DETERMINATION OF THE EPHEMERIS ACCURACY FOR AJISAI, LAGEOS AND ETALON SATELLITES, OBTAINED WITH A SIMPLIFIED NUMERICAL MOTION MODEL USING THE ILRS COORDINATES

I. V. Kara

Astronomical Observatory of the I. I. Mechnikov Odessa National University
Shevchenko Park, 65014, Odessa, Ukraine, *lionkiv@gmail.com*

ABSTRACT. This paper describes a simplified numerical model of passive artificial Earth satellite (AES) motion. The model accuracy is determined using the International Laser Ranging Service (ILRS) high-precision coordinates. Those data are freely available on http://ilrs.gsfc.nasa.gov. The differential equations of the AES motion are solved by the Everhart numerical method of 17th and 19th orders with the integration step automatic correction. The comparison between the AES coordinates computed with the motion model and the ILRS coordinates enabled to determine the accuracy of the ephemerides obtained. As a result, the discrepancy of the computed Etalon-1 ephemerides from the ILRS data is about 10” for a one-year ephemeris.

Introduction

The development of the space industry in Ukraine demands advancement of the artificial Earth satellite (AES) tracking networks. Such networks must provide the AES tracking and safety of their motion. That is a very important task as the AES cost is very high, and loss of a satellite already put into space threatens the country with not only heavy economic losses, but also with losses of international launch contracts. So far the satellite tracking can be performed in several observatories in Ukraine. The accuracy of the obtained observation data has been continuously increasing, and the methods and instruments of observation has been modified and retrofitted. To ensure safe AES operation, numerical motion models for passive object tracking are required as the space debris (SD) poses the greatest threat to satellites.

This paper describes a simplified numerical motion model for the near-Earth space objects. The model is primarily focused on high speed of position computation for objects at altitude above 1,500 km.

AES motion equations

Equations of the near-Earth space object motion in the Earth-centred inertial coordinate system (Cartesian coordinate frame) take the following form:

$$\frac{d^2\vec{r}}{dt^2} = \frac{\partial U}{\partial \vec{r}} + \vec{a}_{MS} + \vec{a}_{Sp}. \quad (1)$$

where $\vec{r}$ – the object position vectors in the indicated coordinate frame; t – time. Summand $\frac{\partial U}{\partial \vec{r}}$ on the right side of the equation is related to the accelerations caused by the Earth’s gravitational field; $\vec{a}_{MS}$ – the total perturbing acceleration by the Moon and the Sun; $\vec{a}_{Sp}$ – the light pressure.

Perturbations due to the Earth gravitational potential

The geopotential value in the International Terrestrial Reference System (ITRS) is of the following form [1, pp. 24-26].

$$U = \frac{GM}{r_\oplus}\left\{\sum_{n=2}^{N}\sum_{m=0}^{n}\left(\frac{r_\oplus}{r}\right)^{n+1} P_n^m(\sin\varphi)\left[C_n^m\cos(m\lambda)+S_n^m\sin(m\lambda)\right]\right\} \quad (2)$$

where $GM_\oplus$ and $r_\oplus$ – the geocentric gravitational constant and the Earth’s equatorial radius; $\vec{r} = (x; y; z)$ – the object position vectors in the ITRS; $P_n^m(\sin\varphi)$ – the associated Legendre functions; φ and λ – the AES latitude and longitude in the same coordinate frame; C_n^m and S_n^m – numerical coefficients of the zonal, tesseral and sectorial harmonic expansion of the Earth's gravitational potential.

As partial derivatives $\frac{\partial U}{\partial \vec{r}}$ are used in equation (1), the calculation of those partial derivatives of geopotential (2) presents the main complexity in practice. There are several commonly used recurrent algorithms for computation of geopotential derivatives, for instance, the algorithm suggested by L.Cunningham [3, pp. 71-74]. Simpler derivation of recurrence relations and also a method of smoothing work with imaginary values away were suggested in the algorithm by A. Drozyner and V. A. Brumberg [4]. The algorithm, developed by K. V. Kholshevnikov in 2005, offers an alternate approach to calculation of geopotential derivatives [6].

In this paper the method of direct calculation of geopotential partial derivatives in the Cartesian coordinate frame with regard to recurrent properties of the Legendre polynomials is used as an alternative method of computation the geopotential perturbations. Such method of perturbation computation showed computation speed comparable with the above-described methods, and it is quite intuitive and simple.

To derive a working equation, it is necessary to accomplish a rather large amount of computations, but that eventually results in a single set of equations ready for being used in software algorithmization for any possible number of harmonics. If expansion in an arbitrary number of harmonics is used (1), a set of equations suitable for computation of geopotential partial derivatives can be obtained as a result of a series of manipulations:

$$\frac{\partial U}{\partial x} = GM_{\oplus}\sum_{n=2}^{N}\sum_{m=0}^{n}\frac{r_{\oplus}^{n}}{r^{n+1}}\left[\begin{array}{l}\dfrac{myP_n^m(\sin\varphi)}{x^2+y^2}\left(C_n^m\sin m\lambda - S_n^m\cos m\lambda\right) - \\ -CS\left\{\dfrac{xz}{r^3}dP_n^m + \dfrac{(n+1)x}{r^2}P_n^m(\sin\varphi)\right\}\end{array}\right]$$

$$\frac{\partial U}{\partial y} = GM_{\oplus}\sum_{n=2}^{N}\sum_{m=0}^{n}\frac{r_{\oplus}^{n}}{r^{n+1}}\left[\begin{array}{l}\dfrac{mxP_n^m(\sin\varphi)}{x^2+y^2}\left(S_n^m\cos m\lambda - C_n^m\sin m\lambda\right) - \\ -CS\left\{\dfrac{yz}{r^3}dP_n^m + \dfrac{(n+1)y}{r^2}P_n^m(\sin\varphi)\right\}\end{array}\right], \quad (3)$$

$$\frac{\partial U}{\partial z} = GM_{\oplus}\sum_{n=2}^{N}\sum_{m=0}^{n}\frac{r_{\oplus}^{n}}{r^{n+2}}CS\left[\frac{(x^2+y^2)}{r}dP_n^m - (n+1)zP_n^m(\sin\varphi)\right]$$

$$dP_n^m = \frac{dP_n^m(\sin\varphi)}{d(\sin\varphi)}, \qquad CS = \left(C_n^m\cos m\lambda + S_n^m\sin m\lambda\right).$$

Although those expressions are cumbersome, it is fairly easy to arrange calculation of their values. It is important to use recurrent expressions for the Legendre polynomials.

Complete definition of instantaneous values C_2^1 and S_2^1 with regard to the inelastic Earth pole tide is performed as follows [7, pp. 57-69]:

$$C_2^1 = \overline{C}_2^1 + \frac{dC_2^1}{dt}\left(t - t_{J2000}\right), \qquad S_2^1 = \overline{S}_2^1 + \frac{dS_2^1}{dt}\left(t - t_{J2000}\right),$$

where $\frac{dC_2^1}{dt} = -0,337\cdot 10^{-11}$ year^{-1} and $\frac{dS_2^1}{dt} = 1,606\cdot 10^{-11}$ year^{-1} – derivatives, determined at epoch J2000. The standardised coefficients at epoch J2000 are the following: $\overline{C}_2^1 = -2,20\cdot 10^{-10}$, $\overline{S}_2^1 = 14,51\cdot 10^{-10}$,

Tidal corrections to the harmonic coefficients of the geopotential expansion are computed using the following formula:

$$\Delta C_n^m - i\Delta S_n^m = \frac{k_{nm}}{2n+1}\sum_{j=2}^{3}\frac{GM_j}{GM_{\oplus}}\left(\frac{r_{\oplus}}{r_j}\right)^{n+1} P_n^m\left(\sin\Phi_j\right)e^{-im\lambda_j}$$

where k_{nm} – nominal values of the Love number; GM_j – gravitational parameter of the Moon $(j=2)$ and the Sun $(j=3)$; r_j – geocentric distance to the Moon or to the Sun; Φ_j – fixed geocentric latitude of the Moon or the Sun; λ_j – fixed West longitude (West of Greenwich) of the Moon or the Sun.

Perturbations by the Moon and the Sun

The next perturbations with the highest value, which affect the body motion in the near-Earth space, are those by the Moon and the Sun. Their perturbing accelerations were computed by formula (4):

$$\vec{a}_{MS} = GM_{Moon}\left(\frac{\vec{r}_M - \vec{r}}{\left|\vec{r}_M - \vec{r}\right|^3} - \frac{\vec{r}_M}{\left|\vec{r}_M\right|^3}\right) + GM_{Sun}\left(\frac{\vec{r}_S - \vec{r}}{\left|\vec{r}_S - \vec{r}\right|^3} - \frac{\vec{r}_S}{\left|\vec{r}_S\right|^3}\right) \quad (4)$$

$\vec{r}_M$, $\vec{r}_S$ – geocentric position vectors of the Moon and the Sun. When integrating equations of motion (1), the Moon's and the Sun's coordinates were computed using the DE405/LE405 model data [9].

Light pressure perturbations

We assume that the rate of solar radiation flux is constant, and the satellite has a spherical shape. With such assumptions the force of direct sunlight pressure on the satellite can be expressed by formula [5, pp. 617-625]:

$$\vec{a}_{Sp} = kqs'\Psi\left(\frac{a_S}{\Delta_S}\right)^2\frac{\vec{r}_S - \vec{r}}{\Delta_S} \quad (5)$$

where $\vec{r}_S$ – geocentric position vector of the Sun; Δ_S – distance between the satellite and the Sun; a_S – astronomical unit (the average distance from the Earth to the Sun); k — parameter that describes the reflective properties of the satellite surface (with $k=1$ — specular reflection, with $k=1,44$ — diffuse reflection), $q = 4,5605\cdot 10^{-6}\,\frac{H}{м^2}$ — the solar constant; s′ — the effective cross-sectional area of the satellite that is the ratio between the cross-sectional area of the satellite and its mass.

When computing perturbations caused by the light pressure, the central problem was to account for the effect of a satellite's entering the Earth's shadow. The model by Ferraz-Mello is applied in this paper [5, p. 622]. He suggested eliminating the problem by introducing the so-called shadow function δ into the perturbing acceleration: with δ = 1, if the satellite is sunlit, δ = 0 - otherwise. In general, the shadow is cone-shaped, but it may be considered as cylindrical shaped due to far distance of the source casting the shadow.

Determination of the ephemeris accuracy using the ILRS coordinates

Integrating of the set of differential equations of the AES motion (1) was performed by the Everhart numerical method of 19th orders in the Cartesian coordinate frame with extended precision. The integration step partition coefficients were calculated independently to improve their accuracy [2]. The integrating is done using variable step [8].

To evaluate the model performance and accuracy, the ILRS (the International Laser ranging System http://ilrs.gsfc.nasa.gov) satellite Cartesian coordinate database is used. The coordinates are presented in the Earth-bound rotating reference frame (ITRF). More detailed information on the coordinate structure can be found in the file on the ILRS official website (http://ilrs.gsfc.nasa.gov/docs/cpf_1.01.pdf). The AES coordinate database is freely available via the ILRS open source FTP (ftp://cddis.gsfc.nasa.gov/pub/slr/cpf_predicts). Using of those coordinates allows of control the satellite tracking accuracy. As those coordinates were obtained with high-precision numerical model for the AES motion, they can be used as reference to evaluate accuracy of other motion models, as well as to control the accuracy of the satellite observations collected [10]. To evaluate the model accuracy, we chose the following AES from the list of the satellites tracked by the ILRS: Etalon-1, Lageos-2 and Ajisai. The main parameters and physical characteristics of those satellites, available on the ILRS official website, are presented in Table 1.

Those are small-sized passive spherical in shape AES. As they move at different altitudes, the evaluation of the model with those satellites will enable to qualitatively assess the accuracy and amplitude of perturbations affecting AES.

According to the description of the ILRS files of the AES coordinate database, each file contains the ITRF coordinates with constant time increment. The inference step for coordinates in a file depends on a satellite as such it can be from several minutes to tens of seconds. Residuals of computed positions were determined by the method of numerical integration of motion equations (1) from the model with reference to the ILRS coordinates. The satellite coordinates and velocity components for initial conditions were determined by the Lagrange interpolation method through 12 points.

The differences between the coordinates obtained and those in the database can define quality of the developed motion model. The obtained absolute values of residuals (O-C) in altitude and computed angular geocentric residuals (O-C) at the end of the intervals of integration are given in Tables 2-4.

Table 1. Orbital parameters of the selected AES

	Diameter, m	Mass, kg	P, min	*i*, degrees	*e*	Perigee, km
Etalon-1	1.294 м	1415	676	64.9	0.00061	19120
Lageos-2	0.6 м	405.38	223	52.64	0.0135	5620
Ajisai (EGS)	2.15 м	685	116	50.0	0.001	1490

Table 2. The (O-C) prediction dynamics for Etalon-1

	6 months	9 months	12 months
Δr [м]	1239	1776	2223
Δα ["]	-6.13 ± 2.99	-8.72 ± 3.66	-10.86 ± 3.65
Δδ ["]	-0.05 ± 7.76	-1.74 ± 11.08	+0.45 ± 13.85

Table 3. The (O-C) prediction dynamics for Lageos-2

	6 months	9 months	12 months
Δr [м]	442	1064	2544
Δα ["]	-7.02 ± 1.18	-10.34 ± 4.36	-28.72 ± 8.33
Δδ ["]	+1.04 ± 2.48	-0.45 ± 14.16	+0.7 ± 31.12

Table 4. The (O-C) prediction dynamics for Ajisai (EGS)

	10 days	20 days	30 days	40 days
Δr [м]	79	437	1065	2011
Δα ["]	-0.76 ± 1.13	-8.43 ± 2.55	-21.18 ± 4.81	-40.04 ± 8.31
Δδ ["]	+0.54 ± 1.71	+0.46 ± 7.62	-0.42 ± 17.84	-1.57 ± 33.52

The mean value of residual O-C in angles for an interval of integration is given in the first part of the above tables, and the largest biases from that mean value are presented in the tables' second part. Thus, it can be seen that the mean value of residual and the bias values gradually increase with time. However, the residual values allow of pointing to the fact that the developed model enables to obtain retrieval ephemerides of a rather high quality online. Such accuracy in computation of the year ephemeris of the AES at altitude above that of Lageos-2 can be sufficient with the telescope filed of view of about 0.5º.

Conclusions

The results show that the prediction accuracy provided by suggested simplified numerical model of the AES motion is quite applicable to plot retrieval one-year ephemerides for satellites at altitude above 1,500 km.

To improve the AES ephemeris accuracy in the motion model, it is necessary to account for weaker perturbations. At present, work is underway on modification of the motion model and accounting for the following perturbations:

- atmospheric braking of the AES with orbit of altitude up to 1000 km;
- usage of the Earth's shadow model of more complex shape when computing perturbations due to the solar radiation pressure;
- accounting for perturbations due to the ocean and atmospheric tidal bulges;
- accounting for perturbations caused by other planets.

The obtained simplified motion model with time intervals of several weeks can be used to search for close approaches of objects. But the model accuracy is not sufficient to do that for the AES with orbit altitude lower than that of Lageos-2. Therefore, the obtained numerical model can be used to search for approaches of the high-orbit AES and geosynchronous objects only. As the geostationary region population has been increasing from year to year, provision of safety of motion of the satellites in operation demands a mechanism for online searching and tracking of space debris, as well as for predicting close approaches of objects.

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