

# MODEL OF DEGENERATE DWARF WITH SPIN-POLARIZED ELECTRON SYSTEM

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**ABSTRACT.** A three-parametric model of a massive degenerate dwarf was proposed. Unlike paramagnetic state of electron system in the standard Chandrasekhar model electrons are considered in a partially spin-polarized state. The parameters of the model are:  $x_0$  – the relativism parameter at stellar centre,  $\mu_e = \langle \frac{Z}{A} \rangle$  – the average chemical composition parameter and  $\zeta$  – the degree of spin polarization of the electron system. The macroscopic characteristics (e.g. mass, radius, energy) as functions of the model parameters were obtained from the solution of the mechanical equilibrium equation. The electron spin polarization was shown to lead to the increase of stellar radius and especially to mass compared with the corresponding characteristics of the standard model. The application of the proposed model to interpreting the stability of massive dwarfs in binary system was discussed.

**Key words:** degenerate dwarf, electron system, spin polarization, relativism parameter, chemical composition parameter, mechanical equilibrium equation.

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## 1. Introduction

In the past two decades dwarfs with different characteristics were discovered with the help of the space telescopes. Therefore the problem of developing the general theory of white dwarfs, including hot dwarfs with great luminosities and dwarfs with great masses, became urgent. The correct consideration of the interparticle interactions and of the general relativity effects leads to the conclusion that the maximum of the dwarf mass, in which the stability is disturbed, is a few percent less than the weight limit, which is derived in standard Chandrasekhar model (Vavrukh et al., 2014). At the same time the observed data indicate the availability in a binary system of dwarfs with masses, which are very close to the Chandrasekhar limit, or exceed it. The search of the mechanism, which can provide the stability of dwarfs with great masses, is one of the urgent tasks for these objects. In this work, we proposed a model of the cold degenerate dwarfs with spin-

polarized electron system, which corresponding magnetic field. The homogeneous magnetic field does not affect directly the star mechanical equilibrium, but it may cause the redistribution of electrons by energies as the result of spins interaction with the field. The result is two subsystems of electrons, with opposite spins, each of which has its Fermi sphere. At absolute zero the temperature has the distribution of electrons by the wave vectors:

$$n_{\mathbf{k},\sigma} = \theta(k_F^\sigma - k), \quad \sigma = \pm 1, \quad (1)$$

where  $k_F^+ > k_F$ ,  $k_F^- < k_F$ ,  $k_F$  – Fermi waveguide number in paramagnetic model with the same general concentration of electrons.

The defining of partial electron concentrations in these two subsystems is determined by the ratios

$$n_+ = \frac{1}{V} \sum_{\mathbf{k}} n_{k,+}, \quad n_- = \frac{1}{V} \sum_{\mathbf{k}} n_{k,-}. \quad (2)$$

We introduce the value

$$\zeta = \frac{1}{n} (n_+ - n_-), \quad n = n_+ + n_-, \quad (3)$$

which determines the degree of spin-polarization of electron system. From the equation (3) we have found that

$$n_+ = \frac{n}{2} (1 + \zeta), \quad n_- = \frac{n}{2} (1 - \zeta). \quad (4)$$

With the ratios (2) we introduce the expression for the waveguides of Fermi numbers for both systems

$$k_F^+ = k_F \lambda_+, \quad k_F^- = k_F \lambda_-, \quad k_F = (3\pi^2 n)^{1/3}, \quad (5)$$

$$\lambda_+ = (1 + \zeta)^{1/3}, \quad \lambda_- = (1 - \zeta)^{1/3}.$$

The model described here was considered in many works, devoted to the equation of solid state physics

both at  $T = 0K$ , and at low temperatures. The electron subsystem here considers not non-relativistic, but thermodynamic characteristics calculated taking into account the electron interactions (Hong & Mahan, 1995; Tanaka & Ichimaru, 1989; Ortiz & Ballone, 1994; Ortiz & Ballone, 1997).

## 2. The dwarf mechanical equilibrium equation

The equation of model (1) – (5) state is derived by a simple generalization equation of state for paramagnetic system (Vavrukh et al., 2011). The equation of state in the spatially two-component homogeneous case of the electron-nuclear model at  $T = 0K$  has a parametric representation:

$$P(x) = \sum_{\sigma} P_{\sigma}(x), \quad P_{\sigma}(x_{\sigma}) = \frac{\pi m_0^4 c^5}{3h^3} \mathcal{F}_{\sigma}(x_{\sigma}),$$

$$\mathcal{F}_{\sigma}(x_{\sigma}) = 4 \int_0^{x_{\sigma}} \frac{dy y^4}{(1+y^2)^{1/2}}, \quad (6)$$

$$\rho(x) = m_u \mu_e \sum_{\sigma} n_{\sigma} = \frac{m_u \mu_e}{6\pi^2} \left( \frac{m_0 c}{\hbar} \right)^3 \sum_{\sigma} x_{\sigma}^3,$$

where  $x_{\sigma} = \hbar k_F^{\sigma} / m_0 c$  – is relativism parameter,  $m_u$  – mass nuclear unit,  $\mu_e = \langle \frac{Z}{A} \rangle$  – the average chemical composition, ( $Z$  – nuclear charge,  $A$  – mass number),  $m_0$  – electron mass,  $c$  – speed of light,  $P_{\sigma}(x)$  – the electron partial pressure,  $\rho(x)$  – mass density of the nuclear subsystem. The pressure ratio in a spin-polarized model to the pressure in the paramagnetic model equals:  $C_1(\zeta) = \frac{1}{2} \sum_{\sigma} \lambda_{\sigma}^4$  at the border  $x \gg 1$  and  $C_2(\zeta) = \frac{1}{2} \sum_{\sigma} \lambda_{\sigma}^5$  at the border  $x \ll 1$ . From this follows that the pressure in the spin-polarized model is greater than the pressure in the paramagnetic model at the same value  $x$ , and the functions  $C_1(\zeta), C_2(\zeta)$  change within the limits:  $1 \leq C_1(\zeta) \leq 2^{1/3}, 1 \leq C_2(\zeta) \leq 2^{2/3}$ .

To obtain the equation of state for inhomogeneous model, we should perform replacement  $x \rightarrow x(r), P_{\sigma} \rightarrow P_{\sigma}(x(r)), \rho \rightarrow \rho(r), x_{\sigma} \rightarrow x_{\sigma}(r)$ . According to the formulas (5),  $x_{\sigma}(r) = x(r) \lambda_{\sigma}$ , where  $x(r) = \hbar k_F(r) (m_0 c)^{-1}$  – is the relativism parameter value in a paramagnetic state.

Let us consider the mechanical equilibrium of star

$$\frac{dP(r)}{dr} = -G\rho(r) \frac{M(r)}{r^2}, \quad \frac{dM(r)}{dr} = 4\pi r^2 \rho(r), \quad (7)$$

where  $P(r)$  – is the pressure on the sphere of radius  $r$ ,  $\rho(r)$  – density on this sphere,  $M(r)$  – mass inside the sphere. The system of equations (7) is reduced to the nonlinear differential equation for  $x(r)$ ,

$$\frac{1}{r^2} \frac{d}{dr} \left\{ r^2 \left[ \frac{\lambda_+^5}{\sqrt{1+x^2(r)\lambda_+^2}} + \frac{\lambda_-^5}{\sqrt{1+x^2(r)\lambda_-^2}} \right] \times \right. \\ \left. \times x(r) \frac{dx}{dr} \right\} = -G(m_u \mu_e)^2 \frac{64\pi^2 m_0^4 c^4}{3(hc)^3} x^3(r), \quad (8)$$

where  $\lambda_+, \lambda_-$  – are the prescribed parameters, and  $\lambda_+ + \lambda_- = 2$ .

## 3. Full polarization case

In the particular case of full polarization, when  $\zeta = 1$  ( $\lambda_- = 0, \lambda_+ = 2^{1/3}$ ), in dimensionless variables

$$\xi = \frac{r}{\lambda}, \quad y_+(\xi) = (\varepsilon_0^+)^{-1} \{ [1 + \lambda_+^2 x^2(r)]^{1/2} - 1 \} \quad (9)$$

the equation (8) coincides its form with the equation of paramagnetic model

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left\{ \xi^2 \frac{dy_+}{d\xi} \right\} = - \left\{ y_+^2(\xi) + \frac{2}{\varepsilon_0^+} y_+(\xi) \right\}^{3/2}, \quad (10)$$

however in this equation parameter  $x_0 \lambda_+$  is used instead of  $x_0$ . Herewith the scale  $\lambda_+$  is determined by the ratio

$$\frac{16G\pi^2}{3(ch)^3} [m_u \mu_e m_0 c^2 \varepsilon_0^+ \lambda]^2 = 1, \quad (11)$$

where  $\varepsilon_0^+ = [1 + x_0^2 \lambda_+^2]^{1/2} - 1$ . The total mass of dwarf is determined by the ratio

$$M(x_0^+, \mu_e) = \frac{\sqrt{2}}{\mu_e^2} M_0 \mathcal{M}(x_0^+),$$

$$\mathcal{M}(x_0^+) = \int_0^{\xi_1(x_0^+)} d\xi \xi^2 \left( y_+^2(\xi) + \frac{2}{\varepsilon_0^+} y_+(\xi) \right)^{3/2}. \quad (12)$$

Herewith  $\xi_1(x_0^+)$  – the dimensionless radius of a dwarf, which is determined from the condition  $y(\xi_1(x_0^+)) = 0$ . From the last formula it follows that the maximum mass of a dwarf which corresponds  $x_0 \gg 1$ , is equal

$$M_{max} = \sqrt{2} \frac{M_0}{\mu_e^2} 2,01824 \dots, \quad (13)$$

that is, it exceeds Chandrasekhar limit by  $\sqrt{2}$  times. The radius of dwarf is equal

$$R(x_0^+, \mu_e) = \xi_1(x_0^+) \lambda = \frac{R_0 \xi_1(x_0^+)}{\mu_e} \frac{\sqrt{2}}{\varepsilon_0^+}. \quad (14)$$

The scales of mass and radius ( $M_0, R_0$ ) are used in the formulas (13), (14) defined by the formulas

$$\begin{aligned} R_0 &= \left(\frac{3}{2}\right)^{1/2} \frac{1}{4\pi} \left(\frac{h^3}{cG}\right)^{1/2} \frac{1}{m_0 m_H}, \\ M_0 &= \frac{m_0 c^2 \lambda \varepsilon_0(x_0)}{G m_H} = \left(\frac{3}{2}\right)^{1/2} \frac{1}{4\pi} \left(\frac{hc}{G m_H^2}\right)^{3/2} m_H. \end{aligned} \quad (15)$$

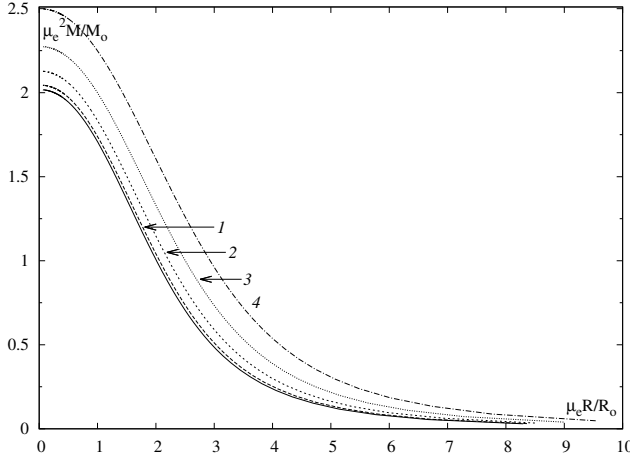


Figure 1: “Mass-radius” relations at different values  $\zeta$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4)

As it can be seen from the ratio (14), the radius of dwarf values in the ultrarelativistic region ( $x_0 \gg 1$ )  $R(x_0^+, \mu_e) \Rightarrow R_0 \cdot 6,89685 \cdot 2^{1/2} (\mu_e x_0 \lambda_+)^{-1}$  exceeds the analogous value at the same concentration in the star centre by  $2^{1/2} \lambda_+^{-1} = 2^{1/6}$  times. In figure 1 the “mass-radius” relations obtained in the standard (solid curve) and spin-polarized models at different values of the parameter  $\zeta$ , are compared to each other.

#### 4. The equilibrium equation solutions at arbitrary polarization

In a general case at arbitrary value of the parameter  $\zeta$ , the equation (8) also can be reduced to the equation (10) using the substitution

$$\sum_{\sigma=\pm} \lambda_\sigma^3 \{(1 + x^2(r) \lambda_\sigma^2)^{1/2} - 1\} = \varepsilon_0^\zeta y(\xi), \quad (16)$$

where

$$\varepsilon_0^\zeta = \sum_{\sigma=\pm} \lambda_\sigma^3 \{(1 + x_0^2(r) \lambda_\sigma^2)^{1/2} - 1\}.$$

To record the right side of the equation (8) by  $y(\xi)$ , we define  $x(r)$  from the ratio (16). We reduce this ratio to the biquadratic equation. Positive and valid root of this equation is written as

$$x(r) = 2^{-1/2} (\lambda_+^8 + \lambda_-^8)^{-1} [b(y) - \varphi(y)]^{1/2}, \quad (17)$$

$$\begin{aligned} b(y) &= 2\{(\lambda_+^8 + \lambda_-^8)[(\varepsilon_0^\zeta y)^2 + 4\varepsilon_0^\zeta y] + \\ &\quad + 4(\lambda_+ \lambda_-)^3 (\lambda_+^5 + \lambda_-^5)\}, \\ \varphi(y) &= \{b^2(y) - 4ac(y)\}^{1/2} = \\ &= 4(2 + \varepsilon_0^\zeta y)(\lambda_+ \lambda_-)^3 \{(\lambda_+ \lambda_-)^2 \times \\ &\quad \times [(\varepsilon_0^\zeta y)^2 + 4\varepsilon_0^\zeta y] + (\lambda_+^5 + \lambda_-^5)^2\}^{1/2}. \end{aligned} \quad (18)$$

At the limit of star  $y(\xi) = 0$ , therefore  $b(0) = \varphi(0) = 8(\lambda_+ \lambda_-)^3 (\lambda_+^5 + \lambda_-^5)$ , and this provides the implementation of the equality  $x(R) = 0$  at the arbitrary value  $\zeta$ .

The equation (8) in a dimensionless form is

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{dy}{d\xi} \right) = -\{\sqrt{2}(\lambda_+^8 + \lambda_-^8)^{-1} \times \\ \times (\varepsilon_0^\zeta)^{-1} [b(y) - \varphi(y)]^{1/2}\}^3. \quad (19)$$

The boundary conditions to  $y(0) = 1$ ,  $y'(0) = 0$ , and the condition  $y(\xi) \geq 0$  corresponds to this equation. The scale  $\lambda$  is determined by the ratio

$$\frac{32\pi^2 G}{3(hc)^3} \left\{ m_u \mu_e m_0 c^2 \lambda \frac{\varepsilon_0^\zeta}{2} \right\}^2 = 1, \quad (20)$$

which at the limit  $\zeta \rightarrow 0$  coincides with the equality in the paramagnetic model.

The equation (19) – is two-parametric, with the parameters  $x_0$  and  $\zeta$ . The equation (19) takes the form of the equation (10) in the case of large values of  $x_0$

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left( \xi^2 \frac{dy}{d\xi} \right) \approx -\left\{ y^2(\xi) + \frac{4}{\varepsilon_0^\zeta} y(\xi) \right\}^{3/2}, \quad (21)$$

the parameter  $\lambda$  is determined by the equality

$$\frac{64\pi^2 G}{3(hc)^3} \{m_0 c^2 \lambda \varepsilon_0^\zeta m_u \mu_e\}^2 (\lambda_+^4 + \lambda_-^4)^{-3} = 1. \quad (22)$$

The equation solutions dependence (19) on the parameters  $x_0, \zeta$  is illustrated figures 2 and 3, and the figure 4 shows the dependence of the dimensionless radius of star  $\xi_1(x_0, \zeta)$  on these parameters.

#### 4. The macroscopic characteristics of dwarfs

The equation solutions (19) determine the macroscopic characteristics of star dependence on the parameters of model  $x_0, \mu_e, \zeta$ . In particular the total mass of star is equal to

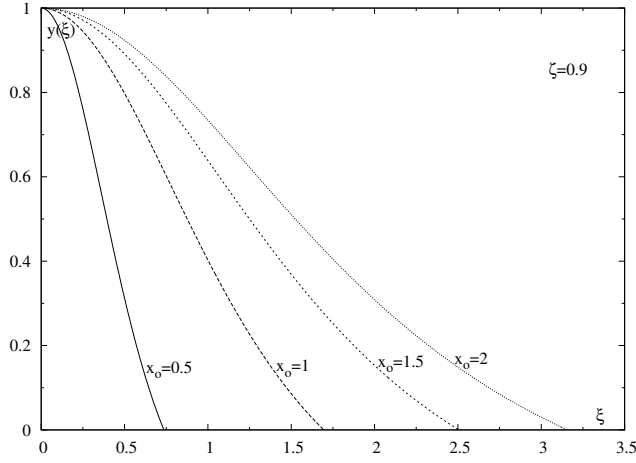


Figure 2: The equation solutions (19) at fixed value  $\zeta = 0.9$

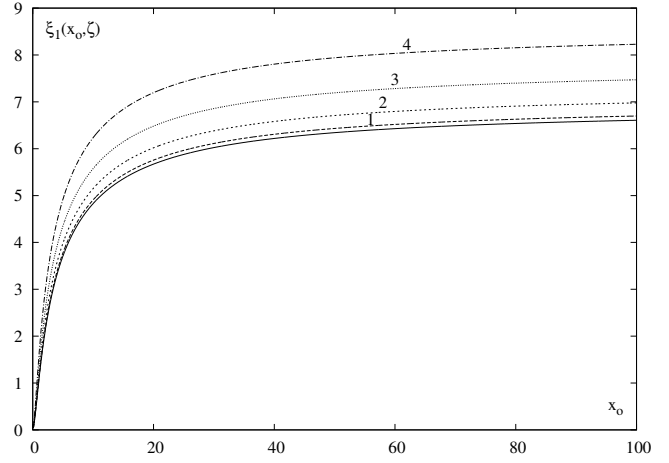


Figure 4: Dependence  $\xi_1(x_0, \zeta)$  of the dimensionless star radius on parameter  $x_0$  at different values  $\zeta$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4)

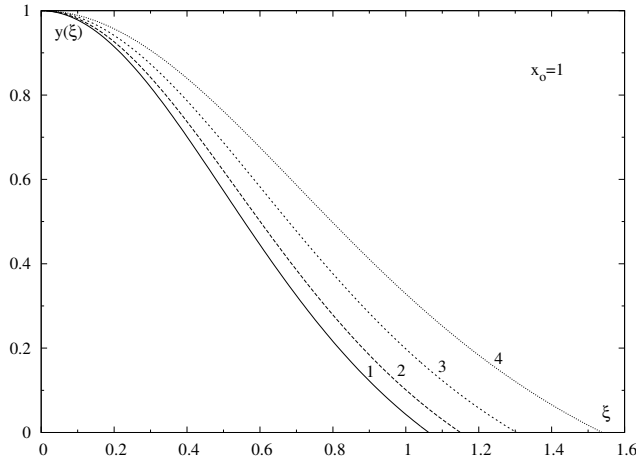


Figure 3: The equation solutions (19) at fixed value  $x_0 = 1$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4)

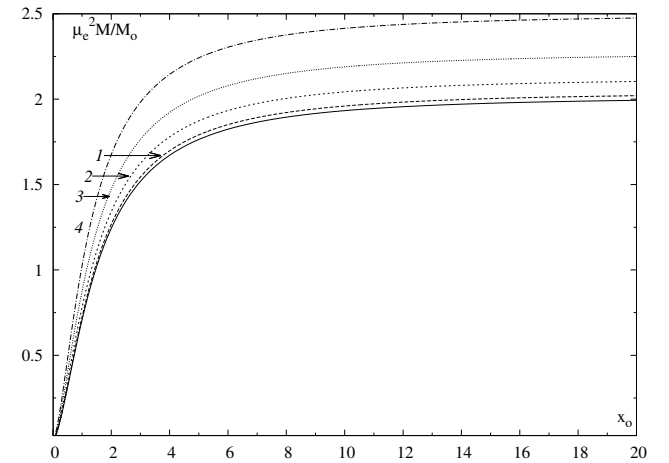


Figure 5: The mass dependence on parameter  $x_0$  at different values  $\zeta$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4)

$$\begin{aligned}
 M(x_0, \mu_e | \zeta) &= \frac{M_0}{\mu_e^2} \mathcal{M}(x_0 | \zeta), \\
 \mathcal{M}(x_0 | \zeta) &= 2\sqrt{2} \{ \varepsilon_0^\zeta (\lambda_+^8 - \lambda_-^8) \}^{-3} \times \\
 &\times \int_0^{\xi_1(x_0 | \zeta)} d\xi \xi^2 [b(y(\xi)) - \varphi(y(\xi))]^{3/2} = \\
 &= \xi_1^2(x_0 | \zeta) \left. \frac{dy}{d\xi} \right|_{\xi = \xi_1(x_0 | \zeta)}. \tag{23}
 \end{aligned}$$

The radius is determined by the ratio

$$R(x_0, \mu_e | \zeta) = \lambda \xi_1(x_0 | \eta) = 2R_0 \frac{\xi_1(x_0 | \zeta)}{\mu_e \varepsilon_0^\zeta}. \tag{24}$$

The  $\mathcal{M}(x_0 | \zeta)$ ,  $R(x_0, \mu_e | \zeta)$  dependence on the parameter  $x_0$  is illustrated in figures 5, 6.

The average value of the electron kinetic energy  $E_{kin}$ , the potential energy of nuclear subsystem  $W$  gravitational interaction, and the total energy (taking into account the electrons rest energy)  $\tilde{E}$  is determined by the ratios

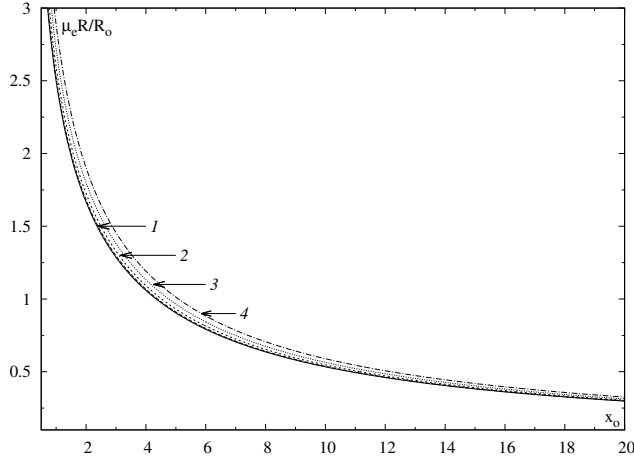


Figure 6: The radius dependence on parameter  $x_0$  at different values  $\zeta$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4 )

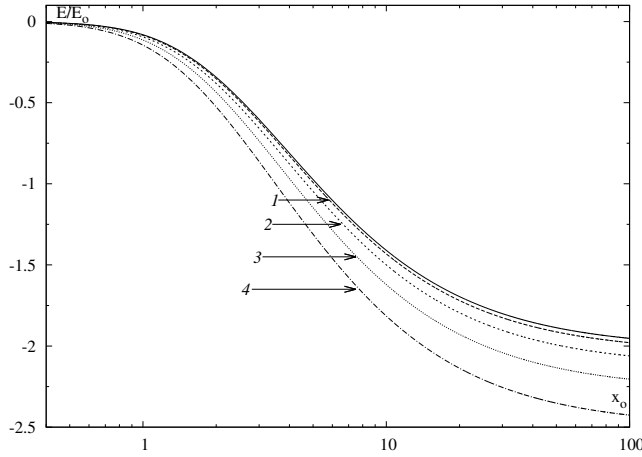


Figure 7: The total energy  $E$  dependence on parameter  $x_0$  at different values  $\zeta$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4 )

$$\begin{aligned}
 E_{kin}(x_0, \mu_e | \zeta) &= \frac{E_0}{\mu_e^3} \left( \frac{2}{\varepsilon_0 \zeta} \right)^3 \int_0^{\xi_1} d\xi \xi^2 \frac{1}{2} \times \\
 &\times \sum_{\sigma} \left\{ x_{\sigma}^3(\xi) [(1 + x_{\sigma}^2(\xi))^{1/2} - 1] - \frac{1}{4} \mathcal{F}_{\sigma}(x) \right\}, \\
 W(x_0, \mu_e | \zeta) &= \frac{E_0}{\mu_e^3} \left( \frac{2}{\varepsilon_0 \zeta} \right)^2 \int_0^{\xi_1} d\xi \xi^3 x^3(\xi) \frac{dy}{d\xi} = \\
 &= -\frac{3}{4} \cdot \frac{E_0}{\mu_e^3} \left( \frac{2}{\varepsilon_0 \zeta} \right)^3 \int_0^{\xi_1} d\xi \xi^2 \frac{1}{2} \sum_{\sigma} \mathcal{F}_{\sigma}(x_{\sigma}(\xi)),
 \end{aligned} \tag{25}$$

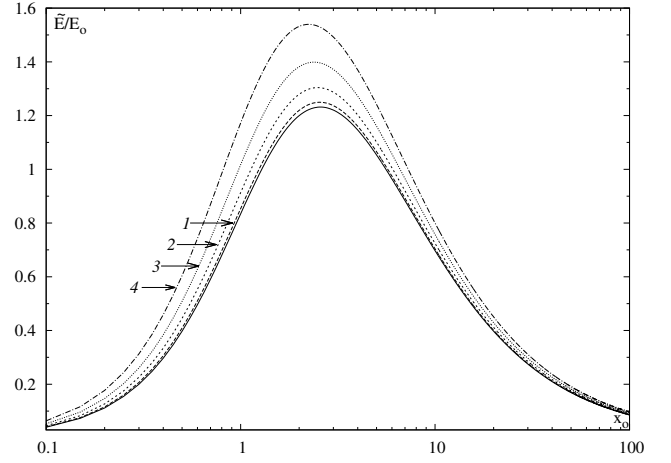


Figure 8: The total energy  $\tilde{E}$  dependence on parameter  $x_0$  at different values  $\zeta$  ( $\zeta = 0$  – solid curve,  $\zeta = 0.2$  – curve 1,  $\zeta = 0.4$  – curve 2,  $\zeta = 0.6$  – curve 3,  $\zeta = 0.8$  – curve 4 )

$$\begin{aligned}
 E(x_0, \mu_e | \zeta) &= E_{kin}(x_0, \mu_e | \zeta) + W(x_0, \mu_e | \zeta) = \\
 &= \frac{E_0}{2\mu_e^3} \left( \frac{2}{\varepsilon_0 \zeta} \right)^3 \int_0^{\xi_1} d\xi \xi^2 \times \\
 &\times \sum_{\sigma} \left\{ x_{\sigma}^3(\xi) [(1 + x_{\sigma}^2(\xi))^{1/2}] - \mathcal{F}_{\sigma}(x_{\sigma}(\xi)) \right\}, \\
 \tilde{E}(x_0, \mu_e | \zeta) &= E_{kin}(x_0, \mu_e | \zeta) + W(x_0, \mu_e | \zeta) + \\
 &+ m_0 c^2 N(x_0, \mu_e | \zeta) = \frac{3}{4} \cdot \frac{E_0}{\mu_e^3} \left( \frac{2}{\varepsilon_0 \zeta} \right)^3 \sum_{\sigma} \int_0^{\xi_1} d\xi \xi^2 \times \\
 &\times \left\{ x_{\sigma}(\xi) [(1 + x_{\sigma}^2(\xi))^{1/2}] - \ln[x_{\sigma}(\xi) + (1 + x_{\sigma}^2(\xi))^{1/2}] \right\}.
 \end{aligned}$$

Herewith  $x_{\sigma}(\xi)$  is determined by the equation (17), where  $y(\xi)$  – the equation solution (19).

It is likely that the model has a physical meaning with a small value of the parameter polarization ( $\zeta \ll 1$ ). In this case the equation (19) is simplified, because with precision to  $\zeta^2$

$$\begin{aligned}
 \lambda_+^5 (1 + x^2(r) \lambda_+^2)^{-1/2} + \lambda_-^5 (1 + x^2(r) \lambda_-^2)^{-1/2} &= \\
 = 2(1 + x^2(r))^{-1/2} \cdot \{1 + \zeta^2 \cdot f(x(r)) + \dots\}, &\tag{26} \\
 f(x) = \frac{5}{9} - \frac{1}{2} x^2 (1 + x^2)^{-1} + \frac{1}{6} x^4 (1 + x^2)^{-2}. &
 \end{aligned}$$

Proceeding to dimensionless variables

$$r = \lambda \xi, \quad \mu(r) = m_0 c^2 \{ (1 + x_0^2)^{1/2} - 1 \} y(\xi), \tag{27}$$

We summarize the equation (19) to this dimensionless form:

$$\frac{1}{\xi^2} \frac{d}{d\xi} \left\{ \xi^2 \frac{dy}{d\xi} \right\} = - \left( y^2 + \frac{2}{\varepsilon_0} y \right)^{3/2} + \frac{1}{6\xi^2} \frac{d}{d\xi} \left\{ \xi^2 \frac{(y^2 + \frac{2}{\varepsilon_0} y)(2y^2 + \frac{4y}{\varepsilon_0} + \frac{1}{\varepsilon_0^2})}{(y + \frac{1}{\varepsilon_0})^4} \frac{dy}{d\xi} \right\}. \quad (28)$$

Herewith the scale  $\lambda$  is determined by the ratio

$$\frac{32\pi^2 G(m_u \mu_e m_0 c^2 \lambda \varepsilon_0)^2}{3(hc)^3} = 1 + \frac{5}{9} \zeta^2. \quad (29)$$

The equation solution (28) can be found by successive approximations for small values  $\zeta$ , using substitution

$$y(\xi) = y_0(\xi) + \zeta^2 y_1(\xi). \quad (30)$$

The mass and radius of dwarf dependence on parameters of model is given by

$$\begin{aligned} M(x_0, \mu_e, \zeta) &\cong \frac{M_0}{\mu_e^2} \left( 1 + \frac{5}{6} \zeta^2 \right) \mathcal{M}(x_0), \\ R(x_0, \mu_e, \zeta) &\cong \frac{R_0}{\mu_e \varepsilon_0} \left( 1 + \frac{5}{18} \zeta^2 \right) \xi_1(x_0), \end{aligned} \quad (31)$$

where  $\mathcal{M}(x_0), \xi_1(x_0)$  are the characteristics of standard model (paramagnetic).

## 5. Summary and conclusions

The degree of spin polarization of the dwarf electron system significantly affects its characteristics as it follows from the calculations, it leads to the increase the mass and radius. At an arbitrary value  $\zeta$  the maximum dwarf mass exceeds this value in the standard model. Within the spin-polarized model the existence of dwarfs in binary systems can be explained, where the dwarfs mass reaches the value  $1,5M_\odot$  and it is at the limit of stability (or beyond it) in terms of the standard model (Vavrukh et al., 2012). Above we considered a

somewhat idealized model, in which the parameters  $\mu_e$  and  $\zeta$  are constants independent of the coordinates. From physical considerations, the degree of spin-polarization electron subsystem depends on the temperature and magnetic field values. Obviously, the global magnetic field has a dipole character, and therefore it is concentrated in the external star regions. Thereby the degree of spin polarization is greater in the outer dwarf regions. In term of strong accretions on the massive magnetic dwarf, in its surface layers thermonuclear reaction can start, which will lead to their heating and reducing the degree of spin polarization. There may be the conditions, in which the mass of star exceeds the permissible critical mass (which is a function of  $\zeta$ ), resulting in the collapse and supernova explosion. Hence follows the need for accurate description of a dwarf within the model with the parameter  $\zeta$ , which is a function of the distance from the star center, as well as the consideration of other factors – interparticle interactions and the effects of general relativity theory.

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