erg/g to $0.4 \cdot 10^{19}$ erg/g. The explosive energy of an MR supernova increases substantially with the mass of the iron core and the initial rotational energy (angular velocity) . The energy released in MR supernova is sufficient to explain core collapse supernova: $0.5 - 2.6 \cdot 10^{51}$erg (Type II and lb supernovae) (Fig.7). The energies of Type Ic supernovae can be higher, probably due to the collapse of more massive cores, of the order of several tens of $M_\odot$.

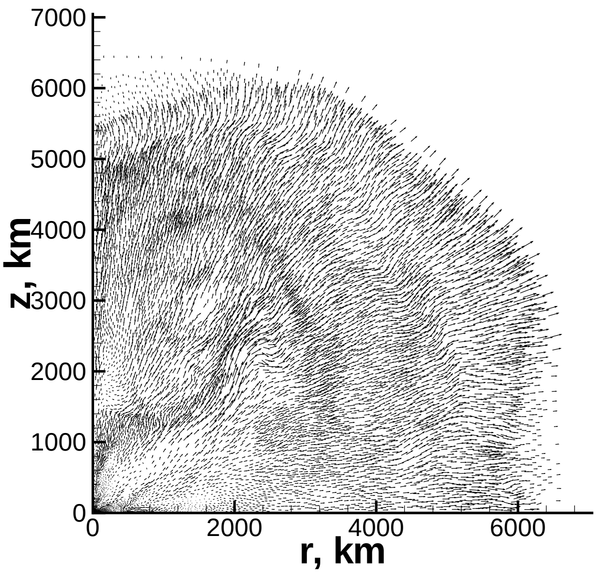

Figure 4: The MR explosion for *dipole-like* initial magnetic field. The velocity field at $t = 0.25s$ after 'switching on' the magnetic field.

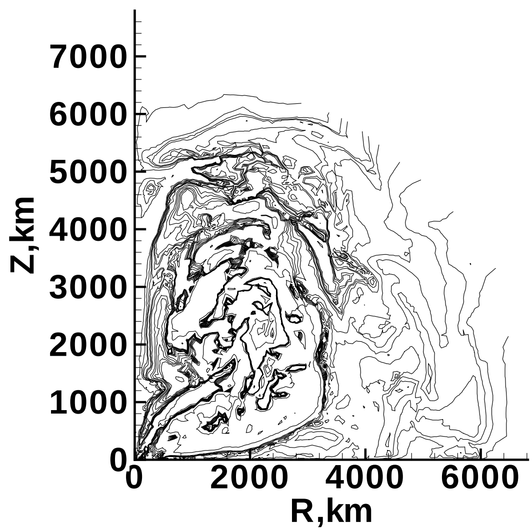

Figure 5: The MR explosion for *dipole-like* initial magnetic field. The specific angular momentum spacial distribution at $t = 0.25s$ after 'switching on' the magnetic field.

3.3 Shen Equation of State(EoS).

We made 2D MR core-collapse supernova simulations by using the tabulated EoS based on the relativistic mean-field theory by Shen et al., 1998. The neutrino cooling was taken into account by a neutrino leakage scheme (Takiwaki et al., 2004).

The simulations of MR supernova explosion for Shen EoS were made using Lagrangian numerical scheme on triangular grid of variable structure. The formulation of the problem of MR supernova explosion, namely the set of equations, initial and boundary conditions, are the same as in the paper by Takiwaki et al., 2004.

We have found a qualitative agreement with the results of analogous simulations achieved by application of ZEUS-2D code in Takiwaki et al., 2004, for the case of extremely strong initial magnetic field ($H_0 = 10^{12}$ G). Namely, in the case of strong initial magnetic field ($H_0 = 10^{12}$ G) we got a prompt MR SN explosion, and formation of a collimated jet. In the case of weaker initial magnetic field ($H_0 = 10^9$ G) (this value of the initial magnetic field is still very strong for presupernova) we obtain a development of Magnito-Differential-Rotational-Instability, delayed explosion in analogy to Takiwaki et al., 2004. The difference in the results of simulations made by Eulerian and Lagrangian numerical methods is in absence of the jet-like ejection in Lagrangian case.

The details of the simulations of MR supernova explosion with the Shen EoS will be published elsewhere.

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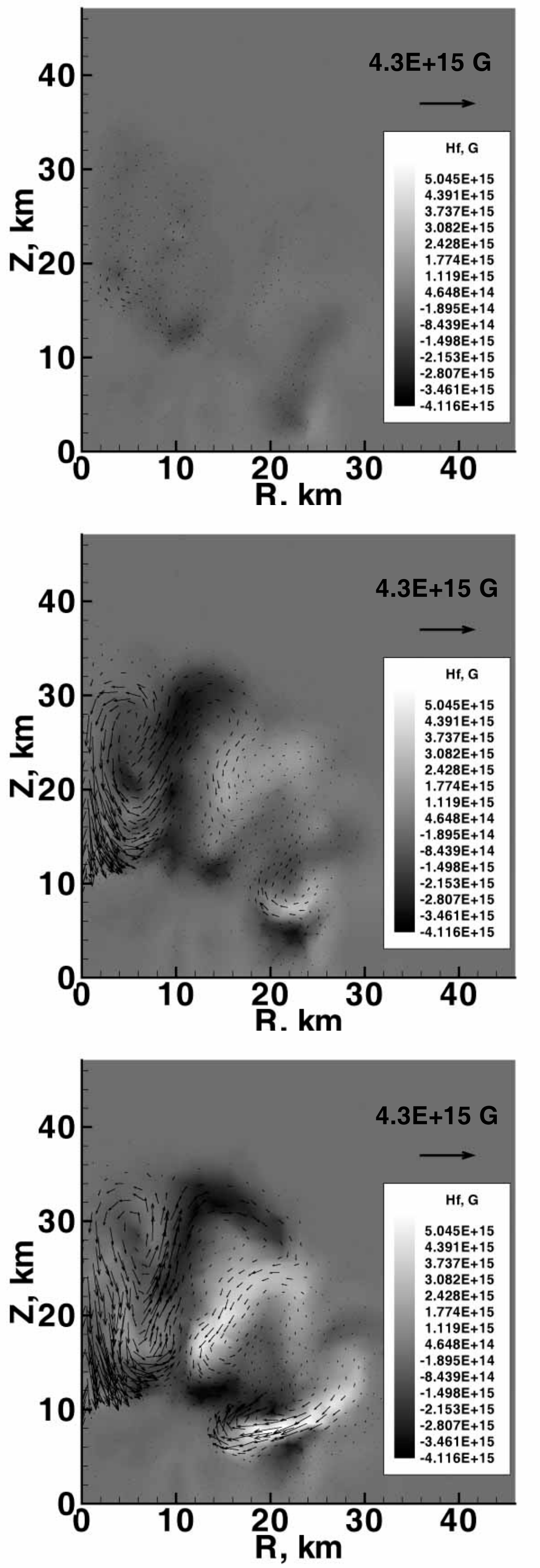

Figure 6: The MDRI development for time moments $t = 0.0045s$, $0.018s$, $0.042s$. Gray scale is the toroidal field H_φ levels. Arrows show a direction and strength of the poloidal magnetic field H_r, H_z.

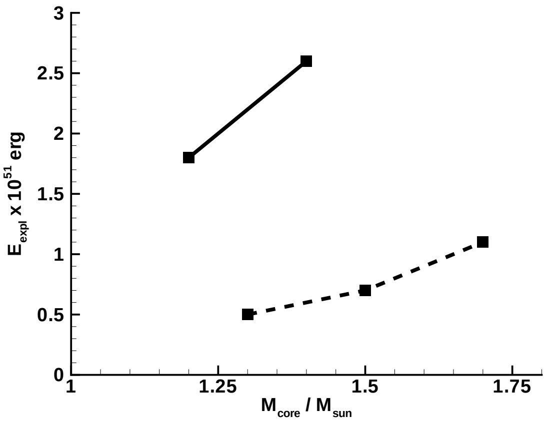

Figure 7: The dependence of the MR supernova explosion energy on the initial core mass for the different values of the specific rotational energy just before the start of the magnetic field evolution $E_{rot}/M_{core} \approx 0.39 - 0.40 \cdot 10^{19}$ erg/g (solid line) $E_{rot}/M_{core} \approx 0.19 - 0.23 \cdot 10^{19}$ erg/g (dashed line) (before collapse).

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