ON THE HEURISTIC RULE FOR PLANETARY DISTANCE DISTRIBUTION

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ABSTRACT. This paper presents a new heuristic rule for the planetary distance distribution in the solar system similar to the Titius-Bode rule of planetary orbit spacing. Application of this universal rule simultaneously for planets and planetary moons has been considered. Natural satellites orbiting around a central body are divided into groups of six satellites in each.

1. Introduction

There is a vast literature on the search of regularities of planetary and moon orbit spacing according to the Titius-Bode – type relation [1–8]. The Titius-Bode Law of Planetary Distances: Its History and Theory by Michael M. Nieto from the Niels Bohr Institute at the University of Copenhagen was issued in 1972. Apparently, the Titius-Bode relation expresses, to some extent, Newtonian mechanics in empirical form: each planet in the solar system is about 1.7 times further from the Sun than the next innermost planet. It was also shown that such regularities are realised in exoplanetary systems [2, 7]. The geometric series for distances follows from Newton's law; however, to perform sufficient simulation and deepen understanding of this phenomenon, it is necessary to rely on the methods of celestial mechanics and apply modern computer technologies. This study presents a new heuristic rule for the spacing of systems of different bodies in the solar system.

2. Rule definition

Natural satellites orbiting around a central body are divided into groups of six moons in each:

$$h_{mn}\,, \quad n=\overline{1.6} \tag{1}$$

where m – the group number; n – the ordinal number of a moon within a group starting with the central body; h_{mn} – the average distance between the central body and moon which equals to the radius of a sphere which has the same area as the planar figure restricted by the moon's orbit.
If $a,\ b$ – the ellipse semi – axes, then the sought radius equals to $\sqrt{ab}$. The distances in the group are approximated with the following formulae (see Table 1). (Here α_m – the group non – dimensional parameter; H_m – the average orbital radius of the 6th moon in the group, which is called the upper boundary of the group and h_{m1} – the lower boundary of a group. The distances h_{mn} within groups of moons are related as follows:

$$h_{m+k\ n} = \beta^k h_{m\ n}\,, \quad k=0,1,2,3... \tag{2}$$

where $m+k$ – the number of a group).

Having the relative values entered, the previous table can be presented as follows (see Table 2). As is evident, here

$$a_{mn} = \frac{h_{mn}}{h_{m6}} \quad \text{and} \quad \alpha_m = \frac{h_{m6}}{h_{m3}} \tag{3}$$

Table 1

Moon number	1	2	3	4	5	6
Distance notation	h_{m1}	h_{m2}	h_{m3}	h_{m4}	h_{m5}	h_{m6}
Distance formula	$\frac{H_m}{\alpha_m^2-1}$	$\frac{H_m}{\alpha_m+1}$	$\frac{H_m}{\alpha_m}$	$\frac{H_m}{\alpha_m-1}$	$\frac{2\alpha_m H_m}{\alpha_m^2-1}$	H_m

Table 2

Moon	a_1	a_2	a_3	a_4	a_5	a_6
Distance formula	$\frac{1}{\alpha^2-1}$	$\frac{1}{\alpha+1}$	$\frac{1}{\alpha}$	$\frac{1}{\alpha-1}$	$\frac{2\alpha}{\alpha^2-1}$	1

Besides, it is supposed that the sequencing axiom is realised for planetary distances with any allowed values of the parameter α:

$$0<\alpha_{m\,n}=\frac{h_n}{h_6}<\alpha_{m\,n+1}\le 1\,,\quad n=\overline{1,5} \qquad (4)$$

from which the left restriction for the parameter α is obtained:

$$1+\sqrt{2}<\alpha_m \qquad (5)$$

From the sequencing axiom, which states that the upper boundary of the group is less than the lower boundary of the next group,

$$h_{m-1\,6}<h_{m\,n}\,,\quad n=1,2,3,...6 \qquad (6)$$

the right restriction is obtained:

$$\alpha<\sqrt{\beta+1} \qquad (7)$$

Let us suppose that there are two Phaetons rather than one hidden in the asteroid belt at the distances of 2.26 and 2.94 AU from the Sun, and that asteroid Chiron (a = 13.65 AU, e = 0.382) is a minor planet (or its remainder). According to the above – formulated rule we receive the following (see Table 3).

Asteroid 538P – L with the average orbital radius of 2.261763 AU, asteroid 1992DT2 with the average radius of 2.9403035 AU and asteroid 1999W140 with the average radius 2.9399417 AU (or the members of the Flora and Eos families) were selected as the fragments of the 5th and 6th planets within the first group.

Thus, the values $\alpha_m\approx 2.94$, $m=1,\ 2,\ 3$; $\beta\approx 13$ rather accurately approximate the relative distances $\frac{h_{m\,k}}{h_{m\,6}},k=\overline{1.6}\,,\ m=\overline{1.3}$, obtained on the basis of actual data.

The majority of the Kuiper belt asteroids are in the region extending between the orbits of the last planet of the second group and the second planet of the third group. The 3rd, 4th, 5th and 6th planets of the third group among trans – Neptunian objects do not belong to families.

3. Description of moon systems

To make it more illustrative, it is more convenient to examine the Neptunian moon system first (see Table 4).

As is seen from Table 4, the first four moons make up a family of the first object within the first group. Positions for the 4th and 5th objects within the first group are empty. The second group is completely empty.

The existence of positions in the second group is determined by the values β, which should meet some additional requirements (see Formula 8). Besides, a definite rule, such as density axiom, can be set: in accordance with this axiom parameters **α** and **β** should take on the least values given that conditions (2) and (6) are fulfilled.

In other words, when distributing moons within the first and third groups, the existence of the second group which fulfils condition (6) follows from condition (2).

Table 3

		PLANET	$h_3=\beta h_2$ β = 13.1	AVERAGE RADIUS	RELATION WITHIN A GROUP	THEORETICAL RELATION	β
1	1	Mercury		.38294034	.130	.130	
	2	Venus		.722332359	.246	.254	
	3	Earth		.99993022	.340	.340	
	4	Mars		1.5203275	.5171	.516	
	5	The Flora family		(2.2616567)	.77	.77	
	6	The Eos family		(2.94)	1	1.	
2	1	Jupiter		5.208709	.134	.13	13.601882
	2	Saturn		9.5300711	.245	.25	13.175292
	3	Chiron		13.208832	.34	.34	13.209754
	4	Uranus		19.18058	.494	.52	12.616084
	5	Neptune		30.068409	.77	.77	13.258811
	6	Pluto		38.855936	1.	1.	13.352555
3	1	(229762) 2007 UK	68.234088	68.69	.139	.13	13.2
	2	(181902) 1999 RD, (82158) 2001FP	124.84393	105.564 157.7	.21 .319	.25	11.08
	3	(148209) 2009 CR	173.0357	169.644	.34	.34	12.85
	4	2004 VN	251.2656	243.365	.493	.52	12.69
	5	(90377) Sedna	393.89616	374.634	.76	.77	12.46
	6	2006 SQ	509.01276	493.24	1	1.	12.69

Table 4: The Neptunian moon system (Neptune's radius 24,764 km).

No	Group number	Object number within a group	Number within a family	Moon names	R, km	R/R	R/R theoretical	β	M, kg
1	I	1	1	Naiad	48 227	0.1359272	0.11		$1.9 \cdot 10^{17}$
2			2	Thalassa	50 075	0.1411358			$3.5 \cdot 10^{17}$
3			3	Despina	52 526	0.1480439			$2.1 \cdot 10^{18}$
4			4	Galatea	61 953	0.1746380			$2.1 \cdot 10^{18}$
5		2		Larissa	73 548	0.2072942	0.24		$4.9 \cdot 10^{18}$
6		3		S/2004 N 1	105 200	0.296505	0.32		
7				Proteus	117 647	0.3315858			$5.0 \cdot 10^{19}$
		4		*163.5*			0.46		
		5		*248.6*			0.70		
8		6		Triton	354 800	1.0000000	1.		$2.1 \cdot 10^{22}$
							α=3.17		
	II	1		*421 – 513.2*					
		2		*783 – 1060*					
		3		*1060 – 1252*					
		4		*1740 – 2108*					
		5		*2645 – 3500*					
		6		*3775 – 4240*					
								β=10.64	
9	III	1		Nereid	4 479 360.7	0.0992068	0.125	9.6401649	$3.1 \cdot 10^{19}$
		2		*11277.8*			0.25		
10		3		Halimede	14 249 954	0.315886	0.3(3)	11.007844	$9.0 \cdot 10^{16}$
11		4	1	Sao	21 924 105	0.4850029	0.5		$6.7 \cdot 10^{16}$
12			2	Laomedeia	22 433 384	0.4972924			$5.8 \cdot 10^{16}$
13		5		Psamathe	37 243 465	0.8255951	0.75		$1.5 \cdot 10^{16}$
14		6		Neso	45 111 053	1.0000000	1.	11.275853	$1.7 \cdot 10^{17}$
							α=3.0		

Now let us examine the Saturnian moon system:

Drawing an analogy between macrocosm and microcosm, in accordance with the planetary model of the atom in which an electron strives to occupy the lowest orbit from the allowed ones, it can be assumed that a similar phenomenon can be observed in macrocosm as it was in the case of the **β** parameter selection during assignment of the second group of the Neptunian moons. It means that the allowed orbits of a central body's moons are determined on the same ground.

As can be seen, the moons of the Saturnian system are divided into three groups.

The moons from the 0^{th} to the 13^{th} form a sub – group located between the orbits of the first and second moons within the first group. This group can be called a family or a sub – group of the first moon within the first group.

One of the criteria by which the moons were assigned to this group, is the moons' sizes given in the last column of the table as it is not feasible to perform any other assignment.

It should be noted that the 9^{th} moon of the first family within the third group of the Saturnian system satisfies the following condition: $h_{3,1,9} = \beta h_{2,1} = 16938$ (see Table 5).

Further let us consider the moon system of Jupiter. Using the same principles as before, we obtain data presented in Table 6. The distance for the first moon of the first group of the Jupiter system is determined by relation $h_1 = \alpha_1 h_6$, although it is less than the central body's radius (see Table 6).

The Uranian moon system can be described with four groups (see Table 7).

In different sources, the solar and planetary parameters vary significantly. Table 8 presents some variations of those parameters, as well as the obtained values of parameters α and β.

Table 5: The Saturnian moon system (Saturn's radius 60,268 km).

No	Group number	Object number within a group	Number within a family	Moon name	R, thsd. km	R/R	R/R theoretical	β	D
1			0	S/2009 S 1	117	0.0957726			0,3
2			1	Pan	133	0.1088697			20
3		1	2	Daphnis	136.5	0.1117347	0.1011		7
5			4	Prometheus	139.4				100
14			13	Enceladus	238.1	0.1949014			499
15			1	Tethys	294.7				1060
16		2	2	Telesto	294.7	0.2412325	0.2326		24
17	**I**		3	Calypso	294.7				19
18			1	Dione	377.4				1118
19		3	2	Helene	377.4	0.3089282	0.3(3)		32
20			3	Polydeuces	377.4				4
21		4		Rhea	527.1	0.4314681	0.4348		1528
		5		*815*			0.6673		
22		6		Titan	1221.643	1. α=3.237	1. **α=3.3**		5150
23		1		Hyperion	1463.9814	0.1138775	0.1161	11.007	266
24		2		Iapetus	3560.1019	0.2769198	0.2439	12.08	1436
		3		*4147*			0.3226	*10.99*	
	II	4		*6122*			0.4762	*11.61*	
25		5	1	Kiviuq	10787.248	0.8390780	0.72	*13.23*	16
26			2	Ijiraq	10835.251	0.8428118			12
27		6		Phoebe	12856.073	1.	1. **α=3.1**	10.524	240
								β = 11.57	
28			1	Paaliaq	14669.37	0.5906256		1 : 10.02	22
29			2	Albiorix	15165.908			1 : 10.36	32
31		1	4	Bebhionn	**16088.201**			1 : 10.99	6
35			8	Skoll	**16626.089**	0.6694005		1 : 11.36	6
36			9	Siarnaq	17136.472	0.6899495			40
39			12	S/2004 S 7	17870.709	0.7195114		1 : 12.2	6
54			27	Farbauti	20170.152	0.8120917		1 : 13.78	5
57	**III**		30	Kari	20729.178	0.8345992		1 : 14.16	7
61			34	Loge	22860.661	0.9204171		1: 15.6	6
62			35	Fornjot	24837.282	1.0		1 : 16.97; 2: 6.98	6
		2		*39450 – 41190*					
		3		*50520 – 47980*					
		4		*70560 – 70832*					
		5		*109100 – 124809*				**β = 11.57**	
		6		*163535 – 148744*				**β^2 = 133.8649**	

($\alpha \approx 3.2$, $\beta \approx 11.57$, $\alpha\beta \approx 37.024$)

Table 6: The moon system of Jupiter (Jupiter's radius 71,492 km).

No	Diameter	Group number	Object number within a group	Number within a family	Moon name	Average radius, thsd. km	R/R	R/R theoretical	β	
			1		*20.49*			0.113		
			2		*43.89*			0.242		
			3		*57.67*			0.318		
		I	4		*84.70*			0.467		
1	~40		5	1	**Metis**	127.69	0.7040459	0.709		
2	~16			2	**Adrastea**	128.69	0.7955960			
3	~146		6		**Amalthea**	181.366	1.0	1.0		
								α=3.14		
4	~98		1		**Thebe**	221.872	0.1178490	0.15	*10.83*	
5	~3630		2		**Io**	421.7	0.2239892	0.26	*9.6*	
6	~3121,6	**II**	3		**Europa**	671.02	0.3564174	0.36	*11.64*	
7	~5262,4		4		**Ganymede**	1070.412	0.5685575	0.56	*12.64*	
			5		*1543.8*			0.82	*12.09*	
8	~4820,6		6		**Callisto**	1882.68	1.0	1.0	10.38	
								α=2.8	**β=11.2**	
			1		*2196 – **2622***	***β ф2485***		0.1153123		
			2		*4634 – 5531*	***β u4723***		0.243309		
9	8		3		Themisto	7309.11	0.3214949	0.3214949	10.89	
10	10		4	1	**Leda**	11108.66				
11	170			2	**Himalia**	11385.86	0.5008128		10.63	
12	86			3	**Elara**	11664.67		0.4739336		
13	36			4	**Lysithea**	11688.92				
14				5	S/2000 J 11	12435.16				
15	1		5	1	S/2003 J 12	16787.83		0.7172426	*10.87*	
16	3			2	Carpo	16814.85	0.7396097		*10.89*	
17	2			1	Euporie	19044.30			10.11	
22	2	**III**		6	Thelxinoe	20074.99			10.66	
32	28			16	**Ananke**	20787.92			/каллисто=11.04	
33	4			17	Hermippe	20898.75			11.10	
34	4		6	18	Thyone	**21055.91**	***β к=21086***		β=10.86	$K_{26}\cdot\beta$
38				22	S/2003 J 10	22027.12			/каллисто=11.69	
51	60			35	**Pasiphae**	22734.76	1.00000000 –	1.0	/каллисто=12.08	
54	46			38	**Carme**	22873.13			/каллисто=12.15	
64	38			48	**Sinope**	23589.52			/каллисто=12.53	
65	4			49	Isonoe	23610.92			/каллисто=12.54	
								α=3.11		
66	5		1	1	Megaclite	24080.92			10.418	
67				2	S/2003 J 2	30018.99			11.6318	
			2		*52900*					
		IV	3		*81900*					
			4		*12750*					
			5		*17300*					
			6		*254600*					
					β=11.02			α=3.11		

(α ≈ 3.04, β ≈ 11.2, αβ ≈ 34.05)

Table 7: The Uranian moon system (the radius of Uranus is 24,800 km).

No	R thsd. km	Group number	Object number within a group	Number within a family	Moon name	r/r	β	β	Theoretical r/r
	13.83	I	1						0.18445
	21.22		2						0.28295
	29.6		3						0.3946
1	49.751000		4	1	Cordelia	0.6510206			0.652
2	53.762629			2	Ophelia	0.7035151			
7	66.097000			7	Portia	0.8649175			
8	69.927000		5		Rosalind	0.9150353			0.935
9	74.800000		6	1	Cupid	0.9788013			1.
10	75.255000			2	Belinda	0.9847553			
11	76.420000			3	Perdita	1. α=2.5342			
							β=6.25		
12	86.004000	II	1		Puck	0.1473864	6.23		0.126
13	97.734000				Mab	0.1674378			
14	129.389950		2		Miranda	0.2217652	6.09		0.251
15	191.019930		3		Ariel	0.327335	6.453		0.334
16	266.298930		4		Umbriel	0.4563838	5.32		0.5
17	435.909790		5		Titania	0.7470.	6.236		0.75
18	583.519630		6		Oberon	1. α=2.99	7.637		1.
		III	1		*618 – 656*				
			2		*819 – 1113*				
			3		*1209 – 1228*				
			4		*1685 – 1743*				
			5		*2438 – 2758*				
			6		*3067 – 3692*				
								β = 6.39	
19	4254.116700	IV	1		Francisco	0.2139946		6.59	
20	7218.710300		2		Caliban	0.3631319		7.469	
21	7961.082700		3	1	Stephano	0.4			
22	8410.678200			2	Trinculo	0.423		6.359	
23	11297.873000		4	1	Sycorax	0.5682881			
24	11316.714000			2	Margaret	0.569		6.519	
25	15801.542000		5	1	Prospero	0.795			
26	16239.657000			2	Setebos	0.8169		6.1	
27	19879.088000		6		Ferdinand	1. . α=**2.42** – 79		5.837	
						α=2.42			

(α ≈ 2.65, β ≈ 6.32, αβ ≈ 16.75)

Table 8: Dynamic parameters of the Sun and solar system planets.

Planetary names	The core temperature, T	Volume (V), cub. m	I_O	I_O*	α	β	αβ
Sun	$1.35 - 1.5 \cdot 10^7$	$1.41 \cdot 10^{27}$	0.171	0.34	2.94	13.1	38.514
Jupiter	$20 - 25 \cdot 10^3$	$14.3 - 15.2 \cdot 10^{23}$	0,20	0.262	3.04	11.2	34.05
Saturn	$11.7 - 20 \cdot 10^3$	$8.27 - 9.23 \cdot 10^{23}$	0,22	0.227	3.2	11.57	37.024
Uranus	$4.737 - 12 \cdot 10^3$	$6.39 - 6.833 \cdot 10^{22}$	0,23	0.212	2.65	6.32	16.75
Neptune	$7 - 14 \cdot 10^3$	$6{,}254 - 6.58 \cdot 10^{22}$	0,26	0.2	3.1	10.64	32.984

Here I_O is the reduced moment of inertia.

Table 9: The S parameter values for the solar system giants and the Sun.

Planetary names	**The core temperature, T**	**Volume (*V*) cub. m**	I_0	**αβ**	**S**	**S~**
Sun	$1.35 \cdot 10^{7}$	$1{,}41 \cdot 10^{27}$	0.34	38.514	$0.07977 \cdot 10^{20}$	$0.08 \cdot 10^{20}$
Jupiter	$25 \cdot 10^{3}$	$1{,}43 \cdot 10^{24}$	0.2	34.05	$0.08399 \cdot 10^{20}$	$0.08 \cdot 10^{20}$
Saturn	$12.15 \cdot 10^{3}$	$8{,}27 \cdot 10^{23}$	0.22	37.024	$0.08356 \cdot 10^{20}$	$0.08 \cdot 10^{20}$
Uranus	$2.45 \cdot 10^{3}$	$6{,}833 \cdot 10^{22}$	0.2	16.75	$0.08325 \cdot 10^{20}$	$0.08 \cdot 10^{20}$
Neptune	$1.2 \cdot 10^{3}$	$6{,}254 \cdot 10^{22}$	0.2	32.984	$0.08302 \cdot 10^{20}$	$0.08 \cdot 10^{20}$

The given values of the parameter S indirectly sustain the planetary spacing rule.

Having the values **T**, **V** and $\mathbf{I_O}$ selected (from Table 8), we see that the parameter **S**, determined by the following formula:

$$S = \frac{V}{T0} \frac{1}{I\alpha\beta}, \qquad (8)$$

takes on close values for planet – giants and the Sun.

4. Conclusions

Formally, α, in the units of the 3rd moon, is the upper boundary of the first group or the distance to the 6th moon. Then, αβ is the distance to the 6th moon within the next group or the upper boundary of the second group.

Thus, a set of values α and β can be determined from formulae (1) – (7) using two radii of the orbits of moons assigned to the given positions. Comparing these values with the values of T, V and I_O in formula (8), the fittest parameter values can be found.

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