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# DIRECT IMAGE RECONSTRUCTION IN MULTI-ELEMENT INTERFEROMETRY

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ABSTRACT. The Ukrainian VLBI system of decameter radio telescopes URAN successfully solves many scientific problems, but implementing aperture synthesis technology has a number of difficulties. One of them is significant phase distortions in this range caused by an inhomogeneous propagation environment. Therefore, the task arose to develop an alternative technology with the conventional name "Interferovision", which would allow us to restore radio images with a limited number of antennas, operate with broadband signals, not require flatness of the objects scene, and have an extended field of view. A method of direct image reconstruction when observing objects in space, using a multi-element interferometer, is proposed. This method is based on a physically based principle that is similar to holography. The wave front is registered by the antennas of the interferometer, and further processing is equivalent to its playback in reverse and registration of the resulting spatial interference image. There are no special requirements for the radiation of individual points of the source, except for its delta correlation. A finite frequency band is considered, and each point can be characterized by its own spectrum, that is, its own autocorrelation function of radiation. The resolution of the direct reconstruction method depends on its width. With the quasi-monochromatic approximation, the autocorrelation functions of the radiation of all the source points degenerate into sinusoids, and the restoration of the image becomes possible only with the use of the Fourier transformation. A theoretical justification of the method for spaces of different dimensions has been obtained. Interferometric systems of the same rank with many antennas are reduced to a same canonical form with fixed number of virtual antennas placed at the origin of the coordinates and at unit distances on the coordinate axes. The ambiguity of the obtained solution is eliminated by using an additional antenna. A simulation of the proposed method of direct image reconstruction for two-dimensional space was carried out. Despite the low conditionality of the system for estimating the distance, when it increases, the angular characteristics are preserved. Therefore, the method is promising for restoring radio images of space radio sources.

**Keywords:** multi-element interferometry, image restoration, wide frequency band, wide field of view.

АНОТАЦІЯ. Українська РНДБ система декаметрових радіотелескопів УРАН успішно вирішує багато наукових задач, але імплементація технології апертурного синтезу стикається з рядом труднощів. Однією з них є значні фазові спотворення в цьому діапазоні, викликані неоднорідністю середовища поширення. Тому постало завдання розробити альтернативну технологію під умовною назвою “Інтерферобачення”, яка дозволила б відновлювати радіозображення при обмеженій кількості антен, оперувала широкосмуговими сигналами, не вимагала площинності об'єктної сцени і мала б розширене поле зору. Запропоновано метод прямої реконструкції зображення при спостереженні об’єктів у просторі з допомогою багатоелементного інтерферометра. Цей метод оснований на фізично обґрунтованому принципі, що нагадує голографію. Хвильовий фронт реєструється антенами інтерферометра, а подальший обробіток еквівалентний його відтворенню в зворотному напрямку та реєстрації утвореного просторового інтерференційного зображення. При цьому не висувається спеціальних вимог до випромінювання окремих точок джерела, окрім його дельта-корельованості. Розглядається скінченна смуга частот, причому кожна точка може характеризуватися власним спектром, тобто власною автокореляційною функцією випромінювання. Від її ширини залежить роздільна здатність методу прямої реконструкції. При квазімонохроматичному наближенні автокореляційні функції випромінювання всіх точок джерела вироджуються в синусоїди і відновлення зображення стає можливим тільки з використанням перетворення Фур’є. Отримано теоретичне обґрунтування способу для просторів різних розмірностей. Інтерферометричні системи одного рангу з багатьма антенами приводяться до єдиного канонічного вигляду з (віртуальними) антенами, розміщеними в початку координат та на одиничних віддалях на осях координат. Двозначність отриманого розв’язку усувається введенням додаткової антени. Проведено моделювання запропонованого способу прямої реконструкції зображення для двовимірного простору. Незважаючи на низьку обумовленість системи стосовно оцінки віддалі при її збільшенні, кутові характеристики при цьому зберігаються, тому спосіб є перспективним для відновлення радіозображень космічних радіоджерел.

**Ключові слова:** багатоелементна інтерферометрія, відновлення зображення, широка смуга частот, широке поле зору.

## 1. Introduction

Aperture synthesis technology, based on the application of the van Zittert-Zernike theorem, has been widely used in modern radio astronomy (Thompson et al., 2017). It boils down to finding the values of the spatial harmonics of the Fourier transform of the image obtained by many two-element interferometers. For this, the antenna system is divided into pairs and the cross correlation function is determined (Scaife, 2019). An infinite number of spatial harmonics is necessary for the correct solution of the inverse problem – image restoration – which is impossible in principle. Therefore, various methods of interpolation and processing of incomplete data are used, for example, compressed sounding and sparse restoration (Garsden et al., 2015). The resolution and quality of the reconstruction results depend significantly on the set of bases of the antenna system and the applied methods. The implementation of aperture synthesis technology requires the satisfaction of a number of approximation conditions, including: approximation of a plane wave, approximation of the delta-correlation of the source points, approximation of the object field of view to the plane and limitation of its dimensions, approximation of quasi-monochromaticity, approximation of the homogeneity of the waves propagation medium, approximation of the complete filling of the area of interferometric bases. Some of these approximations are quite difficult to ensure, and a number of methods are being developed to reduce the impact of their violation, among them – undersampling and deconvolution, isoplanatism (Schwab, 1984), spectral behavior (Taylor et al., 1999), non-coplanarity (Shopbell et al., 2005), direction-dependent calibration effects (Bhatnagar et al., 2013), etc. The difficulties of taking into account the inhomogeneities of the medium of wave propagation, primarily the non-stationary ionosphere, give rise to the so-called phase problem in low-frequency radio astronomy (Kornienko et al., 2008, 2020). In addition, reconstruction of radio images belongs to the class of inverse problems. Therefore, the search for new stable algorithms for its solution is still relevant today. At the same time, despite all the accompanying problems, the interferometers that are functioning today and new ones that are being built or designed are systems of aperture synthesis.

Another approach is based on the use of the longitudinal correlation function (Koshovy et al., 1998) followed by the Radon transform (Koshovy et al., 2002 & Lozynskyy et al., 2023). Image reconstruction is carried out by inverse Radon transform based on a set of received projections for pairs of antennas. With this technology, the approximation of quasi-monochromaticity becomes unnecessary, the resistance to the influence of the inhomogeneity of the propagation medium increases, and the filling of the area of the interferometric bases is improved. But at the same time, to obtain a strict solution, an infinite number of projections is required, which again requires the use of interpolation procedures.

Lozynskyy et al. (2022) considered a fundamentally different view of the problem of image reconstruction in multi-element interferometry, which correctly poses the inverse problem of image reconstruction. They obtained conclusions about the bijectivity of the transformation of spatial (angular) and difference coordinates. In this work, when considering the problem of positioning with the help of a multi-element interferometer, the method of direct image reconstruction is mentioned. But the essence of the method is not sufficiently revealed there.

## 2. Direct reconstruction method

The proposed method consists in the fact that each of the antennas of the interferometer registers the sum of wave fronts from different points of the object in some frequency band. At the same time, radiation from different points of the object takes different paths and takes different delays. Further, during processing, the recordings are played back in the opposite direction, which provides an inversion of these delays. At the points in space that correspond to the positions of the object points, the corresponding part of the radiation will interfere constructively, because the differences in the path of the rays for them become zero. The other part will not create interference in this place.

The essential difference between the direct reconstruction method and the aperture synthesis technology is that with its help, a real image is obtained, it physically exists, and is not determined indirectly through the components of the Fourier transform. It can be registered with a test antenna placed in a suitable location, or even visualized in a certain way.

The method of direct reconstruction resembles holography – after recording a section of the wavefront, it can then be playback, resulting in an image. An important feature of holography is that a small particle of the recorded wavefront appears to carry the necessary information about the object, but with reduced contrast. Consider a similar principle in relation to multi-element interferometry.

### *2.1. Direct problem*

Let us have an object consisting of elements (pixels), the radiation of which are random processes $s_i(t)$, collectively stationary in a wide sense. Let us denote by $R_{ij}(\tau)$ the mutual correlation function the radiation of a pair of source elements and write down the condition of their independence, i.e. delta-correlation:

$$R_{ij}(\tau) = \int s_i(t) s_j(t-\tau)dt = \begin{cases} R_i(\tau), & i = j \\ 0, & i \neq j \end{cases}. \quad (1)$$

Let the radiation of the object be received by a multi-element interferometer (Fig. 1). For the $k$-th element of the interferometer, we write down the sum of the received signals

$$x_k(t) = \sum_i s_i(t - \tau_{ik}) \ . \quad (2)$$

The sum (2) obtained for all $k$ is the starting point for solving the inverse problem.

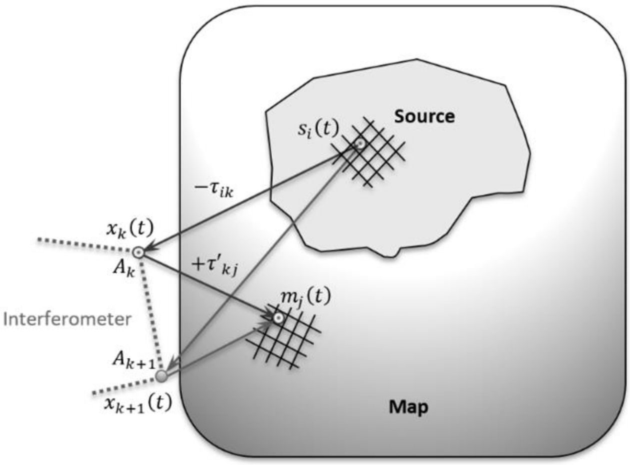


Figure 1: Scheme for determining the intensity of the image element, the resulting beam propagation delay from the $i$-th element of the source to the $j$-th pixel of the image through the $k$-th element of the interferometer is $\tau_{ikj} = -\tau_{ik} + \tau'_{kj}$

*2.2. Inverse problem*

For each ($j$-th) pixel of the image, we will form the sum of the signals taken in reverse time, and we will ask whether they interfere (equivalent to playback by the interferometer elements).

$$m_j(t) = \sum_k x_k\left(t - \tau'_{kj}\right). \quad (3)$$

The intensity of the created signal

$$I\left(m_j\right) = \int\left[\sum_k x_k\left(t - \tau'_{kj}\right)\right]^2 dt. \quad (4)$$

Let's rewrite the square of the sum through the double sum, which will allow us to enter the sign of the integral under the sum

$$I\left(m_j\right) = \sum_l \sum_n \int x_l\left(t - \tau'_{lj}\right) x_n\left(t - \tau'_{nj}\right) dt. \quad (5)$$

After substituting (2) into (5) and noting the resulting beam propagation delay from the $i$-th element of the source to the $j$-th pixel of the image through the $k$-th element of the interferometer by $\tau_{ikj} = -\tau_{ik} + \tau'_{kj}$, we obtain

$$I\left(m_j\right) = \sum_l \sum_n \int\left[\sum_\alpha s_\alpha\left(t - \tau_{\alpha lj}\right)\right] \times \left[\sum_\beta s_\beta\left(t - \tau_{\beta nj}\right)\right] dt. \quad (6)$$

Let's use the notation of the product of sums by a double sum again

$$I\left(m_j\right) = \sum_l \sum_n \sum_\alpha \sum_\beta \int(\bullet), \quad (7)$$

where $\int(\bullet) = \int s_\alpha\left(t - \tau_{\alpha lj}\right) s_\beta\left(t - \tau_{\beta nj}\right) dt$. Note that this integral is similar to the integral in (1). After applying (1), we get

$$I\left(m_j\right) = \sum_l \sum_n \sum_\alpha \sum_\beta R_{\alpha\beta}\left(\tau_{\beta nj} - \tau_{\alpha lj}\right). \quad (8)$$

We consider that on the basis of (1) $R_{\alpha\beta}(\tau)$ is different from zero only if $\alpha = \beta$. This allows you to record the result of transformations in the form

$$I\left(m_j\right) = \sum_l \sum_n \sum_i R_i\left(\tau_{inj} - \tau_{ilj}\right) \quad (9)$$

or

$$I\left(m_j\right) = \sum_l \sum_n \sum_i R_i\left(\tau_{il} - \tau'_{lj} - \tau_{in} + \tau'_{nj}\right). \quad (10)$$

The content of the obtained result (10) is as follows. The radiation of the $i$-th element of the source with a delay $\tau_{il}$ reached the $l$-th element of the interferometer, was recorded and then played back in the reverse direction. At the same time, the delay $\tau_{il}$ changed its sign and became equal $-\tau_{il}$. On the way to the $j$-th pixel of the image, there is an additional delay $\tau'_{lj}$, and the resulting delay will be $\tau_{ilj} = -\tau_{il} + \tau'_{lj}$. Another ray, the $n$-th, which corresponds to the path through the $n$-th element of the interferometer, receives a delay $\tau_{inj} = -\tau_{in} + \tau'_{nj}$ in a similar way. It interferes with the previous one, since it is radiated by the same source element, with an amplitude determined by the corresponding correlation function $R_i(\tau)$ based on (1) for $\tau = \tau_{inj} - \tau_{ilj}$. And so in pairs for all elements of the interferometer and for all elements of the source. Note that $R_i(0)$ corresponds to the radiation intensity of the $i$-th element of the source, that is, the image of the source is formed pixel by pixel in this way.

*2.3. Existence and uniqueness of solution*

Obtaining an image can be conventionally divided into two stages. At the first stage, signal records are obtained, in which a set of delays corresponds to each element of the source with its spatial coordinates. These delays can be interpreted as difference coordinates. At the second stage, the inverse transformation of coordinates is performed.

Lozynskyy et al., (2022) considered the transformation of spatial and difference coordinates and determined the number of interferometer elements necessary for mutual uniqueness in spaces of different dimensions. The number of interferometer elements, arbitrary but greater than or equal to the minimum, is described by a canonical equation of the corresponding rank. That is, all interferometric systems of the same rank with many antennas can be reduced to a single canonical form with (virtual) antennas placed at the origin and at unit distances on the coordinate axes. But as it turned out, in some cases the solution of the inverse problem can be ambiguous - the distance from the object to the interferometer can take two values. A detailed analysis of this shortcoming is beyond the scope of this work, it is enough to note that adding one more element to the interferometer completely eliminates the mentioned problem. With a sufficient number of interferometer elements, the transformation of spatial and difference coordinates is bijective. For two-dimensional space, it is 4, and for three-dimensional – 5 elements. If there is no need to determine the distance to the object, but you can limit yourself only to angular coordinates, which is quite acceptable in radio astronomy, then the minimum number of antennas can be reduced by one.

Thus, based on the bijectivity of the transformation of spatial and difference coordinates (displayed as sets of signal delays in records) and the correspondence of the autocorrelation function at zero displacement to the radiation intensity of the source element, the solution of the problem of image reconstruction exists and is unique.

## 3. Numerical modeling

Numerical modeling of direct and inverse problems was carried out for the case of two-dimensional space. In radio astronomy, such a problem arises when observing radio sources using a 2d interferometer (Lozynskyy et al., 2022). The numerical simulation scheme of the direct reconstruction method is shown in Fig. 2.

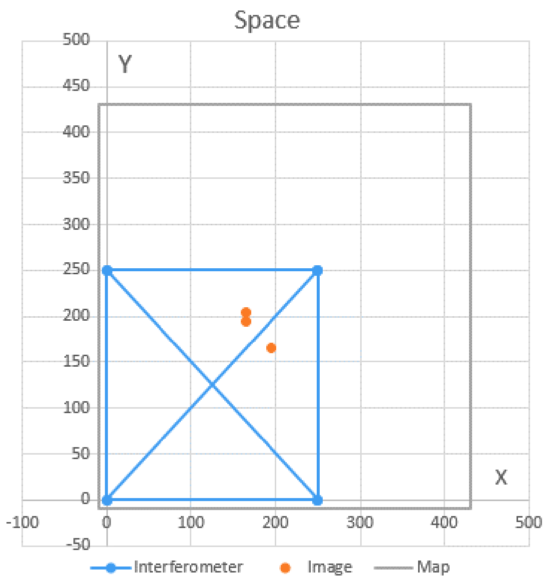


Figure 2: Scheme for numerical modeling of the direct reconstruction method

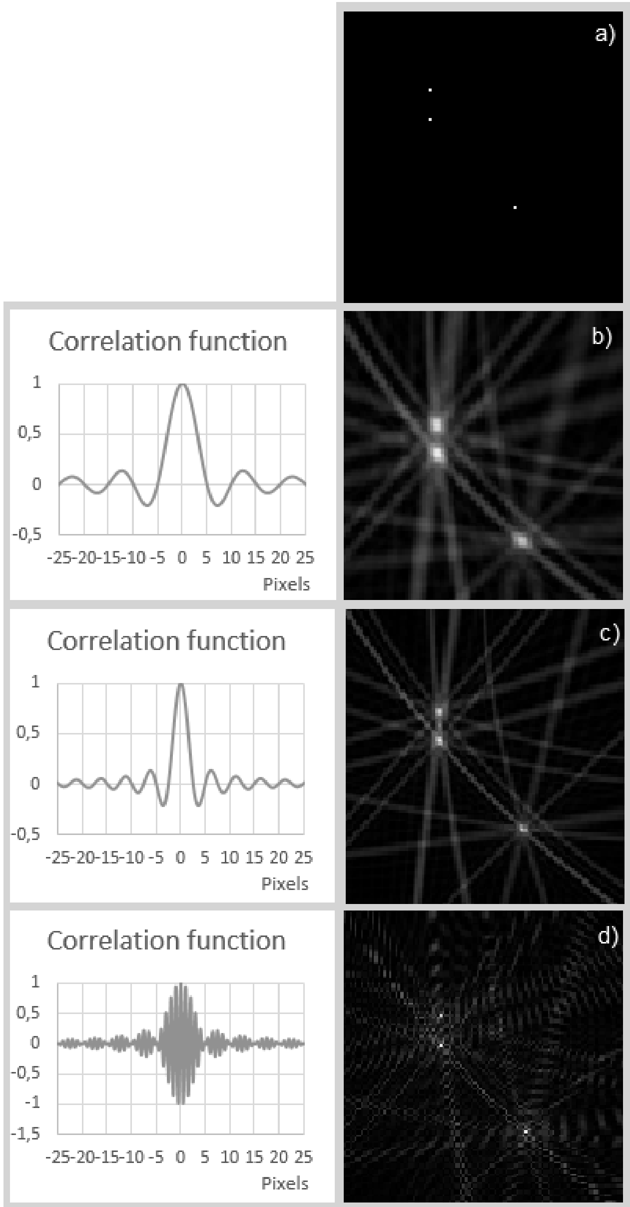


Figure 3: Results of numerical modeling of the direct reconstruction method; a) original image; b-d) reconstructed by setting different autocorrelation functions of the radiation of the object shown on the left

In the space with coordinates *X* and *Y*, 3 pixels of the same intensity are selected, which constitute the model of the original image of the object. 4 interferometer elements are located at the vertices of the square. The part of the space in which the image will be reconstructed is surrounded by a frame.

In Fig. 3 shows the results of numerical modeling of the map section.

## 4. Conclusion

As follows from the given theoretical reasoning and numerical simulation, the direct reconstruction method allows obtaining results with a limited number of interferometer elements. It is clear that with an increase in the number of elements, the relative intensity of artifacts will decrease rapidly. The condition that we know the autocorrelation functions of the object's elements is not difficult to ensure in radio astronomy, since the radio radiation with which radio astronomy operates is mostly broadband. A small frequency band is allocated from it by the receiving systems, and the partial autocorrelation functions of all signals become the same and known in advance. However, the method does not impose any requirements on their uniformity. Continuation of research in this direction will make it possible to determine the individual radiation spectra of individual parts of the image, i.e. spectral indices, or, conventionally speaking, "color". Rejection of the quasi-monochromatic approach, besides this, gives a number of positive consequences. For example, increasing the frequency band is equivalent to increasing the sensitivity and immunity to interference.

Separately, it should be noted that with the quasi-monochromatic approximation, the autocorrelation functions of the radiation of all the source points degenerate into sinusoids of the same frequency and the method seems unsuitable in such conditions. However, the amplitude and phase of the mutual correlation function of signals in pairs of interferometer elements appear, and this is already the scope of the well-known aperture synthesis technology.

When implementing the direct reconstruction method, no requirements for objects distance are imposed, on the contrary, it is determined. Moreover, there are no requirements for the flatness of the object of the scene, for the width of the field of view, the interferometer "sees" the entire available space at once.

An important characteristic of the method is the possibility of reconstruction of images of dynamic objects, since only a portion of signals is accumulated, and then only these instantaneous recordings are used. And this creates the basis for observations through the medium of wave propagation with moving inhomogeneities. In this case, the obtained results will resemble speckles.

When observing objects that are distant compared to the size of the interferometer, blurring is observed in the distance estimation, but the angular characteristics are preserved. This effect becomes clear from the analysis of the form of the canonical equation of a multi-element interferometer, in which the direction cosines enter directly, and the distance – inversely, as a unit divided by the distance. The actual angular distribution of radio radiation intensity is important for radio astronomy, so the proposed method is promising for restoring radio images of space radio sources.

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