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TRAPPED HOT DARK MATTER IN THE SHELL AROUND THE COMPACT OBJECT

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ABSTRACT. The model of the compact object is considered, being alternate to the black hole. In the model, the accreting protons decay at the Planck scale into positrons and hypothetical Planck neutrinos. The energy of the particles is split in two modes, low and high. The electrons, positrons and Planck neutrinos with the high energy move away. The electrons and positrons with the low energy form the compact object. The Planck neutrinos with the low energy form the shell around the compact object. The Planck neutrinos moving away may be interpreted as the hot dark matter (HDM). The Planck neutrinos in the shell around the compact object may be interpreted as the trapped hot dark matter (THDM). In a recent paper, the concept of the free fall pressure was introduced. In the effective gravity, including the Newton gravity and the free fall pressure, the THDM is hidden for the massive particles but makes deflection of the massless particles. The THDM can be seen in the gravitational interaction of two shells. The gravitational interaction of the galaxies in the Virgo cluster (M60, M87, M84, M86, M49) is studied. The motion of the galaxies is defined by the THDM masses in the shells around the compact objects in the centres of the galaxies, the stellar masses of the galaxies and the HDM mass in the galaxies. The velocities of M60 and M49 toward M87 are estimated to be 678 km s^{-1} and 445 km s^{-1} which are 66% and 59% of the observational values respectively.

Keywords: gravitation; dark matter; galaxies: kinematics and dynamics.

АНОТАЦІЯ. Розглядається модель компактного об'єкта, що є альтернативою чорній дірці. У цій моделі акреціруючі протони розпадаються на Планковській шкалі на позитрони та гіпотетичні Планковські нейтрино. Енергія частинок розщеплюється на дві моди, низьку та високу. Електрони, позитрони та Планківські нейтрино з високою енергією віддаляються. Електрони та позитрони з низькою енергією утворюють компактний об'єкт. Планківські нейтрино з низькою енергією утворюють оболонку навколо компактного об'єкта. Планківські нейтрино, що віддаляються, можуть бути інтерпретовані як гаряча темна матерія (ГТМ). Планківські нейтрино в оболонці навколо компактного об'єкта можуть бути інтерпретовані як захоплена в пастку гаряча темна матерія (ЗГТМ). У недавній статті було запроваджено поняття тиску вільного падіння. У моделі ефективної гравітації, що включає гравітацію Ньютона і тиск вільного падіння, ЗГТМ прихована для масивних частинок, але викликає відхилення безмасових частинок. ЗГТМ можна побачити у гравітаційній взаємодії двох оболонок. Розглядається гравітаційна взаємодія галактик у скупченні Діви (M60, M87, M84, M86, M49). Рух галактик визначається масами ЗГТМ в оболонках навколо компактних об'єктів у центрах галактик, зірковими масами галактик та масами ГТМ у галактиках. Швидкості M60 і M49 у напрямку M87 оцінюються в 678 км с$^{-1}$ і 445 км с$^{-1}$, що становить 66% і 59% значень, що спостерігаються, відповідно.

Ключові слова: гравітація; темна матерія; галактики: кінематика та динаміка.

1. Introduction

General relativity (Misner et al., 1973) predicts the formation of a black hole in the gravitational collapse of a body with a mass above $\sim 3\ m_\odot$. The black hole is bounded by the event horizon of the Schwarzschild radius, $r_g = 2Gm/c^2$. The existence of the event horizon breaking the connection of the gravitating mass of the black hole with an outer observer questions the usual physics.

Khokhlov (2017, 2021) developed the model of the compact object as a thin shell near the Schwarzschild radius, being alternate to the black hole. Khokhlov (2017, 2021) assumed that the liberated gravitational binding (kinetic) energy of the matter (protons and electrons) accreting onto the compact object is split in two modes, with the low energy $E_k^- \approx Gm/2r_g$ and with the high energy $E_k^+ \approx 3Gm/2r_g$ as seen in the local frame of the compact object (hereafter we use the unity mass of the particle). Half the accreting matter with the low energy retains at the compact object. The inertial force, $F_{in} \approx 2E_k^-/r_g \approx Gm/r_g^2$, balances the gravity of the compact object thus making it stable. Half the accreting matter with the high energy leaves the compact object overcoming the gravity of the compact object. The protons with the high energy are assumed to decay at the Planck scale into positron and hypothetical Planck neutrinos, the mode of the decay of the proton was proposed in Khokhlov (2011).

The footprints of the decay of the protons at the compact objects may be seen in the astrophysical observations. Annihilation of the positrons arising in the decay of the protons at the compact object Sgr A*, now interpreted as a black hole, may explain (Barbieri & Chapline, 2012) an excess of 511 keV radiation from the centre of the Galaxy (Prantzos et al., 2011). The flux of Planck neutrinos may account for the discrepancy between the accretion rate and the luminosity of Sgr A* (Khokhlov, 2014). The spectrum of ultra high energy cosmic rays may be explained by the Planck neutrinos with the energy corresponding to the decay of the proton at the compact object (Khokhlov, 2020a).

Planck neutrino is a candidate for dark matter particle. The concept of dark matter came into being to explain the discrepancy between the dynamical mass and the baryonic mass in the galaxies and in the clusters of galaxies (Trimble, 1987) and references therein. The modern theory suggests the existence of the cold dark matter (CDM) in the universe (Trimble, 1987) and references therein. Now, the dark matter is described within the framework of the ΛCDM cosmological model (Ostriker & Steinhardt, 1995). Several open problems of the ΛCDM model on galaxy scales were under discussion, e.g. (Weinberg et al., 2015; Kroupa, 2012, 2015) and references therein. Planck neutrinos are assumed to be massless particles (Khokhlov, 2011). Therefore, Planck neutrino may be thought of in terms of hot dark matter (HDM). The model of the galaxy with HDM was developed in Khokhlov (2018, 2020b).

In the present paper, we shall consider the model of the compact object based on the assumption that all the accreting protons decay at the Planck scale into positrons and Planck neutrinos. In this case, the compact object consists of the electrons and positrons with the low energy, and the Planck neutrinos with the low energy are placed in the shell around the compact object. We shall treat the Planck neutrinos in the shell around the compact object as the trapped hot dark matter (THDM).

In a recent paper (Khokhlov, 2021), the effective gravity was considered, including the Newton gravity and the free fall pressure. We shall consider the Planck neutrinos (THDM) in the shell around the compact object within the framework of the effective gravity. In this approach, the THDM is hidden for the massive particles but makes deflection of the massless particles. The THDM can be seen in the gravitational interaction of two shells. The approach may be tested in the gravitational interaction of the galaxies containing the supermassive black holes in their centres while interpreting them as the compact objects.

In the present paper, we shall study the gravitational interaction of the galaxies in the Virgo cluster. Specifically, we shall study the motion of the galaxies M60 and M49 toward the central galaxy M87 caused by the THDM mass in the shells around the compact objects of the galaxies, the stellar masses of the galaxies and the HDM masses of the galaxies.

2. Model of the compact object

Following (Khokhlov, 2017, 2021), consider the model of the compact object, being alternate to the black hole. Khokhlov (2017, 2021) think of the compact object as a thin shell of the radius (near the Schwarzschild radius) at which the falling protons reach the Planck energy. The model is based on the assumption of the decay of the proton at the Planck scale into positron and Planck neutrinos (Khokhlov, 2011). Khokhlov (2017, 2021) assumed that the kinetic energy of the accreting matter (protons and electrons) is split in the high and low energy modes. The accreting matter with the low energy retains at the compact object, with the inertial force balancing the gravity of the compact object. The accreting matter with the high energy leaves the compact object, overcoming the gravity of the compact object. The protons with the high energy decay into the positrons and Planck neutrinos. In what follows, we shall consider the model of the compact object in which all the accreting protons decay into positrons and Planck neutrinos, with the kinetic energy of the electrons, positrons and Planck neutrinos is split in the high and low energy modes.

The concept of the free fall pressure opposing the gravity was introduced in Khokhlov (2021). This leads to the effective gravity, including the Newton gravity and the free fall pressure. The gravitational force is established with the velocity of light c while the inertial force due to free fall pressure is established with a free fall velocity v. In the case of the weak gravity, the inertial force due to free fall pressure is suppressed. At the Schwarzschild radius, the free fall velocity is equal to the velocity of light, and the inertial force due to free fall pressure balances the gravity.

Planck neutrino is assumed to be massless particle (Khokhlov, 2011). Consider an approach to description of the gravitational interaction of the massless particle (photon, Planck neutrino) within the framework of the effective gravity. Consider the massless particle in the gravitational field. In view of the equivalence principle, the massless particle should possess the gravitating mass. Since the massless particle propagates with the velocity of light, the free fall pressure of the massless particle balances its gravity. The effective potential experienced by the massless particle, including the gravitational potential and the inertial potential due to free fall pressure, is zero

$$\Phi_{eff} = \Phi_g + \Phi_{in} = \frac{Gm}{r} - \frac{Gm}{r} = 0 \qquad (1)$$

where G is the Newton constant, m is the mass of the source of gravity. Assume that the perturbation due to free fall pressure comes with a time delay of the size of the massless particle. In this case, a couple of forces due to gravity and free fall pressure act on the massless particle in the gravitational field. The resultant force is zero, but the deflection of the massless particle is twice that of the massive particle. The massless particle is deflected in the gravitational field at an angle

$$\alpha = \frac{4Gm}{c^2 b} \qquad (2)$$

where b is the impact parameter. The deflection of the massless particle eq. (2) is the same as the deflection of light in general relativity (Pauli, 1958).

Consider the behaviour of the electrons, positrons and Planck neutrinos with the low energy. In view of the foregoing reasoning, the massive particles (protons, electrons, positrons) do not experience the gravity of the massless Planck neutrinos. We come to the model of the compact object, consisting of the electrons and positrons with the low energy. The compact object is a shell with the radius close to the Schwarzschild radius, $r_{co} \approx 2Gm_{co}/c^2$. Assume that the positrons and electrons of the compact object are in the chaotic motion in the transverse direction. In the own frame of the compact object, the kinetic energy of the positrons and electrons of the compact object, $E_k^- \approx Gm_{co}/2r_{co}$, produces the centrifugal acceleration of the particles, balancing their gravity. The kinetic energy of the positron (electron) minus its gravitational energy is $E_k^- - E_g \approx -Gm_{co}/2r_{co}$. Therefore, the emission of the electromagnetic radiation by the positrons and electrons of the compact object is suppressed. The remote observer will see the compact object as a black region.

The Planck neutrinos with the low energy are deflected in the gravitational field generated by the mass of the compact object plus the mass of the Planck neutrinos. They are placed in the shell around the compact object. In the frame of the shell, the kinetic energy of the Planck neutrino minus its gravitational energy is $E_k^- - E_g \approx -G(m_{co} + m_{sh})/2r_{sh}$. Assume that the energy (mass) of four Planck neutrinos is $m_\nu = m_p - m_e$ where m_p is the mass of proton, m_e is the mass of electron. The mass of the Planck neutrinos in the shell around the compact object is given by

$$m_{sh} = \frac{(m_p - m_e)}{2m_e} m_{co}. \qquad (3)$$

In view of eq. (2), the radius of the shell is given by

$$r_{sh} = \frac{(m_p - m_e)}{4m_e} r_{co}. \qquad (4)$$

The massive probe particle does not detect the mass of the Planck neutrinos in the shell around the compact object. The mass of the shell eq. (3) is hidden for the massive probe particle. The massless probe particle (photon, Planck neutrino) is deflected by the mass of the shell at an angle eq. (2). The photon lensed by the shell can reveal the hidden mass of the Planck neutrinos. When moving from infinity, the massless probe particle cannot approach the shell closer than twice the radius of the shell. Thus, the massive and massless probe particles experience the Planck neutrinos in the shell around the compact object in different way.

The Planck neutrinos in the shell around the compact object can scatter on the accreting protons. The process of the scattering of four Planck neutrinos on the proton goes at the Planck scale, with the birth of the electron-positron pair (Khokhlov, 2020a). The probability of the scattering of the Planck neutrinos in the shell around the compact object on the accreting protons is given by

$$\frac{1}{\tau} \propto \frac{1}{t_{Pl}} \left(\frac{(E_\nu E_p)^{1/2}}{m_{Pl}} \right)^7 \qquad (5)$$

where t_{Pl} is the Planck time, m_{Pl} is the Planck mass, $E_\nu = (m_p m_{Pl})^{1/2}$ is the energy of four Planck neutrinos of the compact object, $E_p \approx m_p$ is the energy of the accreting proton, taking into account that the relativistic factor of the accreting proton is $v^2/c^2 = r_{co}/r_{sh} \sim 5.5 \times 10^{-3}$. The probability eq. (5) is small thus the interaction of the Planck neutrinos in the shell around the compact object with the accreting protons can be neglected.

Consider the behaviour of the electrons, positrons and Planck neutrinos with the high energy. The positrons and electrons with the high energy can overcome the gravity of the compact object. In the own frame of the compact object, the kinetic energy of the positron (electron) minus its gravitational energy is $E_k^+ - E_g \approx Gm_{co}/2r_{co}$. This energy may be converted into the electromagnetic radiation seen by the remote observer. The luminosity of Sgr A* was explained by the energy $E_k^+ - E_g$ of the electrons leaving Sgr A* (Khokhlov, 2017). The positron from the decayed proton was assumed to have the negligible kinetic energy (Khokhlov, 2017). Later Khokhlov (2020a) showed that the positron and Planck neutrinos share the energy of the decayed proton. In its own frame, the kinetic energy of the positron is the same as that of the electron. Then, half the energy $E_k^+ - E_g$ of the positrons and electrons leaving Sgr A* may explain the luminosity of Sgr A*. The cosmic rays from Sgr A* may carry away the other half of the energy.

The Planck neutrinos with the high energy can overcome the gravity of the compact object plus the shell of the Planck neutrinos. In the own frame of the compact object, the kinetic energy of the Planck neutrino minus its gravitational energy is $E_k^+ - E_g \approx Gm_{co+sh}/2r_{sh}$. They may be thought of as the HDM distributed in the universe. The Planck neutrinos with the low energy are trapped by the compact object plus the shell of the Planck neutrinos thus may be thought of as the THDM. Since the mass of the Planck neutrinos in the shell is much more than the mass of the compact object, the Planck neutrinos with the low energy are mostly trapped by themselves.

Consider the gravitational interaction of two shells of the Planck neutrinos within the framework of the effective gravity (Khokhlov, 2021). Suppose that the mass of the first shell is less than the mass of the second shell. The effective gravitational potential of the shell is a sum of the gravitational potential and the inertial potential due to free fall pressure equal zero

$$\Phi_{eff,sh} = \Phi_{g,sh} + \Phi_{in,sh} = 0. \quad (6)$$

In the gravitational interaction of two shells, the relative inertial potential of the shells is given by

$$\Phi_{in,sh12} = \Phi_{in,sh2} - \Phi_{in,sh1}. \quad (7)$$

In the frame of the first shell, the effective gravitational potential of the second shell is given by

$$\Phi_{eff,sh2} = \Phi_{g,sh2} + \Phi_{in,sh12} = \Phi_{g,sh1}. \quad (8)$$

This gives the velocity of the first shell toward the second shell

$$v_1 = (2\Phi_{eff,sh2})^{1/2} = \left(\frac{2Gm_{sh1}}{R_{12}}\right)^{1/2} \quad (9)$$

where R_{12} is the distance between the shells. In the frame of the second shell, the effective gravitational potential of the first shell is given by

$$\Phi_{eff,sh1} = \Phi_{g,sh1}. \quad (10)$$

This gives the velocity of the second shell toward the first shell

$$v_2 = (2\Phi_{eff,sh1})^{1/2} = \left(\frac{2Gm_{sh1}}{R_{12}}\right)^{1/2}. \quad (11)$$

The relative velocity of the shells is given by

$$v_{12} = v_1 + v_2. \quad (12)$$

So, in the gravitational interaction of two shells of the Planck neutrinos around the compact objects, the inertial potentials of the shells compensate each other by the value of the smaller inertial potential. As a result, the mass of the Planck neutrinos in the shells around the compact objects can be revealed. For both the shells, the value revealed is equal to the mass of the Planck neutrinos of the smaller shell.

3. Gravitational interaction of the galaxies in the Virgo cluster

Consider the motion of the galaxies due to gravitational interaction of the galaxies in the Virgo cluster. We shall interpret the black holes in the centres of the galaxies as the compact objects, consisting of the electrons and positrons. The compact object is surrounded by the shell of the Planck neutrinos. In view of eq. (3), the mass of the shell of the Planck neutrinos around the compact object is more than the mass of the compact object by a factor of $m_p/(2m_e) = 0.9\times10^3$. The mass of the shell is hidden for the massive probe particle and can be revealed in the gravitational interaction of two shells, with the value revealed equal to the mass of the smaller shell eqs. (9),(11). Remind that we treat the Planck neutrinos of the compact object in terms of the THDM.

We shall consider the gravitational interaction of the galaxies defined by the THDM mass around the compact objects in the centres of the galaxies, the stellar masses of the galaxies and the HDM mass in the galaxies. We shall take the stellar masses of the galaxies as the baryonic masses of the galaxies. The stellar masses of the galaxies can be obtained from their luminosities in the V-band taken from NED, adopting the stellar mass-to-light ratio, $m_{*,\odot}/L_{V,\odot} = 6$. In the HDM model of the galaxy (Khokhlov, 2018, 2020b), the dynamical mass of the galaxy, including the baryonic and the HDM mass of the galaxy, is twice the baryonic mass of the galaxy at most. We shall take the dynamical mass of the galaxy twice the sum of the stellar mass of the galaxy and the mass of the smaller shell around the compact object.

We shall model the gravitational interaction of the galaxies with the use of the foregoing formalism of the gravitational interaction of the shells of the Planck neutrinos around the compact objects. Consider two galaxies with the compact objects in their centres. The velocity of the first galaxy toward the second galaxy is defined by the mass centred on the second galaxy as

$$v_1 \approx \left(\frac{2G(m_{thdm} + m_{*2} + m_{hdm2})}{R_{12}}\right)^{1/2} = \left(\frac{4G(m_{thdm} + m_{*2})}{R_{12}}\right)^{1/2} \quad (13)$$

where m_{thdm} is the THDM mass of the smaller compact object, m_* is the stellar mass of the galaxy, m_{hdm} is the HDM mass of the galaxy, R_{12} is the distance between the galaxies.

We shall study the motion of the galaxies M60 and M49 toward the galaxy M87 in the centre of the Virgo cluster, assuming that M87 is at rest. We shall estimate the velocity of M60 toward M87 due to gravitational interaction of M60 with M87, M84, M86, M49 and the velocity of M49 toward M87 due to gravitational interaction of M49 with M87, M84, M86, M60. We shall consider the motion of the galaxies in projection, taking the distances between the galaxies in projection and neglecting the radial distances between the galaxies.

Consider the gravitational interaction of M60 and M87. The distance between the galaxies is 0.97 Mpc. The observational mass of the black hole in the centre of M87 is $m_{bh,M87} = 6.5 \times 10^9$ m$_\odot$ (Event Horizon Telescope Collaboration, 2019) that gives the THDM mass of the shell around the compact object, $m_{thdm,M87} = 5.85 \times 10^{12}$ m$_\odot$. The observational mass of the black hole in the centre of M60 is $m_{bh,M60} = 4.5 \times 10^9$ m$_\odot$ (Shen & Gebhardt, 2010) that gives the THDM mass of the shell around the compact object, $m_{thdm,M60} = 4.05 \times 10^{12}$ m$_\odot$. The luminosity of M87 is $L_{V,M87} = 1.34 \times 10^{11}$ L$_\odot$ that gives the stellar mass of M87, $m_{*,M87} = 8 \times 10^{11}$ m$_\odot$. The mass centred on M87 is a sum of the THDM mass of the shell around the compact object in the centre of M60, the stellar mass of M87 and the HDM mass, $m_{M87} = 2(4.05 \times 10^{12} + 8 \times 10^{11}) = 9.7 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M60 toward M87, 294 km s^{-1}.

Consider the gravitational interaction of M60 and M84. The distance between the galaxies is 1.45 Mpc. The direction of M60 toward M84 is close to the direction of M49 toward M87. The observational mass of the black hole in the centre of M84 is $m_{bh,M84} = 1.5 \times 10^9$ m$_\odot$ (Bower et al., 1998) that gives the THDM mass of the shell around the compact object, $m_{thdm,M84} = 1.35 \times 10^{12}$ m$_\odot$. The luminosity of M84 is $L_{V,M84} = 9.33 \times 10^{10}$ L$_\odot$ that gives the stellar mass of M84, $m_{*,M84} = 5.6 \times 10^{11}$ m$_\odot$. The mass centred on M84 is a sum of the THDM mass of the shell around the compact object in the centre of M84, the stellar mass of M84 and the HDM mass, $m_{M84} = 2(1.35 \times 10^{12} + 5.6 \times 10^{11}) = 3.8 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M60 toward M84, 151 km s^{-1}. Projecting it on the direction of M60 toward M87, one gets the same value, 151 km s^{-1}.

Consider the gravitational interaction of M60 and M86. The distance between the galaxies is 1.4 Mpc. The direction of M60 toward M86 is close to the direction of M49 toward M87. The mass of the black hole in the centre of M86 is unknown. Adopt the value, $m_{bh,M86} = 1.5 \times 10^9$ m$_\odot$, which gives the THDM mass of the shell around the compact object, $m_{thdm,M86} = 1.35 \times 10^{12}$ m$_\odot$. The luminosity of M86 is $L_{V,M86} = 1.57 \times 10^{11}$ L$_\odot$ that gives the stellar mass of M86, $m_{*,M86} = 9.4 \times 10^{11}$ m$_\odot$. The mass centred on M86 is a sum of the THDM mass of the shell around the compact object in the centre of M86, the stellar mass of M86 and the HDM mass, $m_{M86} = 2(1.35 \times 10^{12} + 9.4 \times 10^{11}) = 4.6 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M60 toward M86, 169 km s^{-1}. Projecting it on the direction of M60 toward M87, one gets the same value, 169 km s^{-1}.

Consider the gravitational interaction of M60 and M49. The distance between the galaxies is 1.5 Mpc. The direction of M60 toward M49 is at an angle of 62° to the direction of M60 toward M87. The observational mass of the black hole in the centre of M49 is $m_{bh,M49} = 5.65 \times 10^8$ m$_\odot$ (Loewenstein et al., 2001) that gives the THDM mass of the shell around the compact object, $m_{thdm,M49} = 5.1 \times 10^{11}$ m$_\odot$. The luminosity of M49 is $L_{V,M49} = 1.78 \times 10^{11}$ L$_\odot$ that gives the stellar mass of M49, $m_{*,M49} = 10.7 \times 10^{11}$ m$_\odot$. The mass centred on M49 is a sum of the THDM mass of the shell around the compact object in the centre of M49, the stellar mass of M49 and the HDM mass, $m_{M49} = 2(5.1 \times 10^{11} + 10.7 \times 10^{11}) = 3.2 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M60 toward M49, 136 km s^{-1}. Projecting it on the direction of M60 toward M87, one gets 64 km s^{-1}.

Calculate the total velocity of M60 toward M87 due to gravitational interaction of M60 with M87, M84, M86, M49. The total velocity of M60 toward M87 is estimated to be $294 + 151 + 169 + 64 = 678$ km s^{-1}. The velocity of M60 toward M87, 1030 km s^{-1} (Wood et al., 2017), was obtained from the X-ray data analysis. Thus, the value obtained from the gravitational interaction of the galaxies may explain 66% of the value obtained from the X-ray data. To obtain the more accurate value one needs to include more galaxies in the study.

Consider the gravitational interaction of M49 and M87. The distance between the galaxies is 1.35 Mpc. The mass centred on M87 is a sum of the THDM mass of the shell around the compact object in the centre of M49, the stellar mass of M87 and the HDM mass, $m_{M87} = 2(5.1 \times 10^{11} + 8 \times 10^{11}) = 2.6 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M49 toward M87, 129 km s^{-1}.

Consider the gravitational interaction of M49 and M84. The distance between the galaxies is 1.55 Mpc. The direction of M49 toward M84 is at an angle of 16° to the direction of M49 toward M87. The mass centred on M84 is a sum of the THDM mass of the shell around the compact object in the centre of M49, the stellar mass of M84 and the HDM mass, $m_{M84} = 2(5.1 \times 10^{11} + 5.6 \times 10^{11}) = 2.1 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M49 toward M84, 109 km s^{-1}. Projecting it on the direction of M49 toward M87, one gets 105 km s^{-1}.

Consider the gravitational interaction of M49 and

M86. The distance between the galaxies is 1.55 Mpc. The direction of M49 toward M86 is at an angle of 14° to the direction of M49 toward M87. The mass centred on M86 is a sum of the THDM mass of the shell around the compact object in the centre of M49, the stellar mass of M86 and the HDM mass, $m_{M86} = 2(5.1 \times 10^{11} + 9.4 \times 10^{11}) = 2.9 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M49 toward M86, 128 km s^{-1}. Projecting it on the direction of M49 toward M87, one gets 124 km s^{-1}.

Consider the gravitational interaction of M49 and M60. The distance between the galaxies is 1.5 Mpc. The direction of M49 toward M60 is at an angle of 41° to the direction of M49 toward M87. The luminosity of M60 is $L_{V,M60} = 1.05 \times 10^{11}$ L$_\odot$ that gives the stellar mass of M60, $m_{*,M60} = 6.3 \times 10^{11}$ m$_\odot$. The mass centred on M60 is a sum of the THDM mass of the shell around the compact object in the centre of M49, the stellar mass of M60 and the HDM mass, $m_{M49} = 2(5.1 \times 10^{11} + 6.3 \times 10^{11}) = 2.3 \times 10^{12}$ m$_\odot$. This mass gives the velocity of M49 toward M60, 115 km s^{-1}. Projecting it on the direction of M49 toward M87, one gets 87 km s^{-1}.

Calculate the total velocity of M49 toward M87 due to gravitational interaction of M49 with M87, M84, M86, M60. The total velocity of M49 toward M87 is estimated to be $129 + 105 + 124 + 87 = 445$ km s^{-1}. The velocity of M49 toward M87, 750 km s^{-1} (Gavazzi et al., 1999), was obtained from the kinematics analysis. Thus, the value obtained from the gravitational interaction of the galaxies may explain 59% of the value obtained from the kinematics analysis. To obtain the more accurate value one needs to include more galaxies in the study.

4. Discussion

In the model considered, the mass of the shell of the Planck neutrinos around the compact object, the THDM mass, is hidden for the massive probe particle but may be seen in the gravitational interaction of the shells of the Planck neutrinos around the compact objects in the centres of the galaxies. We have calculated the velocity of M60 toward M87 due to gravitational interaction of M60 with M87, M84, M86, M49 and the velocity of M49 toward M87 due to gravitational interaction of M49 with M87, M84, M86, M60. The gravitational interaction of the galaxies is defined by the masses of the shells of the Planck neutrinos around the compact objects in the centres of the galaxies (THDM mass), the stellar masses of the galaxies and the HDM mass in the galaxies.

Estimate the contribution of the DM mass, including the THDM and HDM mass, and the stellar mass in the gravitational interaction of the galaxies. The velocity of M60 toward M87 is defined by the THDM mass 36%, the stellar mass 14%, the HDM mass 50%. The velocity of M49 toward M87 is defined by the THDM mass 20%, the stellar mass 30%, the HDM mass 50%. In total, the DM is responsible for 86% of the velocity of M60 toward M87 and for 70% of the velocity of M49 toward M87.

The relative velocity of the Milky Way and M31 was explained by the baryonic mass and the HDM mass in the galaxies (Khokhlov, 2020b). The observational masses of the black holes in the centres of the galaxies are $m_{bh,MW} = 4 \times 10^6$ m$_\odot$ (Ghez et al., 2008; Gillessen et al., 2009) in the Milky Way and $m_{bh,M31} = 2 \times 10^8$ m$_\odot$ (Bender et al., 2005) in M31. Accordingly, the THDM masses of the shells around the compact objects are $m_{thdm,MW} = 3.6 \times 10^9$ m$_\odot$ in the Milky Way and $m_{thdm,M31} = 1.8 \times 10^{11}$ m$_\odot$ in M31. The THDM mass revealed in the gravitational interaction of the shells is equal to that in the smaller shell in the Milky Way which can be neglected in comparison with the baryonic masses in the Milky Way, $m_{b,MW} = 10^{11}$ m$_\odot$ (Khokhlov, 2018), and in M31, $m_{b,M31} = 1.3 \times 10^{11}$ m$_\odot$ (Khokhlov, 2020b). Thus, the THDM mass is not seen in the gravitational interaction of the Milky Way and M31. The DM is responsible for 50% of the relative velocity of the Milky Way and M31.

The light rays are deflected by the THDM. Therefore, the THDM mass may be seen in the lensing of the galaxies. One can measure the total masses of the galaxies in the galaxy-galaxy weak lensing. van Uitert et al. (2011) measured halo masses of the galaxies in the weak lensing analysis, using data for the galaxies from RCS2 and SDSS ($0.08 < z < 0.48$) of stellar masses in the range $m_* \sim 10^{10} - 10^{12}$ m$_\odot$. The halo masses of the galaxies determined in the weak lensing analysis increase with the stellar masses of the galaxies. For the early type galaxies, the halo mass grows from $m_h^e = 1.35 \times 10^{11} h^{-1}$ m$_\odot$ at $m_* = 2.0 \times 10^{10}$ m$_\odot$ to $m_h^e = 7.2 \times 10^{13} h^{-1}$ m$_\odot$ at $m_* = 1.2 \times 10^{12}$ m$_\odot$. For the late type galaxies, the halo mass grows from $m_h^l = 5.6 \times 10^{10} h^{-1}$ m$_\odot$ at $m_* = 2.0 \times 10^{10}$ m$_\odot$ to $m_h^l = 2.2 \times 10^{13} h^{-1}$ m$_\odot$ at $m_* = 6.9 \times 10^{11}$ m$_\odot$. The lensing halo masses of the early type galaxies are greater than those of the late type galaxies. This may be explained by the smaller masses of the shells of the Planck neutrinos around the compact objects (THDM masses) in the centres of the late type galaxies.

Compare the total masses, including the stellar, THDM and HDM masses, of the Milky Way, M31, M87 with the lensing halo masses, corresponding to the stellar masses of the galaxies. Estimate the total mass of the Milky Way, taking the stellar mass of the Milky Way, $m_{*,MW} = 10^{11}$ m$_\odot$, and neglecting the THDM mass. Calculation gives $m_{MW} = 1 + 1 = 2 \times 10^{11}$ m$_\odot$. The corresponding lensing halo mass of the late type galaxy is $m_h^l = 1.4 \times 10^{11}$ m$_\odot$ which is consistent with the total mass of the Milky Way. Estimate the total mass of M31, taking the stellar mass of M31, $m_{*,M31} = 1.3 \times 10^{11}$ m$_\odot$. Calculation gives $m_{M31} = 2(1.3+1.8) = 6.2\times10^{11}$ m$_\odot$. The corresponding lensing halo mass of the late type galaxy is $m_h^l = 2.1 \times 10^{11}$ m$_\odot$ which is lower than the total mass of M31. Estimate the total mass of M87, taking the stellar mass of M87, $m_{*,M87} = 8 \times 10^{11}$ m$_\odot$. Calculation gives $m_{M87} = 2(0.8 + 5.85) = 13.3 \times 10^{12}$ m$_\odot$. The corresponding lensing halo mass of the early type galaxy is $m_h^e = 9.4 \times 10^{13}$ m$_\odot$ which is higher than the total mass of M87.

Consider the gravity of M87 experienced by the massive probe particle within the framework of the HDM model (Khokhlov, 2018, 2020b). The THDM mass is hidden for the massive probe particle. It can experience the baryonic and HDM mass. In the HDM model, the baryonic matter of the galaxy is embedded into the HDM, with the HDM density constant with radius. At some radius r_0, the HDM mass is equal to the baryonic mass. Estimate the radius r_0 for the stellar mass of M87. For the HDM density $\rho_{hdm} = 5\times10^{-3}$ m$_\odot$ pc^{-3} (Khokhlov, 2018, 2020b), the HDM mass is equal to the stellar mass of M87, $m_{*,M87} = 8 \times 10^{11}$ m$_\odot$, at the radius $r_0 = 34$ kpc. At $r < r_0$, the probe particle moves along the elliptic orbit. At $r \geq r_0$, the probe particle moves along the parabolic orbit. At $r \geq r_0$, the enclosed dynamical mass is twice the enclosed baryonic mass.

Estimate the enclosed dynamical mass of M87 at 100 kpc from the velocity dispersion profile. Longobardi et al. (2018) used a sample of planetary nebulas to map the velocity dispersion profile of the M87 outer halo, subtracting the contribution of the intra-cluster planetary nebulas. The radial velocity dispersion of the M87 halo at 100 kpc is $\sigma_r \sim 200$ km s^{-1} (Longobardi et al., 2018). In the HDM model (Khokhlov, 2018, 2020b), the enclosed dynamical mass at $r \gg r_0$ is defined by the radial velocity which is twice the radial velocity dispersion, $v_r = 2\sigma_r$. Then, the enclosed dynamical mass of M87 at 100 kpc is $m_{dyn,M87}(< 100 \text{ kpc}) = v_r^2 r/2G = 1.9 \times 10^{12}$ m$_\odot$. The enclosed baryonic mass of M87 at 100 kpc is $m_{b,M87}(< 100 \text{ kpc}) = 9.5 \times 10^{11}$ m$_\odot$. This is comparable with the stellar mass of M87, $m_{*,M87} = 8\times10^{11}$ m$_\odot$. The radial velocity dispersion of the intracluster planetary nebulas at 100 kpc is $\sigma_r \sim 800$ km s^{-1} (Longobardi et al., 2018). It is reasonable to think that the intra-cluster baryonic matter is dragged by the motion of the galaxies.

So, the THDM mass of the shell around the compact object in the centre of the galaxy manifests itself in different way depending on the observation method. It is hidden for the objects tracing the gravity of the galaxy. It may be seen in the gravitational interaction of the shells of the Planck neutrinos around the compact objects in the centres of the galaxies thus influencing the motion of the galaxies. In lensing, the THDM mass of the shell around the compact object in the centre of the galaxy gives contribution to the lensing mass of the galaxies.

5. Conclusion

We have considered the model of the compact object, being alternate to the black hole. The model is based on the assumption that the accreting protons decay at the Planck scale into positrons and Planck neutrinos. The energy of the particles is split in two modes, low and high. The compact object consists of the electrons and positrons with the low energy. The Planck neutrinos with the low energy are placed in the shell around the compact object. The mass of the shell is more than the mass of the compact object by a factor of $m_p/(2m_e) = 0.9 \times 10^3$. The electrons, positrons and Planck neutrinos with the high energy move away, overcoming the gravity of the compact object and the shell of the Planck neutrinos. The Planck neutrinos with the high energy may be interpreted as the HDM.

We have considered Planck neutrinos within the framework of the effective gravity, including the Newton gravity and the free fall pressure. The gravitational potential of the Planck neutrinos in the shell around the compact object is balanced by the inertial potential due to free fall pressure. The mass of the Planck neutrinos is hidden for the massive particles. The massless particles (photons, Planck neutrinos) are deflected by the mass of the Planck neutrinos. In effect, the Planck neutrinos in the shell around the compact object are trapped and may be interpreted as the THDM. The THDM mass may be seen in the lensing of the galaxies containing the central compact object (black hole).

In the gravitational interaction of two shells of the Planck neutrinos around the compact objects, the inertial potentials of the shells compensate each other by the value of the smaller inertial potential, revealing the masses of the Planck neutrinos (THDM masses) in the shells, equal to the mass of the Planck neutrinos of the smaller shell. We have studied the gravitational interaction of the galaxies in the Virgo cluster, M60 with M87, M84, M86, M49 and M49 with M87, M84, M86, M60. The gravitational interaction of the galaxies is defined by the revealed THDM masses in the shells around the compact objects in the centres of the galaxies, the stellar masses of the galaxies and the HDM mass in the galaxies. The total velocity of M60 toward M87 due to gravitational interaction of M60 with M87, M84, M86, M49 is 678 km s^{-1} being 66% of that obtained from the X-ray data, 1030 km s^{-1} (Wood et al., 2017). The total velocity of M49 toward M87 due to gravitational interaction of M49 with M87, M84, M86, M60 is 445 km s^{-1} being 59% of that obtained from the kinematics analysis, 750 km s^{-1} (Gavazzi et al., 1999).

References

Barbieri J., Chapline G.: 2012, *Phys. Lett. B*, **709**, 114.

Bender R., Kormendy J., Bower G. et al.: 2005, *ApJ*, **631**, 280.

Bower G.A., Green R.F., Danks A. et al.: 1998, *ApJ*, **492**, 111.

Event Horizon Telescope Collaboration, Akiyama K., Alberdi A., Alef W. et al.: 2019, *ApJ*, **875**, L6.

Gavazzi G., Boselli A., Scodeggio M., Pierini D., Belsole E.: 1999, *MNRAS*, **304**, 595.

Ghez A.M., Salim S., Weinberg N.N. et al.: 2008, *ApJ*, **689**, 1044.

Gillessen S., Eisenhauer F., Trippe S. et al.: 2009, *ApJ*, **692**, 1075.

Khokhlov D.L.: 2011, *Open Astron. J.*, **4 (SI 1)**, 151.

Khokhlov D.L.: 2014, *Phys. Lett. B*, **729**, 1.

Khokhlov D.L.: 2017, *Int. J. Mod. Phys. Appl.*, **4**, 8.

Khokhlov D.L.: 2018, *Open Astron.*, **27**, 294.

Khokhlov D.L.: 2020a, *Open Astron.*, **29**, 40.

Khokhlov D.L.: 2020b, *Odessa Astron. Publ.*, **33**, 11.

Khokhlov D.L.: 2021, *Int. J. Adv. Astron.*, **9**, 28.

Kroupa P.: 2012, *PASA*, **29**, 395.

Kroupa P.: 2015, *Can. J. Phys.*, **93**, 169.

Loewenstein M., Mushotzky R.F., Angelini L., Arnaud K.A., Quataert E.: 2001, *ApJ*, **555** L21.

Longobardi A., Arnaboldi M., Gerhard O., Pulsoni C., Söldner-Rembold I.: 2018, *A&A*, **620**, A111.

Misner C.W., Thorne K.S., Wheeler J.A.: 1973, *Gravitation*, Freeman, San Francisco.

Ostriker J.P., Steinhardt P.J.: 1995, *Nature*, **377**, 600.

Pauli W.: 1958, *Theory of Relativity*, Pergamon Press, London.

Prantzos N., Boehm C., Bykov A.M. et al.: 2011, *Rev. Mod. Phys.*, **83**, 1001.

Shen J., Gebhardt K.: 2010, *ApJ*, **711**, 484.

Trimble V.: 1987, *ARA&A*, **25**, 425.

van Uitert E., Hoekstra H., Velander M., Gilbank D.G., Gladders M.D., Yee H.K.C.: 2011, *A&A*, **534**, A14.

Weinberg D.H., Bullock J.S., Governato F., de Naray R.K., Peter A.H.G.: 2015, *Proc. Nat. Acad. Sci.*, **112**, 12249.

Wood R.A., Jones C., Machacek M.E. et al.: 2017, *ApJ*, **847**, 79.