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ILLUMINATION OF ARTIFICIAL EARTH SATELLITES IN CIRCULAR ORBITS

A. Strautman[1], O. Bazyey[2]

[1] Department of Physics and Astronomy, FMPIT, Odesa I. I. Mechnikov National University
42 Pastera St, Odesa, Ukraine, *albinstrautman@gmail.com*

[2] Department of Mathematical Analysis, FMPIT, Odesa I. I. Mechnikov National University
2 Zmiienka Vsevoloda St, Odesa, Ukraine, *o.bazyey@onu.edu.ua*

ABSTRACT. The purpose of the work is to build an updated model of the illumination of artificial satellites in circular Earth orbits and to study the duration and nature of solar illumination in orbits with different inclinations and altitudes throughout the year.

The mathematical model uses the equation of the circular cone of the shadow, built taking into account the movement of the Sun relative to the Earth. The center of the cross section of the base of the cone coincides with the center of the Earth. The motion of the satellite is simulated by Kepler's orbit. The computer model makes it possible to determine with a given accuracy the duration of the satellite's stay in the Earth's shadow.

Simulation of the duration of illumination of satellites at two altitudes has been performed: 5,000 km and 35,786 km (geosynchronous orbit altitude) throughout the year. Curves of the duration of the satellites' stay in the shadow are given. The shape of the curves varies from a nearly straight line for inclined orbits 25°, then they become periodic, and then divide into two parts, resembling the shape of a parabola. Among all the possible inclinations of the orbits of satellites, extreme ones have been detected. These are orbits with an angle of inclination 23°26′, which defines a straight orbit. On them, an artificial satellite falls into the Earth's shadow throughout the year at each orbit. The second group of extreme orbits are orbits with inclinations, in which the satellite falls into the shadow only near the time of the equinoxes. Shortest duration of stay of satellites in the shadow moving in orbits with an angle of inclination 113°26′. Falling into the shadow lasts from 15.02 to 23.04 and from 19.08 to 27.10 for an altitude of 5,000 km, and from 12.03 to 28.03 and from 14.09 to 01.10 for an altitude of 35,786 km.

The results of the simulations will allow us to clarify the effect of sunlight and solar wind pressure on the motion of satellites over time. This will allow the use of additional satellite accelerations resulting from radiative impact to change the orbits of space debris and clean up near-Earth space.

Key words: Earth orbit, artificial satellite, shadow cone, duration of stay in the shadow.

АНОТАЦІЯ. Метою роботи є побудова уточненої моделі освітленості штучних супутників на колових навколоземних орбітах та дослідження тривалості та характеру сонячного освітлення на орбітах з різними нахиленнями та висотами протягом року.

У математичній моделі використане рівнянні колового конуса тіні, побудованого з урахуванням руху Сонця відносно Землі. Центр перерізу основи конуса збігається з центром Землі. Вплив атмосфери не враховується. Рух супутника моделюється Кеплеровою орбітою. Комп'ютерна модель дає змогу визначити з заданою точністю тривалість перебування супутника у тіні Землі.

Виконано моделювання тривалості освітленості супутників на двох висотах: 5 000 км та 35 786 км (висота геосинхронної орбіти) протягом року. Наведені криві тривалості перебування супутників у тіні. Форма кривих змінюється від практично прямої лінії для орбіт з нахиленням 25°, далі набувають періодичного характеру, а потім діляться на дві частини, що нагадують форму параболи. Серед усіх можливих нахилень орбіт супутників виявлені екстремальні. Це орбіти з кутом нахилення 23°26′, що визначає пряму орбіту. На них штучний супутник протягом усього року на кожному витку потрапляє у тінь Землі. Друга група екстремальних орбіт – це орбіти з нахилами, при яких супутник потрапляє у тінь лише поблизу часу рівнодень. Найменша тривалість перебування супутників у тіні, які рухаються по орбітах з кутом нахилу 113°26′. Потрапляння у тінь триває від 15.02 до 23.04 та від 19.08 до 27.10 для висоти 5000 км, та від 12.03 до 28.03 та від 14.09 до 01.10 для висоти 35786 км.

Результати моделювання дозволять уточнити вплив тиску сонячного світла та сонячного вітра на рух супутників протягом тривалого часу. Це дозволить використати додаткові прискорення

супутників, що виникають внаслідок радіативного впливу, для змін орбіт космічного сміття та очищення навколоземного простору.

Ключові слова: навколоземна орбіта, штучний супутник, конус тіні, тривалість перебування у тіні.

1. Introduction

Since the first launch of an artificial satellite in 1957, the number of spacecraft and the degree of contamination of near-Earth space has been constantly increasing. Space debris, consisting of broken satellites, spent rocket stages and debris from collisions, poses a serious threat to working vehicles in orbit. The main characteristic of this phenomenon is the concentration of space debris, which depends on altitude, type of orbit and time. Two approaches are used to predict the evolution of garbage - deterministic (modeling the orbits of individual objects) and stochastic (modeling the distribution of garbage concentration). However, both methods demonstrate a steady increase in the amount of debris, which over time can lead to a cascading effect - an avalanche-like increase in debris. The solution of this problem is possible through the development of new models of the motion and interaction of objects in near-Earth space, including taking into account radiation effects (sunlight pressure, solar wind, albedo, etc.), which can be used to change the orbits of "passive" objects. The purpose of the study is to build an updated model of the illumination of near-Earth satellites, determine the duration of stay in the Earth's shadow, as well as analyze the dependence of the duration of stay in the Earth's shadow on the inclination of the circular orbit at different altitudes throughout the year.

2. Mathematical model and software implementation

The Sun, illuminating the Earth, forms a cone of shadow in the opposite direction. For an observer at a satellite-centric point, the Sun is completely covered by the Earth's dark disk. To model the shadow, we use a simplified approximation of a circular cone, whose equation

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$$

(Selezniova et al., 2016), and we describe the movement of the satellite in Kepler's orbit. The Earth is taken for a sphere without an atmosphere: polar compression and refraction of light in the atmosphere are not taken into account. The model uses the equation of the shadow cone, built taking into account the movement of the Sun relative to the Earth. The center of section of the base of the cone coincides with the center of the Earth.

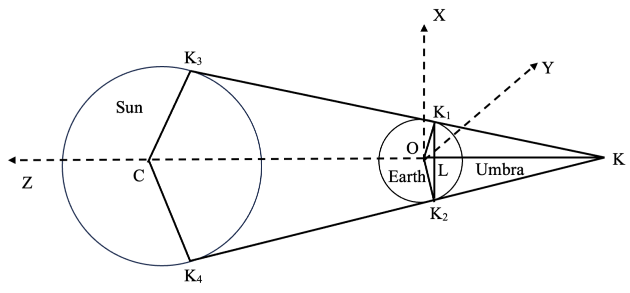

Figure 1: Shadow cone model.

As a result of the Earth's annual motion, the axis of the shadow cone, which is marked as the Z axis in Figure 1, also rotates. The center of the Sun remains on the Z axis, so the shadow cone equation

$$x^2 + y^2 = \frac{(z+217)^2}{47088},$$

where the unit of measurement of distances is the average radius of the Earth. Let's prove that the equation of the cone looks like this using Figure 1. The Sun is at a point with coordinates C(0, 0, 1 a.u.) Radius of the Sun $R_s \approx 109$ r. e. (1 a.u. $\approx$ 23 481 r. e. (r. e. – Radius of the Earth)). The Earth is at a point with coordinates O(0, 0, 0). Radius of the Earth R_e = 1 r. e. CO $\approx$ 23 481 r. e.; $OK_1 = OK_2 = 1$ r. e.; $CK_3 = CK_4 \approx 109$ r. e.; C(0, 0, 1 a.u.); O(0, 0, 0); KK_1K_2 – shadow cone; Let us recall the form of the cone equation for our case, given that the vertex of the cone is not at the center of the coordinate system at the point K(x_B, y_B, z_B):

$$\frac{(x-x_B)^2}{a^2} + \frac{(y-y_B)^2}{b^2} - \frac{(z-z_B)^2}{c^2} = 0$$

the equation of the shadow cone (we will find a, b, c, K(x_B, y_B, z_B).

Consider $\triangle KOK_1$ and $\triangle KCK_3$: OK_1 and CK_3 – these are the radii, $OK_1 \perp KK_3$ and $CK_3 \perp KK_3$ (with proper tangents and radii), $OK_1 \parallel CK_3$, $\triangle KOK_1 \sim \triangle KCK_3$ (according to the lemma about similar triangles),

$$\frac{CK_3}{OK_1} = \frac{CK}{KO} = \frac{109}{1},$$

[$\frac{KO+CO}{KO}$ = 109, 109 KO = KO + 23481, KO $\approx$ 217 r. e.; K(0; 0; -217); $cos < KOK_1 = \frac{OK_1}{KO} = \frac{1}{217} = \frac{OL}{OK_1}$, [$OL = \frac{1}{217} r.e.$, KL = KO - OL = 217 - $\frac{1}{217} = \frac{47088}{217} =$ c, $sin < KOK_1 = \frac{LK_1}{OK_1} = LK_1$ (as $OK_1 = 1$ r.e.); $sin < KOK_1 = \sqrt{1 - cos < KOK_1^2} = \sqrt{1 - \frac{1}{217^2}} = \sqrt{\frac{47088}{47089}} = a = b$ (as the Earth is a ball); Shadow Cone Equation:

$$\frac{47089x^2}{47088} + \frac{47089y^2}{47088} - \frac{47089(z+217)^2}{47088^2} = 0,$$

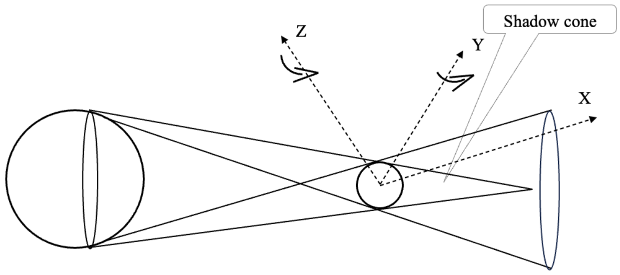

Figure 2: Rotation of axes to account for the movement of the Sun.

$$x^2 + y^2 = \frac{(z + 217)^2}{47088}.$$

Since the Sun revolves around the Earth, you need to reduce the equation of the cone to the form that we gave above. To bring the shadow cone equation to the form we gave above, we, as shown in Figure 2, rotate the coordinate system around the Z axis and the Y axis counterclockwise so that the axes of the coordinate system are arranged in the same way as in Figure 1, so that the cone equation is the same as we gave above.

According to the specified Keplerian elements of the orbit, the coordinates of the satellite are calculated over the entire period of rotation with a given step. The resulting coordinates are compared with the equation of the shadow cone, which makes it possible to determine the illumination of the satellite: in the shadow or illuminated by the Sun. The developed software allows you to determine the duration of the satellite's stay in the shadow on one orbit with a given accuracy throughout the year. The distance from the Sun to Earth is modeled by Keplerian elements of the orbit of the mean Sun. We used the following values of the elements of the Sun's orbit:
e = 0,0167133
a = 1,00000261 a.u.
ω = 102,93768193°
Ω = 0
i = -0,00001531°
(https://ssd.jpl.nasa.gov/planets/approx_pos.html)

3. Conditions and results of modeling

The study was carried out for two values of the altitudes of circular orbits: 5000 km and 35786 km, the inclinations of which to the plane of the Earth's equator vary from 0 to 180° in increments of 5°, the longitude of the ascending node and the average anomaly are zero. The study was also carried out for the INTELSAT 10 (IS-10) satellite with the following orbital elements:
e = 0,0002520
a = 35786 km
ω = 186,593°
Ω = 253,512°
i = 0,018°
(https://celestrak.org/NORAD/elements/geo.txt)

We got the dependence of the duration of the satellite's stay in the shadow on the calendar date. Figures 3 and 5 show the duration of stay in the shadow of satellites with an inclination of the orbital planes from 20 to 120°, that is, these are mainly orbits with direct motion. Figures 4 and 6 show the duration of stay in the shadow of satellites with an inclination of the orbital planes from 120 to 20°, that is, these are mainly orbits with reverse motion. Figure 7 shows the duration of stay in the shadow on one branch each day for the INTELSAT 10 (IS-10) satellite. The inclination of the satellite's orbit is measured from the equatorial plane in increments of 5°, and the inclination of the Earth's (Sun's) orbit is not a multiple of this value. Because of this, we obtained asymmetric pairs of graphs in Figures 3 and 5, and 4 and 6. If the pitch of inclination of the satellite's orbits is chosen as a multiple of 23.44°, you can get completely similar pairs of graphs in the above figures.

The shape of the curves varies from an almost straight line for orbits with an inclination of 25°, become periodic, and then divide into two parts, resembling the shape of a parabola. Tilts of 0 and 180° obviously give the same curves, since both orbits lie in the plane of the Earth's equator. The difference between them lies solely in the direction of movement of the satellite.

In general, the graphs show the dependence of the duration of the stay of an artificial satellite in the shadow on one orbit on the calendar date at different altitudes above the Earth's surface and different inclinations of the orbital planes. The graphs are plotted in 5° pitch increments. The figures do not indicate slopes for curves that overlap each other. They are in the intervals: in Figure 3 – from 70° to 120°, in Figure 4 – from 120° to 160°, in Figure 5 – from 40° to 120°, in Figure 6 – from 120° to 80°.

The resulting shape of the graphs is determined by the inclination of the plane of the ecliptic to the plane of the celestial equator. Throughout the year, the declination of the Sun changes from +23°26′ to -23°26′ (Karttunen et al., 2007). On the days of the equinoxes, the declination of the Sun is 0, respectively, the axis of the cone of the shadow is parallel to the plane of the celestial equator, so the satellite stays in shadow at this time for the longest time. On the days of the solstices, the axis of the cone of the shadow is inclined at the greatest angle 23°26′, the artificial satellite for most orbital inclinations does not fall into the shadow at all. At this time, the duration of stay in the shade is the shortest.

For the INTELSAT 10 (IS-10) satellite, the same dependence of being in the shadow by analogy is

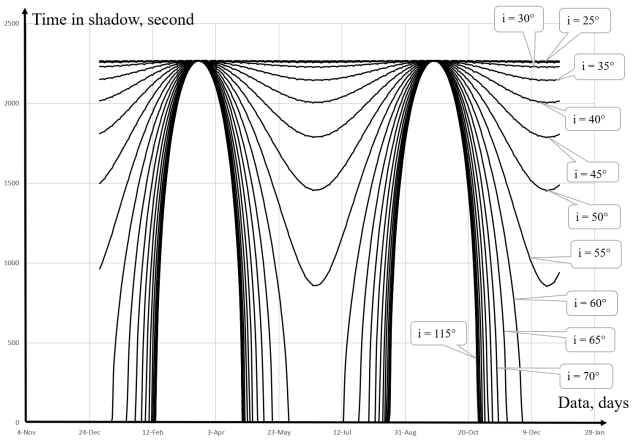

Figure 3: Time in the shadows, height above the Earth surface 5000 km, inclination 20-120°.

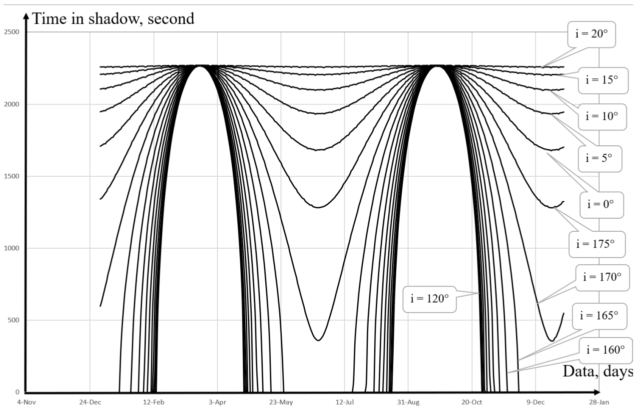

Figure 4: Time in the shadows, height above the Earth surface 5000 km, inclination 120-20°.

obtained as for satellites for geostationary orbit with zero inclination of the orbit relative to the Earth's equator, but the difference is only one thing: the maximum stay in the shadow for this satellite is greater (about 4,000 seconds).

4. Conclusion

In Figures 3 and 5, almost straight lines show that satellites in orbits with an inclination of 25° are in the Earth's shadow throughout the year for 2300 seconds at each revolution. This is because the planes of their orbits are very close to the plane of the ecliptic. As the inclination increases, the duration of stay in the shadow decreases as the date approaches the moments of the solstices. Starting from an inclination of 60° to an inclination of 165°, dates appear when the satellites are illuminated continuously. The longest duration of this period is in satellites moving in orbits with inclinations

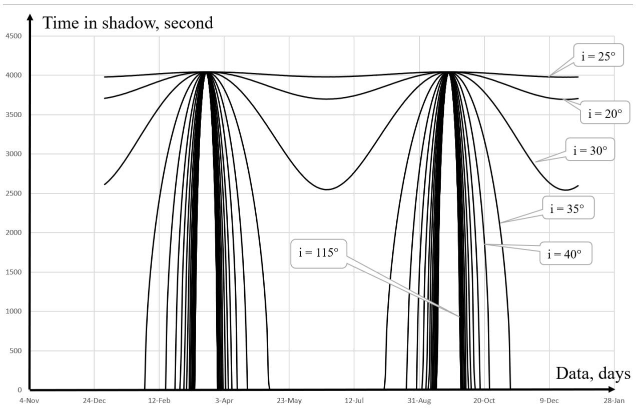

Figure 5: Time in the shadows, height above the Earth surface 35786 km, inclination 20-120°.

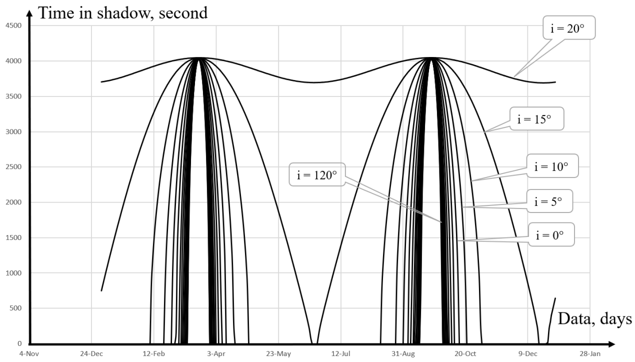

Figure 6: Time in the shadows, height above the Earth surface 35786 km, inclination 120-20°.

of 115-120°. Such orbits are almost perpendicular to the plane of the ecliptic. Thus, among all the possible inclinations of satellite orbits, there are extreme ones. This is the angle 23°26′, which defines the direct orbit in which an artificial satellite falls into the Earth's shadow throughout the year at each orbit. The second group of extreme orbits are orbits with inclinations, in which the satellite falls into the shadow only near the time of the equinoxes. The shortest duration of stay in the shadow is for satellites moving in orbits with an angle of inclination of 113°26′. Falling into the shadow lasts from 15.02 to 23.04 and from 19.08 to 27.10 for an altitude of 5000 km, and from 12.03 to 28.03 and from 14.09 to 01.10 for an altitude of 35786 km.

A practical example with the INTELSAT 10 (IS-10) satellite proves that our calculations of common cases that we have considered are correct and our example is one of our common cases.

The developed illumination model can be used for further studies of the effect of radiation effects on the orbital motion of objects. In particular, it is proposed to use the discontinuity of solar pressure (due to being in the shadow) to control the movement of space debris.

If satellites or debris have variable reflectivity (albedo) or the ability to orient relative to the Sun, this opens up the prospect of changing orbits without additional fuel consumption – that is, cleaning up near-Earth space by natural forces.

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