DEPRESSION IN THE CONTINUOUS SPECTRUM OF SOLAR RADIATION IN THE REGION 6500 – 8200 Å

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ABSTRACT. Based on the authors' computations of cross-sections of the basic processes which form continuous absorption in the photospheres of solar-type stars, the spectral dependence of the solar radiation intensity in the continuous spectrum in the visible and infrared regions has been investigated. It is shown that the depression observed in the continuous spectrum of solar radiation in the region (6500 – 8200) Å is caused by the processes of photoionization of neutral hydrogen atoms excited up to the energy levels with the principal quantum number $n = 3$. The results of computation are in good agreement with the data observed.

Key words: cross-sections, continuous absorption, solar radiation intensity.

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1. Introduction

As is known, base on observations, Milne (1922) established general properties of the continuous absorption coefficient $æ_c(\lambda)$ of the Sun in the visible and infrared parts of the spectrum – with maximum near 9000 Å, deep minimum at 16400 Å and the subsequent growth at larger wavelengths. The suggestion made by Wildts (1939) that photoionization of negative hydrogen ions is the main mechanism of the continuous absorption explained the behaviour of $æ_c(\lambda)$ at $\lambda < 16400$ Å. Smith & Burch (1959), aimed to obtain precise measurement of the relative cross section $\sigma(\lambda)/\sigma(\lambda_0)$ (at $\lambda_0 = 5280$ Å) of the hydrogen ion photoionization under laboratory experimental conditions, found the presence of a fine structure in the region (6500 – 8200) Å. It should be noted that in fact the effective cross-section of the hydrogen atom photoionization was determined during that experiment, and the H^- photoionization is just one of the processes, which, however, make a major contribution. The results of observations by Münchs (1948) are also indicative of specific features of $æ_c(\lambda)$ in the neighborhood of the maximum. Computations of $æ_c(\lambda)$ performed by Chandrasekhar & Breen (1946), base on Wildt's suggestion, showed no fine structure in the vicinity of the maximum. More accurate computations of the relative cross-section of the photoionization of H^- ion, which were carried out with multiparameter variational wave functions by Geltman's (1962), appeared to be very close to the results obtained by Smith & Burch (1959) throughout all regions, except that one of maximum where the deviation was of the order of 2%. Neither Geltman nor other authors of later theoretical works (John (1988), Rau (1996)) paid any attention to these differences.

From general physical considerations it is clear that the photoionization of excited hydrogen atoms in the states with principal quantum number $n \geq 3$ should affect the continuous absorption in the small region at $\lambda < 8200$ Å. Primarily, it regards the ionization of atoms with $n = 3$, for which the ionization energy is equal to 0,1111... Ry that is twice as high as the H^- ionization energy (0,0555... Ry). Although the photoionization of such excited atoms is analogous to that one of the H^- ions, these processes can not make a great contribution in quantitative terms as the concentration of excited atoms under the solar conditions is low.

Over the recent decades, the investigation of spectral dependence of the solar radiation intensity was a subject of a significant number of works performed with high accuracy (Burlov-Vasil'ev, Vasil'eva & Matveev, 1996). This raises the problem of detailed calculation of the continuous absorption coefficient for the purpose of interpreting the observed solar spectrum in the vicinity of the region (6500 – 8200) Å, as well as clearing up the role of the excited hydrogen atom photoionization processes in the formation of spectra of stars similar to the Sun.

2. Cross-sections of the basic processes and continuous absorption coefficient

The continuous absorption coefficient in the visible and infrared regions of the spectrum for solar-type stars has the form:

$$æ_c(\lambda) = \{1 - \exp[-hc/\lambda T k_B]\} \times \tag{1}$$
$$\times \left\{ \frac{N_{H^-}}{V}\sigma_{H^-}(\lambda) + \sum_{n\geq 1} \frac{N_H^{(n)}}{V}\sigma_H^{(n)}(\lambda) + \right.$$

$$+\frac{N_e}{V}\sigma_{ee}(\lambda)+\sum_a \frac{N_a}{V}\sigma_a(\lambda)\Bigg\},$$

where $\sigma_{H^-}(\lambda)$ is the cross-section of the photoionization process of isolated H^- ion; N_{H^-}/V is the concentration of H^- ions; $\sigma_H^{(n)}(\lambda)$ is the cross-section of photoionization of a neutral hydrogen atom in the state with the principal quantum number n; $N_H^{(n)}/V$ is the concentration of such atoms; $\sigma_{ee}(\lambda)$ is the cross-section of photon absorption by either a free electron which is in the field of hydrogen atoms in different states, or protons, or electrons, or other charged and neutral particles; N_e/V is the concentration of electrons; $\sigma_a(\lambda)$ is the cross-sections of photon interaction with other micro-particles (atoms and ions of helium, metals, etc.); N_a/V is the concentration of particles of the corresponding class. The sum

$$\sigma_{eff}(\lambda)\equiv\sigma_{H^-}(\lambda)+\sum_{n\geq1}\frac{N_H^{(n)}}{N_{H^-}}\sigma_H^{(n)}(\lambda),\qquad \sigma_H^{(n)}(\lambda)=\sum_0^{n-1}\sigma_{n,l}(\lambda) \tag{2}$$

can be interpreted as an effective cross-section of hydrogen in visible and infrared regions os the spectrum as it is defined by bound-free transition in the H^- ions and neutral hydrogen atoms. This value per se corresponds to the results of measurements obtained by Smith & Burch (1959).

In this study we used the cross-sections $\sigma_{H^-}(\lambda)$ computed within the framework of basic approach in Vavrukh & Stelmakh (2013). As can be seen in Figure 1, it is very close to the most accurate calculation by Geltmans (1962). The thin curve in Fig. 1 represents the $\sigma_{eff}(\lambda)=\sigma_{H_-}(\lambda)$ aproximation.

For convenience, we will henceforth use the dimensionless wavelength $\lambda_*=\lambda/\lambda_0$, where $\lambda_0=4\pi a_0\alpha_0^{-1}=$ 911,27... Å (α_0 is the fine structure constant, a_0 – the Bohr radius). The cross-section $\sigma_{H^-}(\lambda)$ has zero asymptotics near the red limit of $\Lambda_*=(\Delta\varepsilon)^{-1}\cong$ 18,018... (where $-\Delta\varepsilon$ is the H^- ion ionization potential in Rydberg);

$$\sigma_{H^-}(\lambda)\sim(\lambda_*-\Lambda_*)^{3/2}\quad\text{at}\quad\lambda_*\to\Lambda_*. \tag{3}$$

Such behavior of $\sigma_{H^-}(\lambda)$ is conditioned by the fact that the photoelectron wave function in the weak field of a neutral hydrogen atom is close to the plane wave. The asymptotics of cross-section of the isolated hydrogen atom photoionization, computed by the photoelectron wave functions in the proton's Coulomb field (Karzas & Latter, 1961) are different:

$$\sigma_{n,l}(\lambda)\sim\begin{cases} x_n^{7/2+l} & \text{at}\quad x_n\ll1,\\ x_n^3\Theta(1-x_n) & \text{at}\quad x_n\to1,\end{cases} \tag{4}$$

where $x_n=\lambda_*/n^2$, $\Theta(z)$ is a single theta-function; l is the orbital quantum number.

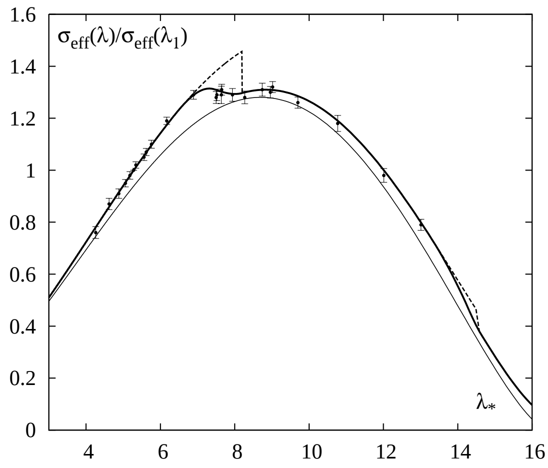

Figure 1: The spectral dependence of the effective relative cross-section in different approximations. Marks on the graph show the experimental results by Smith & Burch (1959).

Fig. 2 illustrates the spectral dependence of cross-section $\tilde{\sigma}_{3,l}(\lambda)=(\alpha_0a_0^2)^{-1}\sigma_{3,l}$. The dashed curve in the Fig. 1 shows the effective cross-section in the vicinity of the maximum in approximation:

$$\sigma_{H^-}(\lambda)+\frac{N_H^{(3)}}{N_{H^-}}\sum_{l=0}^{2}\sigma_{3,l}(\lambda)+\frac{N_H^{(4)}}{N_{H^-}}\sum_{l=0}^{3}\sigma_{4,l}(\lambda). \tag{5}$$

As is seen from Fig. 1, such an approximation describes the results of Smith & Burch (1959) quite well; however, the deviation in the region (6500 − 8200) Å is essential. It can be caused by the fact that the final stage of the the hydrogen atom photoionization in the vacuum differs from that one in the partially ionised

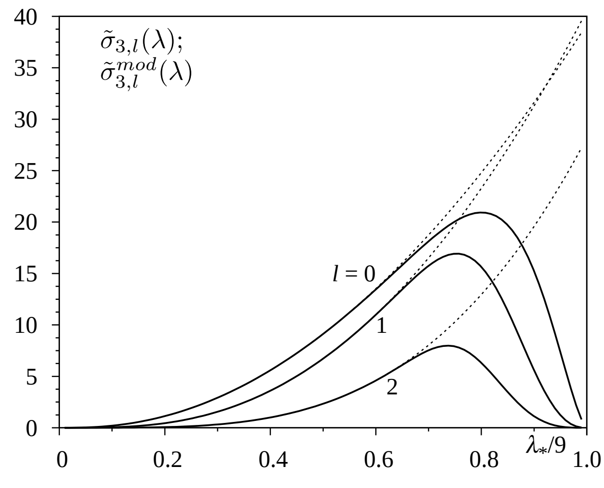

Figure 2: Dashed curves represent spectral dependences of the functions $\tilde{\sigma}_{3,l}(\lambda)$ while solid curves correspond to the model cross-sections $\tilde{\sigma}_{3,l}^{mod}(\lambda)$ at $b_{n,l}=$ 4,0.

plasma. In vacuum the photoelectron is in the protons' Coulomb field. The duration of the photoelectron stay in the vicinity of the parent atom in the partially ionised photospheric plasma

$$\tau_1 = R/v_e \tag{6}$$

can exceed the recombination time of the free electron with a proton:

$$\tau_2 = R/v. \tag{7}$$

Here R is of the order of the Debye radius R_D, and the photoelectron speed is determined by the condition

$$\frac{m}{2}v_e^2 = \hbar\omega - \Delta E_n = \frac{e^2}{2a_0}(\omega_* - \Delta\varepsilon_n), \tag{8}$$

where $\Delta\varepsilon_n$ is the ionization energy in Rydberg; and v is the mean square speed at the given temperature ($v^2 = 3k_BT/m$). Given that $\tau_1 > \tau_2$ ($v_e^2 < v^2$), we find that

$$\lambda_* > \lambda_*^{max}\left\{1 + \frac{3}{2}\lambda_*^{max}t\right\}^{-1} = \lambda_*^0, \tag{9}$$

where $t = k_BT/\,Ry$ is the dimensionless photospheric temperature; $\lambda_*^{max} = n^2$. For $n = 3$ at the effective solar temperature $6 \cdot 10^3$K we have $\lambda_*^0 \approx 6$, for $n = 4$ – $\lambda_*^0 \approx 8$, while $\lambda_*^0 \approx 10$ for $n = 5$. As it follows from formula (9), the photoelectron with low energy most of the time spends in the weak field of neutral hydrogen atom rather in the proton's Coulomb field. Therefore, at $\lambda_* < \lambda_*^0$ the correct cross-sections $\sigma_{n,l}(\lambda)$ are those ones calculated with the Coulomb photoelectron functions while at $\lambda_* > \lambda_*^0$ – those calculated by the functions close to plane waves. The latter have zero asymptotic at $\lambda_*^{max} = n^2$. As an example, we present the asymptotic of the cross-sections, calculated by the plane waves at $n = 3$

$$\sigma_{3,l}(\lambda) \approx \pi\alpha_0 a_0^2 2^{8+2l}3^{3-l}(x)^{7/2+l}(1-x)^{3/2+l}, \tag{10}$$

where $x = \lambda_*/9$. To approximately factor in the effects of recombination processes, the joining of the cross sections from the study by Karzas & Latter (1961) with those from (10) was used in the study by Vavrukh & Stelmakh (2013). In this paper we used the model cross-sections:

$$\begin{aligned}\sigma_{n,l}^{mod}(\lambda) &= \sigma_{n,l}(\lambda) \times \\ &\times \left\{1 - \exp\left[-b_{n,l}\left(\frac{1-\lambda_*/n^2}{1-\lambda_*^0/n^2}\right)^{3/2+l}\right]\right\},\end{aligned} \tag{11}$$

where $\sigma_{n,l}(\lambda)$ are the cross-sections calculated by the Coulomb functions by Karzas & Latter (1961); and parameters $b_{n,l} \sim 1$ are chosen in such a way that $\sigma_{n,l}^{mod}(\lambda)$ are close to the expression (10) near a red limit (Fig. 2). The thick curve in Fig. 1 corresponds to approximation (5) where $\sigma_{n,l}^{mod}(\lambda)$ is used instead of $\sigma_{n,l}(\lambda)$. The computation results correspond to the temperature $T = 6\cdot 10^3$ K and the dimensionless barion concentration $\rho a_0^3(m_H)^{-1} = 5 \cdot 10^{-8}$.

When calculating the cross-section of photon absorption by free electrons, we took into account the presence of protons and neutral hydrogen atoms in the ground and excited states. Approximately, the potential energy of an electron in the field of these particles can be presented in the form

$$\begin{aligned}V_e(\mathbf{r}) \cong \frac{1}{V}\sum_{\mathbf{q}}\{&V_{ep}(\mathbf{q})S_p(-\mathbf{q}) + \\ &+ \sum_{n\geq 1} V_{eH}^{(n)}(\mathbf{q})S_H^{(n)}(-\mathbf{q})\}e^{i(\mathbf{q},\mathbf{r})},\end{aligned} \tag{12}$$

where $S^p(-\mathbf{q}) = \sum_{j=1}^{N_p}\exp[-i(\mathbf{q},\mathbf{R}_j^p)]$ is the structure factor of a proton subsystem; $\mathbf{R}_j^p$ is the radius vector of the j-th proton; $S_H^{(n)}(-\mathbf{q})$ is the structure factor of a neutral hydrogen atoms, which are in the state with the principal quantum number n;

$$\begin{aligned}V_{ep}(\mathbf{q}) &= -4\pi e^2 a_0^2\{q_*^2 + \xi^2\}^{-1} \equiv \\ &\equiv 4\pi e^2 a_0^2 v_{ep}(q_*)\end{aligned} \tag{13}$$

is a Fourier energy of the electron-proton interaction taking into account screening effects ($q_* = qa_0$, $\xi = a_0/R_D$ is the dimensionless reciprocal screening radius), with only s-states for centrally-symmetric potentials of electron-atom interaction accounted for. We obtain the following expressions:

$$\begin{aligned}&V_{eH}^{(n)}(\mathbf{q}) = 4\pi e^2 a_0^2 v_{eH}^{(n)}(q_*);\\ &v_{eH}^{(1)}(\mathbf{q}_*) = -(8+q_*^2)(4+q_*^2)^{-2};\\ &v_{eH}^{(2)}(\mathbf{q}_*) = \{6 + 2q_*^2 + 3q_*^4 + q_*^6\}(1+q_*^2)^{-4};\ldots.\end{aligned} \tag{14}$$

In the Bohr approximation and at chaotic spatial distribution of particles when $< S_p(\mathbf{q})S_p(-\mathbf{q}) >= N_p$, $< S_H^{(n)}(\mathbf{q})S_H^{(n)}(-\mathbf{q}) >= N_H^{(n)}$, $< S_H^{(n)}(\mathbf{q})S_p(-\mathbf{q}) >= 0$,

$$\begin{aligned}\sigma_{ee}(\lambda) = \alpha_0 a_0^2\pi^{3/2}\lambda_*^3\frac{2^9}{3}\int_0^\infty dk\, k\, e^{-k^2/t}\int_{z_-}^{z_+} dz\, z \times\\ \times\left\{\frac{N_p}{V}a_0^3 v_{ep}^2(z^{1/2}) + \sum_{n=1,2}\frac{N_H^{(n)}}{V}[v_{eH}^{(n)}(z^{1/2})]^2 + \ldots\right\},\end{aligned} \tag{15}$$

where

$$\begin{aligned}&z_- = (k-k_0)^2, \quad z_+ = (k+k_0)^2,\\ &k_0 = (k^2 + \lambda_*^{-1})^{1/2};\end{aligned} \tag{16}$$

N_p/V is the proton concentration. At the ionization equilibrium under conditions of solar photosphere, the contribution into cross-section $\sigma_e(\lambda)$ made by an electron-proton interaction is of the same order as the contribution caused by interaction of electrons with hydrogen atoms in the ground state. The contribution

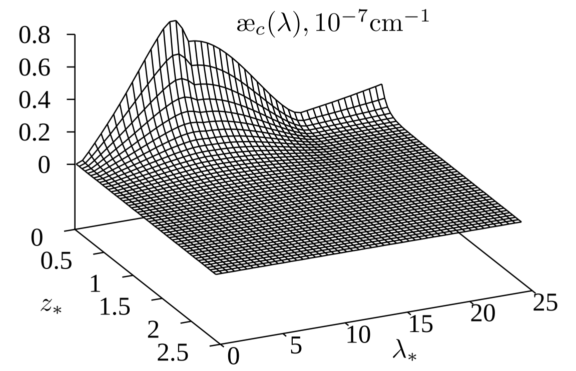

Figure 3: The spectral and coordinate dependence of the continuous absorption coefficient for the Sun.

due to the interaction of electrons with excited atoms is small. Generally, the contribution of the processes of photon absorption by free electrons to the continuous absorption coefficient in the region of maximum $æ_c(\lambda)$ is small. However, this contribution is essential in the wavelength region $\lambda > 14 \cdot 10^3$ Å.

3. Solar radiation intensity in the continuous spectrum

For the center of the solar disc the intensity of radiation in the continuous spectrum was calculated by the standard formula

$$I_\lambda = \int_0^{z_0} dz\, æ_c(\lambda|z)\, B_\lambda(z) \exp\left\{ - \int_z^{z_0} dz\, æ_c(\lambda|z) \right\}, \tag{17}$$

where the Planck distribution function $B_\lambda(z) = \frac{4\pi c^2 \hbar}{\lambda_0^5} \frac{1}{\lambda_*^5} \{\exp[-\lambda_*^{-1} t(z)^{-1}] - 1\}^{-1}$ which accounts for the sum over photons polarization is used as the source function. We have chosen a plane-parallel model of atmosphere with the thickness $z_0 = 240$ km (Aller, 1971), in which the density and temperature distribution along the radius are approximated in the following form:

$$\rho(z) = \rho_0 \exp\left\{ -\frac{a z_*}{t(z_*)} \right\}, \tag{18}$$
$$t(z_*) = t_0 \exp\left\{ -\frac{b z_*}{1 + c z_*} \right\},$$

where $t_0 = T_0 k_B (e^2/2a_0)^{-1}$; $\rho_0 = 0{,}450 \cdot 10^{-6}$ g/cm^3; $T_0 = 7{,}3 \cdot 10^3$ K; $a = 2{,}45$; $b = 0{,}36$; $c = 0{,}33$, $z_* = z \cdot 10^{-7}$ cm. The continuous absorption coefficient $æ_c(\lambda|z)$ for this atmosphere is shown in Fig. 3. The equilibrium concentration of particles for the given density and temperature werw calculated by the self-consistent method, which is examined in details for the hydrogen-helium model by Stelmakh (2014).

The results of numerical calculation I_λ in terms of 10^{14} J /s m^3 sr. are shown in Fig. 4 (curve 1). The Figure also presents the results of the intensity measurement by Burlov-Vasil'ev, Vasil'eva & Matveev (1996) in the region $(6 - 11) \cdot 10^3$ Å. Curve 2 was calculated on the base of $æ_c(\lambda|z)$, not accounting for the cross-sections of the excited hydrogen atom photoionization at $n = 3$ and $n = 4$. As is seen from the Figure, the spectral behavior of curve 1 is similar to that of the curve, ploted by the observation data. Both curves have a deflection in the region $(6500 - 8200)$ Å relative to curve 2.

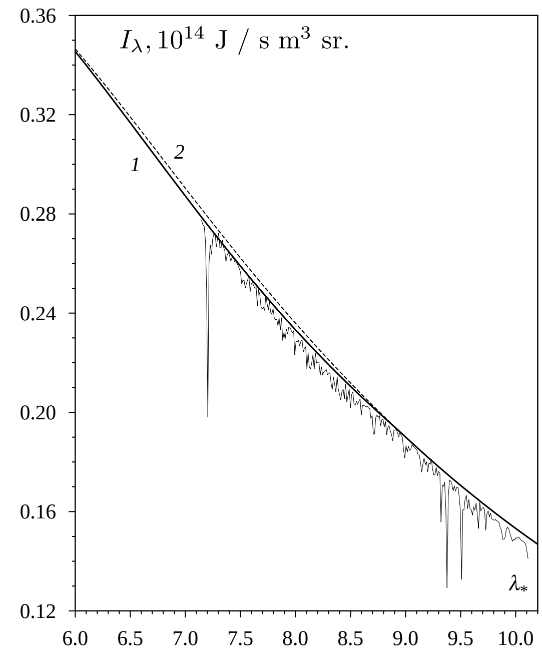

Figure 4: The continuous radiation intensity for the solar disk center.

As follows from our computations, the depression on the curve for the solar radiation intensity spectral distribution in the region $(6500 - 8200)$ Å is caused by the photoionization of neutral hydrogen atoms excited to the energy levels of the principal quantum number $n = 3$.

If the cross-sections of excited hydrogen atom photoionization, calculated by the Coulomb functions, are used, then there is a jump of the calculated curve of the solar radiation intensity in the continuous spectrum in the region 8200 Å; that is inconsistent with the data observed. The model cross-sections are in good agreement, which is indicative of their correct qualitative spectral dependence.

As is known, the intensity of solar radiation in the ultraviolet region depends on the phase of activity. In the phase of maximum it is 2-3 times higher than in the minimum intensity phase. Therefore, the concentration of hydrogen atoms, excited to the level with $n = 3$, can vary over 11-year cycle, and that can affect the depth of depression in the region $(6500 - 8200)$ Å.

Besides the point, the measurements of the solar radiation intensity, which provided the basis of paper by Burlov-Vasil'ev, Vasil'eva & Matveev (1996), had been performed during the period of minimum solar activity.

Depression, which is discussed in this paper, must also be observed in the spectra of stars of other spectral classes, in which the photoionization of negative hydrogen ions is the main mechanism of continuous absorption formation. In the spectra of the hotter stars, such depression should be stronger while its exhibitions will be weak and irregular in the stars with $T_{eff} < 5{\cdot}10^3$ K.

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