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MELTING HADRONS, BOILING QUAKRS AND GLUONS

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ABSTRACT. Resonance spectra (Hagedorn distribution) are critically revised with emphasis on the saturation of hadron states and possible transition to a soup of quarks and gluons. Previous studies in this direction are extended by use of non-linear, complex Regge trajectories whose limited real part supports the idea of saturation of high-mass/spin resonance production.

Keywords: hadrons, resonances, Hagedorn spectrum, Regge trajectories, quarks, gluons.

АННОТАЦІЯ. В роботі проведено аналіз адронних спектрів (розподіл Хагедорна) з акцентом на можливість насичення адронних станів та переходом їх до “варева” (супу) з кварків та глюонів. Основні результати даної роботи такі:

1) досліджено спектр мезонів та баріонів в області великих мас;

2) з’ясована роль критичної температури в статистичній моделі Хагедорна;

3) проаналізовані наслідки конечності спектра Хагедорна;

4) дана статистична інтерпретація фазового переходу від адронів до кварків та глюонів в термінах рівняння стану. Унікальною рисою отриманого рівняння стану є наявність в ньому від’ємної температури та переохолодження речовини.

В більшості відомих теорій та моделей траєкторії Редже є лінійні та реальні функції, що не допускає застосування їх для реальних резонансів. Результати попередніх аналізів в цьому напрямку нами розширено завдяки застосуванню нелінійних комплексних траєкторій Редже, в яких зростання реальної частини є обмежений, що підтверджує насичення процесу народження резонансів з великими масами. Приведені конкретні моделі таких траєкторій, а саме: траєкторії з кореневими порогами та такі, що слідують з дисперсійних співвідношень. Параметри приведених моделей траєкторій Редже верифіковано за допомогою порівняння з експериментальними даними про мезонні та баріонні резонанси. В цілому, робота є внеском в дослідження однієї з важливих та загадкових проблем фізики високих енергій – конфайнменту кварків та глюонів на прикладі переходу важких адронних резонансів в новий агрегатний стан речовини, можливо – кварк-глюонну плазму, властивості якої ще до кінця не з’ясовано.

Ключові слова: Адрони, резонанси, спектр Хагедорна, траєкторії Редже, кварки, глюони.

1. Introduction

The spectrum of hadron resonances is among the central problems of high-energy physics since the properties of highly excited resonances are intimately connected with the problem of confinement.The idea of the resonance gas was suggested by Belenky and Landau, who used the Bethe-Uhlenbeck method for non-ideal gases. The field was revitalized in a series of papers by Rolf Hagedorn (Hagedorn), and followers who introduced the notion of limited temperature, that later received various modifications and interpretations . In a recent paper (Szanyi, 2023) we extended Hagedorn’s approach by combining statistical physics with the analytic $S-$matrix theory, realized by the Regge-pole model.

Here we the spectrum of hadronic resonances by relating seemingly two different approaches: statistical - that of Hagedorn and Regge.

The density of hadron states follows the law

$$\rho(m) = f(m)\exp(m/T_H), \tag{1}$$

where $f(m)$ is a slowly varying function of mass and T_H is the Hagedorn temperature, originally considered as the limiting temperature but later re-interpreted as the temperature of the color deconfinement phase transition where hadrons "boil"transforming matter into a boiling quark-gluon soup.

Over 50 years after the publication of R. Hagedorn’s paper (Hagedorn) on the spectrum of resonances many important details still remain open. In spite of many efforts, the calculated value of the Hagedorn temperature shows a surprisingly widespread from $T_H = 141$ MeV to $T_H = 340$ MeV, depending on the parametrization and the set of data (baryons, mesons) used. The discrepancies may have different origin, in particular: a) the large uncertainties in the specification and identification of heavy resonances, b) the analytical form of the Hagedorn spectrum, in particular, the form of the function $f(m)$ multiplying Hagedorn's exponential. In the present paper, we address both issues.

Our approach here is limited to the world of observed resonances, summarized by the Particle Data Group and theoretical methods based on analyticity, unitarity, and duality.

Crucial in our paper is the identification of the function $f(m)$ with the derivative of the relevant Regge trajectory. In the spirit of the analytic S-matrix approach, Regge trajectories encode an essential part of the strong interaction dynamics, they are building blocks of the theory. There were many attempts to find analytic forms of the non-linear complex Regge trajectories, based on mechanical analogues (strings), quantum chromodynamics etc. Below we rely on duality and constraints based on analyticity and unitarity, constraining the threshold and asymptotic behaviour of the trajectories. An important constraint, affecting the spectrum near its critical point is the upper bound on the real part of Regge trajectories', coming from dual models with Mandelstam analyticity. Construction of explicit models of the trajectories satisfying the above constraints is a non-trivial problem. In the present paper, we propose explicit models of such trajectories allowing explicit calculations and compatible with the data on resonances.

Below we argue that while Hagedorn's exponential rise comes mainly from the proliferation of spin and isospin degeneracy of states with increasing mass, that can be counted directly, the prefactor $f(m)$ reflects dynamics, encoded in Regge trajectories given that $f(m) = \alpha'(m)$, where $\alpha'(m)$ is the slope of the trajectory.

The low-mass, $m < 1.8$ GeV spectra do not exhibit any surprise by following Hagedorn's exponential. The only open questions are the value of the Hagedorn temperature T_H and possible differences between the spectra for various particles. The spectra beyond $m = 1.8$ GeV are different: on the experimental side, the high-mass resonances tend to gradually disappear, their status becoming uncertain.

The most important issue is the existence of a "melting point"where the resonances are transformed to a continuum of a boiling soup of quarks and gluons. This critical region/point is studied by various methods: statistics and thermodynamics, quantum.

2. Melting hadrons

In this Section we study the relation between the mass density of hadronic states given by the Hagedorn spectrum and the dynamics emerging from Regge pole models, inspecting non-linear Regge trajectories.

In spite of the huge number of papers, the subject remains a topical problem of hadron dynamics with numerous open questions. We address the following issues:

- the behavior of the meson mass spectrum in the high-mass region;
- the role of the critical temperature and the prefactor in $\rho(m)$ in the Hagedorn model of hadronic spectra;
- the finiteness of the Hagedorn spectrum and its consequences.

The expression for pressure in this thermodynamic approach in the Boltzmann approximation is given by:

$$p = \sum_i g_i p(m_i) = \int_{M_1}^{M_2} dm\, \rho(m)\, p(m), \qquad (2)$$

with

$$p(m) = \frac{T^2 m^2}{2\pi^2} K_2\big(\frac{m}{T}\big), \qquad (3)$$

where M_1 and M_2 are the masses of the lightest and heaviest hadrons, respectively, and g_i-s are particle degenerations.

For fixed isospin and hypercharge a cubic density of states, $\rho(m) \sim m^3$, fits the data. Moreover, the cubic spectrum can be related to collinear Regge trajectories. Indeed, following the arguments of Burakovsky and Horowitz (Burakovsky & Horowitz) , on a linear trajectory with negative intercept, $\alpha(t) = \alpha' t - 1$, some integer values of $\alpha(t) = J$ correspond to states with negative spin, $J = \alpha(t_J)$, with squared masses $m^2(J) = t_J$. Since a spin-J state has multiplicity $2J+1$, the total number of states with spin $0 \le J \le j$ at $t = m(j)^2$ is given by

$$N(j) = \sum_{J=0}^{j} (2J+1) = (j+1)^2 = \alpha'^2 m^4(j). \qquad (4)$$

Hence the density of states per unit mass interval is obtained as the derivative of this cumulative quantity,

$$\rho(m) = \frac{dN(m)}{dm} = 4\alpha'^2 m^3, \qquad (5)$$

and it grows as the cubic power of the mass. Consequently for a finite number of collinear trajectories, N, the corresponding mass spectrum is given as

$$\rho(m) = 4N\alpha'^2 m^3. \qquad (6)$$

A different view on the spectra was advocated by E. Shuryak (Shuryak), who suggested to use a quadratic parametrization, completely different from the conventional form:

$$\rho(m) \sim m^2.$$

In both the statistical bootstrap model and in the dual resonance model, the resonance spectrum takes the form of Eq. (1). In the dual resonance model $f(m) \sim \frac{d}{dm}\Re\alpha(m^2)$. We use non-linear complex Regge trajectories to determine this pre-factor as discussed in the next subsections.

The meson and baryon spectra differ, in particular by their slopes. More important is the question of the asymptotic behaviour of $\rho(m)$ for large masses. In theory, Hagedorn's exponential may rise indefinitely, however, starting from $m \approx 2.5$ GeV resonances are not observed. The question arises whether it is a "technical"issue (the resonances gradually fade becoming too wide to be detected) or there is a critical point where they melt to a continuum transforming the hadron matter to a "boiling soup". This point can be illuminated by means of Regge trajectories, as we demonstrate it in what follows.

We concentrate on the meson spectrum, more specifically that of ρ and its excitations. This familiar trajectory is chosen just as a representative example. Other trajectories, including baryonic ones as well as those with heavy (c and b) flavors will be studied later. We are interested in the high-mass behavior, starting from $m \approx 1.8$ GeV. Beyond this value the exponential behavior of the Hagedorn spectrum is expected to change drastically. We concentrate on its behaviour above 1.8 GeV.

Note that rather than comparing the density of states $\rho(m)$ to the data it is customary to accumulate states of masses lower than m,

$$N_{exp} = \sum_i g_i \Theta(m - m_i), \tag{7}$$

where g_i is the degeneracy of the i-th state with mass m_i in spin J and isotopical spin I, i.e.,

$$g_i = \begin{cases} (2J_i+1)(2I_i+1), & \text{for non-strange mesons} \\ 4(2J_i+1), & \text{for strange mesons} \\ 2(2J_i+1)(2I_i+1), & \text{for baryons} \end{cases}$$

The theoretical equivalent of Eq. (7) is

$$N_{theor}(m) = \int_{m_\pi}^{m} \rho(m')dm', \tag{8}$$

where the lower integration limit is given by the mass of the pion. We identify $f(m)$ in $\rho(m)$ with the slope of the relevant non-linear complex Regge trajectory $\alpha'(m)$. In the next subsection we discuss the properties of these trajectories following from the analytic $S-$matrix theory and duality, and present an explicit example of such a trajectory.

3. Regge trajectories

At low and intermediate masses, light hadrons fit linear Regge trajectories with a universal slope, $\alpha' \approx 0.85$ GeV^2. As masses increase, the spectrum changes: resonances tend to disappear. The origin and details of this change are disputable.

Termination of resonances, associated with a "ionization point"was also studied in a different class of dual models, based on logarithmic trajectories [?].

Possible links between the Hagedorn spectra and Regge trajectories appear in the statistical bootstrap and dual models, according to which the pre-factor $f(m)$ in Eq. (1) depends on the slope of the relevant Regge trajectory, $\alpha'(m^2)$, which is constant for linear Regge trajectories.

We extend the Hagedorn model by introducing the slope of relevant non-linear Regge trajectories. Anticipating a detailed quantitative analysis, one may observe immediately that a flattening of $\Re\alpha(s = m^2)$ [1], results in a decrease of the relevant slope $\alpha'(m)$ and a corresponding change in the Hagedorn spectrum. Following Eq. (1) we parametrize

$$\rho(m) = \left(\frac{d}{dm}\Re\alpha(m^2)\right) \times \exp(m/T_H). \tag{9}$$

Based on the decreasing factor $\Re\alpha'$ in Eq. (9) the exponential rise of the density of states slows down near to the melting point around $m \approx 2 - 2.5$ GeV. The cumulative spectrum Eqs. (7) and (8), accordingly tends to a constant value.

Any Regge trajectory should satisfy the followings:

- threshold behavior imposed by unitarity;
- asymptotic constraints: the rise of real part of Regge trajectories is limited, $\Re\alpha(s) \leq \gamma\sqrt{t}\ln t, \quad s \to \infty$;
- compatibility with the nearly linear behavior in the resonance region (Chew-Frautschi plot).

The threshold behavior of Regge trajectories is constrained by unitarity. t-channel unitarity constrains the Regge trajectories near the threshold, $t \to t_0$ to the form

$$\Im\alpha(t) \sim (t - t_0)^{\Re\alpha(t_0)+1/2}. \tag{10}$$

Here t_0 is the lightest threshold, *e.g.* $4m_\pi^2$ for the meson trajectories. Since $\Re\alpha(4m_\pi^2)$ is small, a square-root threshold is a reasonable approximation to the above constraint.

[1]We use the (here positive) variables s or t interchangeably with crossing-symmetry in mind.

In the resonance region below flattening near $m = \sqrt{s} < 2.5$ MeV the meson and baryon trajectories are nearly linear (Chew-Frautschi plot). Fixed-angle scaling behavior of the amplitude constrains the trajectories even more, down to a logarithmic behavior.

There are several reasons why the non-linear and complex nature of the Regge trajectories is often ignored, namely: 1) the observed spectrum of meson and baryon resonances (Chew-Frautschi plot) seem to confirm their linearity; 2) in the scattering region, $t < 0$, the differential cross-section, $d\sigma/dt \sim \exp((2\alpha(t) - 2)\ln s)$ is nearly exponential in t; 3) Dual models, *e.g.* the Veneziano amplitude are valid only in the narrow-width approximation, corresponding to linear Regge trajectories (hadronic strings). Deviation from linearity is unavoidable, but its practical realization is not easy.

4. Models of Regge trajectories

Unitarity imposes a severe constraint on the threshold behaviour of the trajectories:

$$\Im m\alpha(t)_{t\to t_0} \sim (t - t_0)^{\Re e\alpha(t_0)+1/2}, \tag{11}$$

while asymptotically the trajectories are constrained by dual amplitude with Mandelstam analyticity

$$\left|\frac{\alpha(t)}{\sqrt{t}\ln t}\right|_{t\to\infty} \leq \text{const.} \tag{12}$$

The above asymptotic constraint can be still lowered to a logarithm by imposing wide-angle power behaviour for the amplitude.

The above constraints are restrictive but still leave much room for model building.

While the parameters of meson and baryon trajectories can be determined both from the scattering data and from the particles spectra, this is not true for the pomeron (and odderon) trajectory, known only from fits to scattering data (negative values of its argument). An obvious task is to extrapolate the pomeron trajectory from negative to positive t-values to predict glueball states at $J = 2, 4, ...$, for which, however, no experimental evidence exists so far. Given the nearly linear form of the pomeron trajectory, known from the fits to the (exponential) diffraction cone, little room is left for variations in the region of particles ($t > 0$).

4.1. Additive thresholds

Apart from the Pomeron trajectory, the direct-channel f trajectory is essential in the proton-proton system. Guided by conservation of quantum numbers, we include two f trajectories, labelled f_1 and f_2, with mesons lying on these trajectories.

The real and imaginary part of the f_1 and f_2 trajectories can be derived from the parameters of the f-resonances.

To be consistent with the meson trajectories, the linear term is replaced by a heavy threshold mimicking linear behaviour in the mass region of interest (M < 5 GeV),

$$\alpha_P(M^2) = \alpha_0 + \alpha_1(2m_\pi - \sqrt{4m_\pi^2 - M^2}) + \\ +\alpha_2(\sqrt{M_H^2} - \sqrt{M_H^2 - M^2}), \tag{13}$$

with M_H an effective heavy threshold $M = 3.5$ GeV.

The $f_0(500)$ resonance. The experimental data on central exclusive pion-pair production measured at the energies of the ISR, RHIC, TEVATRON and the LHC collider all show a broad continuum for pair masses $m_{\pi^+\pi^-} < 1$ GeV/c^2. The population of this mass region is attributed to the $f_0(500)$. This resonance $f_0(500)$ is of prime importance for the understanding of the attractive part of the nucleon-nucleon interaction, as well as for the mechanism of spontaneous breaking of chiral symmetry. In spite of the complexity of the $f_0(500)$ resonance, and the controversy on its interpretation and description, we take here the practical but simple-minded approach of a Breit-Wigner resonance

$$A(M^2) = a\,\frac{-M_0\Gamma}{M^2 - M_0^2 + iM_0\Gamma}. \tag{14}$$

The Breit-Wigner amplitude of Eq. (14) is used below for calculating the contribution of the $f_0(500)$ resonance to the Pomeron-Pomeron cross section.

4.2. Dispersion relations

Meson trajectories. The nearly linear real part of the meson trajectory can be elated to its imaginary part by

$$\Re e\,\alpha(s) = \alpha(0) + \frac{s}{\pi}PV\int_0^\infty ds'\,\frac{\Im m\,\alpha(s')}{s'(s'-s)}. \tag{15}$$

In Eq. (15), PV denotes the Cauchy Principal Value of the integral. The imaginary part is related to the decay width by

$$\Gamma(M_R) = \frac{\Im m\,\alpha(M_R^2)}{\alpha'\,M_R}. \tag{16}$$

The quantity α' in Eq. (16) denotes the derivative of the real part, $\alpha' = \frac{d\Re e\,\alpha(s)}{ds}$. The relation between $\Gamma(M)$ and $\Im m\,\alpha$(s) requires $\Im m\,\alpha(s) > 0$. In a simple analytical model, the imaginary part is chosen as a sum of single threshold terms

Table 1: Parameters of resonances belonging to the f_1 and f_2 trajectories.

	I^G	J^{PC}	traj.	M (GeV)	M^2 (GeV^2)	Γ (GeV)
$f_0(980)$	0^+	0^{++}	f_1	0.990±0.020	0.980±0.040	0.070± 0.030
$f_1(1420)$	0^+	1^{++}	f_1	1.426±0.001	2.035±0.003	0.055± 0.003
$f_2(1810)$	0^+	2^{++}	f_1	1.815±0.012	3.294±0.044	0.197± 0.022
$f_4(2300)$	0^+	4^{++}	f_1	2.320±0.060	5.382±0.278	0.250± 0.080
$f_2(1270)$	0^+	2^{++}	f_2	1.275±0.001	1.6256±0.003	0.185± 0.003
$f_4(2050)$	0^+	4^{++}	f_2	2.018±0.011	4.0723±0.044	0.237± 0.018
$f_6(2510)$	0^+	6^{++}	f_2	2.469±0.029	6.096±0.143	0.283± 0.040

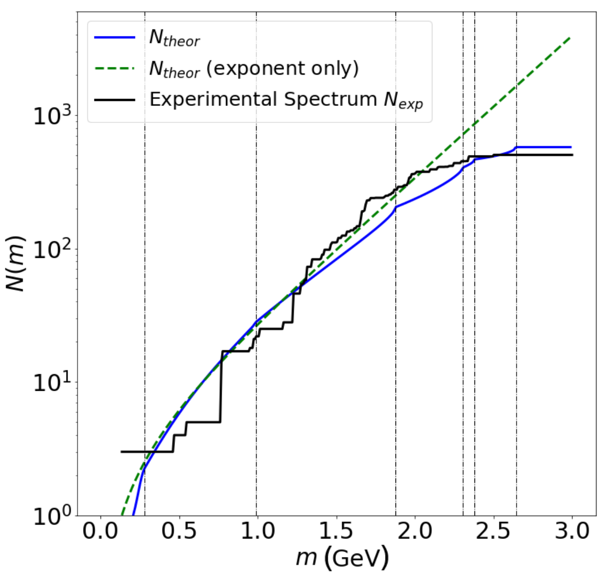

Figure 1: Hadron mass spectrum N_{theor} from Eq. (8) compared with the data. Additionally, N_{theor} obtained from a simple exponential mass density is also shown (dashed line).

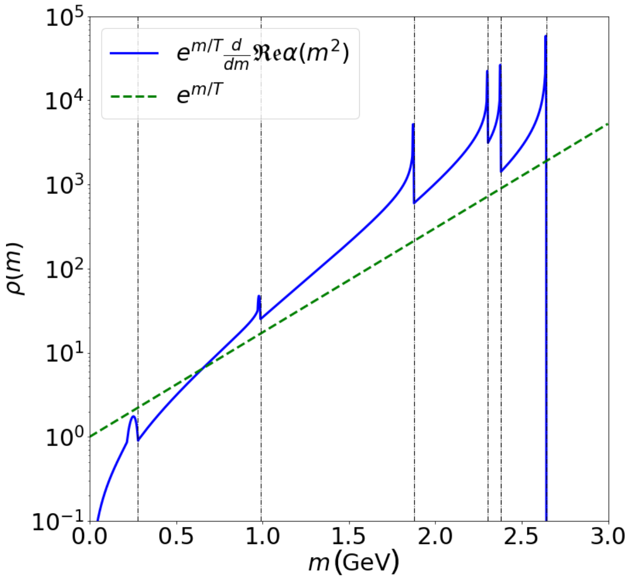

Figure 2: Mass density $\rho(m)$ calculated by using the derivative of the smoothed ρ-meson trajectory as a prefactor. The exponential density without any prefactor is also shown (dashed line), using the same temperature and normalization. Above the highest threshold the derivative, and hence the density vanishes (not seen due to the logarithmic scale).

$$\Im m\,\alpha(s) = \sum_n c_n (s-s_n)^{1/2} \big(\frac{s-s_n}{s}\big)^{|\Re e\,\alpha(s_n)|}\theta(s-s_n). \quad (17)$$

The imaginary part of the trajectory shown in Eq. (17) has the correct threshold and asymptotic behaviour. The highest threshold, higher than all the resonance masses lying on the trajectory, is chosen as an effective threshold. This highest threshold ensures that $\Re e\,\alpha(s)$ tends to a constant value for $s \to \infty$.

Calculated mass spectra and mass densities are shown in Fig. 1 and Fig. 2.

Baryon trajectories. The Pomeron-proton channel, $Pp \to M_X^2$ couples to the proton trajectory, with the $I(J^P)$ resonances: $1/2(5/2^+)$, F_{15}, m = 1680 MeV, Γ = 130 MeV; $1/2(9/2^+)$, H_{19}, m = 2200 MeV, $\Gamma = 400$ MeV; and $1/2(13/2^+)$, $K_{1,13}$, $m =$ 2700 MeV, $\Gamma = 350$ MeV. The status of the first two is firmly established, while the third one, $N^*(2700)$, is less certain, with its width varying between 350 ± 50 and 900 ± 150 MeV . Still, with the stable proton included, we have a fairly rich trajectory, $\alpha(M^2)$.

We use the explicit form of the trajectory, ensuring correct behaviour of both its real and imaginary parts. The imaginary part of the trajectory can be written in the following way:

$$\mathcal{I}m\,\alpha(s) = s^{\delta}\sum_n c_n\left(\frac{s-s_n}{s}\right)^{\lambda_n}\cdot\theta(s-s_n)\,, \qquad (18)$$

where $\lambda_n = \mathcal{R}e\ \alpha(s_n)$. Eq. (18) has the correct threshold behaviour, while analyticity requires that $\delta < 1$. The boundedness of $\alpha(s)$ for $s \to \infty$ follows from the condition that the amplitude, in the Regge form, should have no essential singularity at infinity in the cut plane.

The real part of the proton trajectory is given by

$$\mathcal{R}e\,\alpha(s) = \alpha(0) + \frac{s}{\pi}\sum_n c_n\mathcal{A}_n(s)\ , \qquad (19)$$

where

$$\mathcal{A}_n(s) = \frac{\Gamma(1-\delta)\Gamma(\lambda_n+1)}{\Gamma(\lambda_n-\delta+2)s_n^{1-\delta}}\,{}_2F_1\left(1, 1-\delta; \lambda_n-\delta+2; \frac{s}{s_n}\right)\times$$
$$\times\theta(s_n-s) + \left\{\pi s^{\delta-1}\left(\frac{s-s_n}{s}\right)^{\lambda_n}\cot[\pi(1-\delta)]-\right.$$
$$\left.-\frac{\Gamma(-\delta)\Gamma(\lambda_n+1)s_n^{\delta}}{s\Gamma(\lambda_n-\delta+1)}\,{}_2F_1\left(\delta-\lambda_n, 1; \delta+1; \frac{s_n}{s}\right)\right\}\theta(s-s_n)\ .$$

The proton trajectory, also called N^+ trajectory, contains the baryons N(939) $\frac{1}{2}^+$, N(1680) $\frac{5}{2}^+$, N(2220) $\frac{9}{2}^+$ and N(2700) $\frac{13}{2}^+$. In the fit, the input data are the masses and widths of the resonances. The quantities to be determined are the parameters c_n, δ and the thresholds s_n. We set $n = 1, 2, x$ and $s_1 = (m_\pi + m_N)^2 = 1.16\ \mathrm{GeV}^2$, $s_2 = 2.44\ \mathrm{GeV}^2$ and $s_x = 11.7\ \mathrm{GeV}^2$.

Other parameters of the trajectory, obtained in the fit, are summarized below: $\alpha(0) = -0.41$, $\delta = -0.46 \pm 0.07$, $c_1 = 0.51 \pm 0.08$, $c_2 = 4.0 \pm 0.8$ and $c_x = (4.6 \pm 1.7)\cdot 10^3$. Taking the central values of these parameters we obtain the following values for the λ's: $\lambda_1 = 0.846$, $\lambda_2 = 2.082$, $\lambda_x = 11.177$.

A typical curve of meson mass spectrum calculated by means of trajectory with additive threshold is shonw in Fig. 1. The resulting mass density is shown in Fig. 1.

5. Boiling quarks and gluons

In the previous section, we inspected the spectrum of resonances by combining two different approaches - statistical (Hagedorn) and dynamical (Regge). We have focused on the region of heaviest resonances, the region where hadrons may melt transforming in a boiling "soup"of quarks and gluons. Melting may happen in different ways, characterized by the details for a phase transition of colorless hadronic states into a quark-gluon soup. In terms of hadron strings this process corresponds to breakdown (fragmentation) of a string. Lacking any theory of confinement providing a quantitative description of interacting string, we will not pursue this model. Instead we use thermodynamics adequate in this situation. To complement the previous section, we present our arguments below related to the possible change of phase from a different, thermodynamic perspective.

The Hagedorn exponential spectrum of resonances Eq. (1) results in a singularity in the thermodynamic functions at critical temperature $T = T_c$ and in an infinite number of effective degrees of freedom in the hadronic phase. Furthermore, the Hagedorn-like mass spectrum is incompatible with the existence of the quark-gluon phase. To form a quark phase from the hadronic phase, the hadron spectrum cannot grow more quickly than a power. This is possible in case of a simple power parametrization $\rho \sim m^k$, compatible with $k \approx 3$, for the observable mass spectrum in the interval 0.2–1.5 GeV. Assuming ideal contributions to thermodynamical quantities we hence take energy density in the form

$$\epsilon = \int_0^\infty \rho(m)T^4\sigma(m/T)dm = \lambda_k T^{k+5}, \qquad (20)$$

and obtain the corresponding pressure and sound velocity square as follows[2]:

$$p = \frac{\lambda_k}{k+4}T^{k+5}, \quad c_s^2 = 1/(k+4). \qquad (21)$$

It can be shown that the existence of the forward cone in hadronic interactions with non-decreasing total cross sections, *i.e*, pomeron dominance, confirmed by numerous experiments at high energies, results in an asymptotic, $T \gg m$ EOS $p(T) \sim T^6$ where m is a characteristic hadron (e.g. pion) mass. The inclusion of non-asymptotic (secondary) Regge terms produces a minimum in the $p(T)$ dependence at negative pressure, with far-reaching observable consequences.

The unorthodox $p \sim T^6$ asymptotic behavior is orthogonal to the "canonical"(perturbative QCD) form $\sim T^4$. Still, it cannot be rejected e.g. when assuming a screening of the action of large-distance van der Waals forces at high temperatures and densities.

The asymptotic form $\sim T^6$ can be extended to lower temperatures by adding non-asymptotic Regge-pole exchanges. The resulting EOS is

$$p(T) = aT^4 - bT^5 + cT^6, \qquad (22)$$

where a, b, c are parameters connected with Regge-pole fits to high-energy hadron scattering. The

[2]Note that the definition of entropy density s, energy density ϵ and velocity of sound c_s in case of $\mu = 0$:

$$s(T) = p'(T),\ \epsilon(T) = Ts - p,\ \ c_s^2 = \frac{dp}{d\epsilon} = \frac{p'}{Tp''} = \frac{s}{Ts'}.$$

remarkable property of this EOS, apart from the non-standard asymptotic behavior, $\sim T^6$, is the appearance of the non-asymptotic term T^5 with negative sign, creating a local minimum with negative pressure. This metastable state with negative pressure can produce inflation of the universe.

The standard bag equation of state assuming, for simplicity, vanishing chemical potential, $\mu = 0$:

$$p_q(T) = \frac{\pi^2}{90}\nu_q T^4 - B, \tag{23}$$

$$p_h(T) = \frac{\pi^2}{90}\nu_h T^4, \tag{24}$$

where $p_q(T)$ and $p_h(T)$ are pressure in the quark-gluon plasma (QGP) and in the hadronic gas phase, respectively, B is the bag constant, and $\nu_{q(h)}$ is the number of degrees of freedom in the QGP (hadronic gas),

$$p_c = B\nu_h/(\nu_q - \nu_h), \;\; T_c = [90B/\pi^2(\nu_q - \nu_h)]^{1/4}. \tag{25}$$

Since $s(T) = dp(T)/dT$, the relevant formula for the entropy density can be rewritten as

$$s(T) = (2\pi^2 T^3/45)\nu_h[1 - \Theta(T - T_c) + \nu_q\Theta(T - T_c). \tag{26}$$

$$s^{(T)} = s(T)/T^3, \;\; s_c^* = s_c/T_c^3, \;\; s_c = \frac{\pi^2 T_c^3}{45}(\nu_h + \nu_q). \tag{27}$$

The above simple bag model EOS can also be modified (Boyko, 1990) by making the bag "constant" T-dependent, $B(T) = AT$, to produce a metastable QGP state with negative temperature:

$$p_q(T) = (\pi^2/90)\nu_q T^4 - AT, \quad p_h(T) = (\pi^2/90)\nu_h T^4. \tag{28}$$

Another important feature of this EOS is that for $\Gamma - \gamma = 0$ it describes second order phase transitions, with singular behavior of the thermal capacity at $T = T_c$. Really, in this case, we have $T - T_c \sim (\Delta s^*)^3$ near $T = T_c$.

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