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TOWARDS THEORETICAL MODELLING OF DISTRIBUTION OF MATTER IN UNIVERSE

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ABSTRACT. The description of the distribution of matter in the universe requires not only accurate observations but also adequate approaches to their theoretical interpretation. This paper proposes a method of parameterization of distributions with a morphologically complex topology using structural invariants (for example, Euler), based on which it is possible to distinguish clusters with different topologies (in the observation plane). Based on the introduced classification and appropriate scaling, it becomes possible to estimate the distribution of matter, for example, within the framework of the mean-field model. To study the kinetics of the evolution of matter distribution, it is proposed to introduce the appropriate ordering parameter, which is built based on the calibrating and current values of the Euler-Poincaré invariants. This approach, by constructing phase diagrams for the ordering parameter, allows you to track the details of the kinetics of the evolution of matter distributions, studying, in particular, the temporal hierarchy of relaxation times of intermediate states. The temporal kinetics of this approach are described using simple kinetic equations that describe the relaxation of the ordering parameter field, and the values of invariants determined with the help of appropriate measurements (observations) appear as initial conditions. This approach can be seen as an alternative to other approaches to parameterization of structurally complex systems, such as Voronoi methods, graphs, etc. A comparative analysis of the results of various alternative approaches to the parameterization of topologically complex distributions of matter, which will be conducted in the future, should contribute to the deepening of existing ideas about the nature of the distribution of matter in the Universe.

Keywords: matter distribution, Euler-Poincaré structure invariants, order parameter relaxation.

АБСТРАКТ. Опис розподілу матерії у Всесвіті вимагає не тільки прецизійних спостережень, але й адекватних підходів до їх теоретичної інтерпретації. У нашій роботі пропонується метод параметризації розподілів із морфологічно складною топологією за допомогою структурних інваріантів (наприклад, інваріантів Ейлера-Пуанкаре), на основі якого можна розрізняти кластери з різною топологією (у площині спостереження). На основі такої класифікації та відповідного масштабування з’являється можливість вивчення розподілу речовини, (наприклад, за допомогою моделі середнього поля). Для вивчення кінетики еволюції розподілу речовини пропонується ввести відповідний параметр впорядкування, який будується на основі калібрувальних (асимптотичних) та поточних значень інваріантів Ейлера-Пуанкаре. Такий підхід, шляхом побудови фазових діаграм для параметру впорядкування дозволяє відслідковувати деталі (і, зокрема, немонотонний характер) кінетики еволюції розподілів матерії вивчаючи, наприклад, часову ієрархію часів релаксації проміжних станів глобально неоднорідної системи. Часова кінетика про цьому підході описується за допомогою простих кінетичних рівнянь, які описують релаксацію поля параметру впорядкування, а в якості початкових умов фігурують визначені за допомогою відповідних вимірів (спостережень) значення інваріантів. Запропонований підхід може розглядатися як альтернативний до інших підходів до параметризації структурно складних систем, таких, як методи побудов Вороного, графів та ін. Порівняльний аналіз результатів різних альтернативних підходів до параметризації топологічно складних розподілів речовини, який буде проведено у майбутньому має сприяти поглибленню існуючих уявлень про характер розподілу речовини у Всесвіті.

Ключові слова: розподіл матерії, структурні інваріанти Ейлера-Пуанкаре, релаксація поля параметра впорядкування.

1. Introduction

We explore unexpected connections between cosmological objects and micromechanical systems (granular materials), drawing attention to some common features. To model and understand these phenomena, we focus on using the Euler-Poincaré invariants to describe scaling relationships.

Earlier (Gerasymov, 2015; 2022) has been considered a simple but effective model that characterizes the kinetics of the compaction field in granular materials. This model related packing density to parameters such as the number of impacts or other perturbations. We assumed that these methods, originally developed for granular systems, can be applied to the study of the distribution of matter in the Universe (Gerasymov & Kudashkina, 2022).

The ultimate goal of our study is to quantitatively describe the geometric properties of materials with complex internal structures at different scales. We draw an analogy between the distribution of masses on a cosmic scale, such as in supergalactic clusters, and the basic structure of granular materials. To do this, we use stereological methods that allow us to qualitatively describe the morphology of objects by analyzing sectional planes and extracting three-dimensional information (Gerasimov & Spivak, 2020).

1) The research addresses the task of quantitatively describing the geometric characteristics of materials with complex internal morphology at micro- and macro-scales.

2) An analogy between the distribution of masses in large-scale systems, such as supergalactic clusters, and the structure of granular materials is examined.

3) The application of stereology methods for qualitative description of the morphology of objects analyzed on sectional planes to extract three-dimensional information is discussed.

2. Method of parameterization of distributions with a morphologically complex topology

Since measurement of absolute density is often impossible both in the Universe and in bulk materials, we turn to various functions and invariants to describe changes in relative density, compare scales, and study dependencies between different systems. Some of the key factors we consider include mean density, density relative to the mean, structural patterns, Euler-Poincaré invariants, modeling, and statistical distributions.

We introduce the concept of the Euler-Poincaré connection as an essential stereological characteristic that helps to describe the morphology of an object. We present basic connectivity parameters for various spatial dimensions such as 3D, 2D, and 1D. This Euler-Poincare connection is the foundation for analyzing the geometric and topological features of structures at different scales.

For example, the scaling dependence for the Universe and granular material can be expressed in terms of the Euler-Poincaré invariants.

$$\frac{\chi_U}{\chi_S} = h(\rho_U, \rho_S).$$

Here f and g are functions that describe the dependence of the characteristic curvature on density and volume for the Universe (U) and for a granular system (S), respectively:

$$\chi_U = f(\rho_U, V_U),$$
$$\chi_S = g(\rho_S, V_S).$$

The scaling dependence ceases to be a power law, that is, limited to one specific form. The use of Euler-Poincaré invariants makes it possible to take into account more complex structural and geometric aspects depending on the dependence between the densities of the objects under consideration.

2.1. Euler-Poincaré invariants

Based on the observed external similarity of clustering, which is characterized by the formation of specific thread-like clusters, both in granular matter and in the distribution of matter in the Universe, it can be assumed that Euler-Poincaré invariants can be used to analyze the topology of particle clusters both in granular materials and groups of galaxies in the Universe.

Euler-Poincaré invariants are numerical characteristics associated with the topological properties of spatial structures. They can be calculated for different spatial regions. Let's consider how these characteristics can be used to analyze the topological properties of both space structures and structures of bulk materials.

Let's say we have a data set that represents the distribution of galaxies in a certain region of space. We can partition this space into segments and then use Euler-Poincaré invariants to analyze the topological properties of these segments. The relationship between the parameters in Euler's formula is expressed:

$$\chi = N_1 - N_2 + N_3,$$

N_1 - the number of facets (cells, grid cells) that fall within the perimeter of the object;

N_2 - the number of grid lines that are cut off by the perimeter of the object and that are located inside the perimeter of the object;

N_3 – number of points of intersection of the object's perimeter with the grid lines.

For example, in Figure 1 the Euler characteristic is $\chi = 7$.

In the context of cosmology, the following correspondence can be established:

N_1 – number of large galaxy clusters or superclusters;

N_2 – void-type structures;

N_3 – boundaries of large structures.

Let us have two segments: one with a high density of galaxies and the other with a low density. For each segment, we can calculate the Euler characteristic χ. If we have a segment with a high density of galaxies, for example, 1 component and 0 voids (if the segment is compact), thus, C=1, H=0, then:

$$C - H = \chi = 1\,.$$

Now, if we have two segments: one with a high density of galaxies, and the other with a low density, then we can compare the resulting values of the Eulerian characteristic.

Let a segment with a high density of galaxies have a characteristic χ_1 and a segment with a low-density χ_2, then:

– If $\chi_1 > \chi_2$, then this may indicate a more complex topology of the segment with a high density of galaxies, possibly the presence of additional components or voids.
– If $\chi_1 < \chi_2$, then the segment with a high density of galaxies may have a simpler topology.

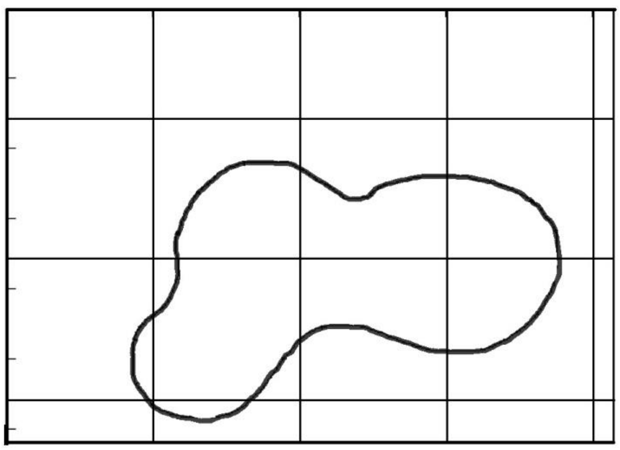

Figure 1: N_1 = 8, N_2 = 11, N_3 = 10, the Euler characteristic is χ =7.

Quantitative differences in topological properties can give us information about the structure and distribution of galaxies. For example, if a segment with a high density of galaxies has many components, this may indicate the presence of various clusters (clusters or superclusters of galaxies) in that region of space. If a segment with a low density of galaxies has one component, this may indicate a more uniform distribution of galaxies without a clear structure. The number of voids may indicate the presence of regions with a low density of galaxies (Luminet, 2008).

By analyzing topological properties, the presence of filaments and different patterns in the distribution of galaxies can be revealed. You can also analyze the shape of clusters, their boundaries, and relative positions.

Topological properties may change over time, which may indicate dynamic processes of fusion or the formation of new structures.

To assess the connectivity of components, for example, in a flat slice, you can use the connectivity measure:

$$\gamma = \frac{2C}{n(n-1)},$$

where n is the number of galaxies in the slice.

To analyze the shape of structures, you can approximate the shape using the relationship

$$A = \frac{l_1}{l_2},$$

where is l_1 and l_2 – the length of the axes of the ellipse fitted to the structure.

Filaments and patterns can be identified by analyzing the density gradient and looking for local maxima.

Euler-Poincaré characteristics are also suitable for analyzing more complex topological structures, such as waves, which also occur in both granular materials and the Universe (Gerasimov, 2022).

2.2. Order parameter

Euler-Poincaré invariants provide information about the topological characteristics of a structure but do not themselves contain a density or order parameter.

We can try to include the concept of "order" through local characteristics of the structure, such as the ratio of the number of neighbors to the total number of particles in a certain region. This can serve as a measure of organization or structural order in a given area. In the context of granular materials, the concept of "order" may be associated with parameters describing the degree of packing of particles or the characteristics of their interactions (Gerasimov et al., 2021).

In its most general form, the order parameter is defined as follows. Let us have some structure, represented by a set of points in space P_i (i=1, …, N). You can enter a parameter that will evaluate the degree of ordering of this structure. For example, it could be the ratio of the average number of nearest neighbors to the total number of points:

$$\eta_0 = \frac{1}{N}\sum_{i=1}^{N} n_i\,,$$

where is n_i – number of nearest neighbors of a point P_i.

For a granular material, the nearest neighbors can be considered the particles with which the particle has contact. In the structure of the Universe, the order parameter can be related similarly to the local organization of galaxies.

Using the Euler-Poincaré characteristics, we give a phenomenological definition of the order parameter.

$$\eta = \frac{\chi - \chi_1}{\chi_{AS} - \chi_1},$$

where are χ – experimentally observed invariant value; χ_1 – some value specified in the experimental conditions; χ_{AS} – the value to which the value of the invariant asymptotically tends.

The introduced definition of the order parameter allows us to track the kinetic stages of the evolution of a distribution parameterized in terms of Euler invariants. This method allows us to interpret the non-monotonicity of the corresponding phase diagram for the order parameter in terms of the hierarchy of characteristic relaxation times of inhomogeneous anisotropic states.

3. Conclusion

A stereological analysis methodology is presented, enabling the assessment of properties of materials and structures with complex morphology based on Euler-Poincaré connectivity.

An analogy between the geometry of mass distribution in the large-scale Universe structure and the morphology of granular materials is identified.

The results provide opportunities for a deeper understanding of the Universe's structure and materials on various scales.

The results presented allow a rough classification of the observed mass distributions on the scale and in the observation plane. Advantages include the possibility of parameterizing arbitrarily complex distributions and the possibility of large-scale extrapolation of topological invariants with allowance for scaling. The disadvantages are the dependence of the obtained results on the choice of the observation plane and the need for aperture scanning to refine the values of the invariants describing the observed distributions.

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