

## COSMOLOGY, GRAVITATION, ASTROPARTICLE PHYSICS, HIGH ENERGY PHYSICS

DOI 10.18524/1810-4215.2021.34.244245

ON THE STRUCTURE OF THE CONFIGURATION SPACE  
OF CHARGED BLACK HOLES

V.D. Gladush

Oles Honchar Dnipro National University,  
Dnipro, Ukraine, *vgladush@gmail.com*

**ABSTRACT.** Some properties of the configuration space (CS) of charged black holes (BH) we are considered. A reduced action for the spherically symmetric configuration of the gravitational and electromagnetic fields is constructed. We restrict ourselves to considering of T-region, where the studied fields have a dynamic meaning. Using the Hamiltonian constraint, we exclude the nondynamic degree of freedom. This leads to the action of the system in the CS with the corresponding supermetric. It turns out that the CS is flat, and its metric admits a two-parametric group of motions. This group generates conservation laws for the geodesic equations. The first law is the charge conservation law, and second is the mass conservation law (the mass function). Using the Hamiltonian constraint, they allow one to find momenta as a function of the field variables and calculate the action as a function of the conserved quantities and field variables in CS. We emphasize that to find this action, we use only the integrability condition for a differential form. The quantization of the system is reduced to the quantization of a free particle in a three-dimensional pseudo-Euclidean space. The natural measure corresponding to the CS metric is used to construct the Hermitian DeWitt and mass operators. Based on the self-consistent solution of quantum DeWitt equations and equations for the eigenvalues of the mass and charge operators, the wave function for the spherically symmetric configuration of the gravitational and electromagnetic fields in the T-region is constructed. As a result, we get a model of charged BH with continuous mass and charge spectra.

**Keywords:** spherically symmetric configurations, configuration space, Hamiltonian constraint, DeWitt, mass and charge operators, conditions of self-consistent.

**АНОТАЦІЯ.** Розглядаються деякі властивості конфігураційного простору (КП) заряджених чорних дір (ЧД). Будується редукована дія для сферично-симетричної конфігурації гравітаційного і електромагнітного полів. Ми обмежуємося розглядом Т-області, де досліджувані поля мають

динамічний зміст. Використовуючи гамільтонову в'язь, ми виключаємо нединамічний ступінь вільності. Це призводить до дії системи в КП з відповідною суперметрикою. Виявляється, що КП є плоским, а його метрика допускає двопараметричну групу рухів, що породжує відповідні закони збереження геодезичних рівнянь. Перший закон - це закон збереження заряду, а другий - закон збереження маси (масова функція). Використовуючи гамільтонову в'язь, знаходимо імпульси як функції польових змінних і обчислюємо дію як функцію польових змінних і величин, що зберігаються в КП. Підкреслимо що для знаходження цієї дії ми використовуємо лише умову інтегрованості диференціальної лінійної форми. Квантування системи зводиться до квантування вільної частинки в тривимірному псевдо-евклідовому просторі. Природна міра, що відповідає метриці КП, використовується для побудови ермітових операторів Девітта і масового операторів. На основі самоузгодженого розв'язку системи квантових рівнянь Девітта і рівнянь на власні значення операторів маси і заряду побудована хвильова функція для сферично-симетричної конфігурації гравітаційного і електромагнітного полів в Т-області. В результаті ми отримуємо модель зарядженої ЧД з безперервним спектром маси і заряду.

**Ключові слова:** сферично-симетричні конфігурації, конфігураційний простір, Гамільтонова в'язь, оператори Девітта, маси і заряду, умова самоузгодженості.

## 1. Introduction

One of the most important tools of quantum gravity is the method of the Feynman continuum integral. However, it turns out that the gravitational path integral is dominated by metrics which describe virtual gravitational instantons. One therefore could believe that the divergence of the two loop amplitude is due to a general failure of perturbation theory in

general relativity [Schulz 2014]. For example, Hawking writes in [Hawking 1993]: "Attempts to quantize gravity ignoring the topological possibilities and simply drawing Feynman diagrams around flat space have not been very successful. It seems to me that the fault lies not with the pure gravity or supergravity theories themselves, but with the uncritical application of perturbation theory to them. In classical relativity, we have found that perturbation theory has only a limited range of validity. One cannot describe a black hole as a perturbation around flat space. Yet this is what writing down a string of Feynman diagrams amounts to."

Therefore, when studying black holes, we must use a non-perturbative quantization approach. It also implies the timeliness and relevance of research in the field of the quantum theory of black holes.

The proposed work is devoted to the study of the properties of the configuration space (CS) of spherically-symmetric (SS) systems of electromagnetic and gravitational fields. We restrict ourselves to considering the  $T$ -region of space-time (ST), where the studied fields have a dynamic meaning. The transition to the CS is carried out by excluding the non-dynamical degree of freedom from the constructed reduced action using the Hamiltonian constraint. In this way, the CS metric is constructed and its properties are studied. It turns out that the CS is flat, and its metric admits a 2-parameter Abelian group of motions. It corresponds to the conservation laws of the equations of the geodesic. One of them is associated with the law of conservation of charge  $Q$ , the other - with the law of conservation of mass  $M$  (mass function). These conservation laws, together with the Hamiltonian constraint, allow us to explicitly find the momenta as functions of the field variables, as well as  $M$  and  $Q$ . Further, using the integrability condition for a linear differential form, the action of the system in the CS is constructed as a function of conserved quantities and field variables. The Poisson brackets of the constructed dynamic quantities are calculated. The quantization of the system is reduced to the introduction of the DeWitt operators, total mass and charge. The Hermitian DeWitt and total mass operators with respect to the natural measure associated with the introduced CS metric are used. Note that the differential part of the introduced DeWitt operator coincides with the Laplace-Beltrami operator defined on the CS. As a result, we come to the consideration of the self-consistent problem for the eigenvalues of the DeWitt operators, total mass and charge. The obtained solution leads to a model of a charged BH with a continuous mass and charge spectrum.

## 2. Classic model description

As is known, the ST metric  $M^4$  for the SS configuration of the electromagnetic and gravitational fields admits the Killing vector. The region  $R \subset M^4$ , where this vector is timelike, is called the  $R$ -region and the region  $T \subset M^4$ , where this vector is spacelike, is called  $T$ -region. We only touch on  $T$ -regions where the fields under study have a dynamic meaning. We write the action of gravitational and the electromagnetic field system in the standard form

$$S_{tot} = -\frac{1}{16\pi c} \int \left( \frac{c^4}{\kappa} {}^{(4)}R + F^{\mu\nu} F_{\mu\nu} \right) \sqrt{-g} d^4x, \quad (1)$$

where  ${}^{(4)}R$  is the scalar curvature,  $F_{\mu\nu} = A_{\nu,\mu} - A_{\mu,\nu}$  is the electromagnetic field tensor,  $g = \det |g_{\mu\nu}|$ . In the SS case for the  $T$ -region, after dimensional reduction, it is reduced to the action of the form (Gladush 2018)

$$S = \frac{lc^3}{2\kappa} \int \left\{ \frac{1}{\sqrt{hf}} \left[ \frac{\kappa}{c^4} R^2 E^2 - R_{,0} (hR)_{,0} \right] + \sqrt{hf} \right\} dx^0$$

for the metric

$$ds^2 = f(x^0)(dx^0)^2 - h(x^0)(dx^1)^2 - R^2(x^0)d\sigma^2, \quad (2)$$

where  $E = F_{01} = A_{1,0} = \varphi_{,0}$ . We proceed to the characteristic variables  $\{\xi, R\}$ , which have a number of advantages (Gladush 2019)

$$f = N^2 \frac{R}{\xi}, \quad h = \frac{\xi}{R}. \quad (3)$$

In this case, the CS metric associated with the kinetic part of the Lagrangian coincides with the minisuperspace metric arising when  $N$  is excluded from the total action.

In the new variables, the metric (2) takes the form

$$ds^2 = \frac{R}{\xi} (N dx^0)^2 - \frac{\xi}{R} (dx^1)^2 - R^2 d\sigma^2. \quad (4)$$

Note that in the  $T$ -region the variable  $R$  has a temporary character, therefore, to emphasize this fact, we will sometimes write  $R = cT$ , where  $T$  is a variable with the dimension of time.

Action can now be rewritten as follows

$$S = \int L dx^0, \quad L = \frac{l}{2c} (N^{-1} \mathfrak{T} + NU) \quad (5)$$

where  $L$  is Lagrange function of a reduced system with kinetic and potential parts

$$\mathfrak{T} = -\frac{c^4}{\kappa} \dot{\xi} \dot{R} + R^2 \dot{\varphi}^2, \quad U = \frac{c^4}{\kappa}. \quad (6)$$

where  $\dot{\xi} = \xi_{,0}$ ,  $\dot{R} = R_{,0}$ ,  $\dot{\varphi} = \varphi_{,0} = \partial\varphi/\partial x^0$ .

2.1. The transition to the configuration space

The Lagrange function (5) implies the constraint

$$\frac{\delta L}{\delta N} = \frac{\partial L}{\partial N} = \frac{l}{2c} \left\{ -\frac{\mathfrak{T}}{N^2} + U \right\} = 0. \quad (7)$$

Hence follows

$$N = \sqrt{\frac{\mathfrak{T}}{U}} = \frac{\sqrt{\kappa}}{c^2} \sqrt{\mathfrak{T}}. \quad (8)$$

Substituting the obtained expression for  $N$  into the action (5) defined in the  $T$ -region of the ST, we obtain the action  $S_T$  for the curve defined in the CS:

$$S_{\mathfrak{T}} = \int L|_{H=0} dx^0 = \mu \int \sqrt{\mathfrak{T}} dx^0 = \mu \int d\Omega, \quad (9)$$

where

$$d\Omega^2 = G_{ab} dq^a dq^b = -\frac{c^4}{\kappa} d\xi dR + R^2 d\varphi^2 > 0 \quad (10)$$

is CS metric in coordinates:  $q^a = \{q^1 = \xi, q^2 = R, q^3 = \varphi\}$ ,  $(a, b = 1, 2, 3)$ . Wherein  $\mu = cl/\sqrt{\kappa}$ . Note that (9) implies

$$dS_{\mathfrak{T}} = \mu \sqrt{\mathfrak{T}} dx^0 = \mu d\Omega. \quad (11)$$

The variational principle for the new action  $S_T$  (9) defines the equations of the geodesic in CS with the metric  $d\Omega^2$  (10). The expression (9) is analogous to the Jacobi action of classical mechanics (Landau 1988). The above procedure was used in (Gladush 2018) to study some of the classical and quantum aspects of the geometrodynamics of SS Einstein-Maxwell systems. Note also that the continual version of the geodesic action (9) is an important construct in quantum gravity (Barbour 2002, Kiefer 2007, Anderson 2013).

Determinant of the metric tensor  $G_{ab}$  CS:

$$G = \det \|G_{ab}\| = -\frac{c^8}{4\kappa^2} R^2 < 0. \quad (12)$$

It defines an volume element that specifies the natural measure in CS:

$$dV = \sqrt{-G} dq^1 dq^2 dq^3 = \frac{c^4}{2\kappa} R d\xi dR d\varphi. \quad (13)$$

Note that CS metric (10) admits two Killing vectors

$$\vec{\eta}_{\xi} = \eta_{(\xi)a} \frac{\partial}{\partial q^a} = \frac{\partial}{\partial \xi}, \quad \vec{\eta}_{\varphi} = \eta_{(\varphi)a} \frac{\partial}{\partial q^a} = \frac{\partial}{\partial \varphi}, \quad (14)$$

which correspond to the conservation laws of the geodesic equations in the CS:

$$\eta_{(\xi)a} u^a = -\frac{c^4}{2\kappa} \frac{dR}{d\Omega} = const. \quad (15)$$

$$\eta_{(\varphi)a} u^a = R^2 \frac{d\varphi}{d\Omega} = const. \quad (16)$$

Here  $u^a = dq^a/d\Omega = \{d\xi/d\Omega, dR/d\Omega, d\varphi/d\Omega\}$  are components of the vector tangent to the geodesic.

2.2 Dynamic quantities

From the Lagrangian (5), taking into account (8) and (11), we obtain the momenta

$$P_{\xi} = \frac{\partial L}{\partial \dot{\xi}} = -\frac{c^3 l}{2\kappa N} \dot{R} = -\frac{c^5 l}{2\kappa \sqrt{\kappa}} \frac{dR}{d\Omega}, \quad (17)$$

$$P_R = \frac{\partial L}{\partial \dot{R}} = -\frac{c^3 l}{2\kappa N} \dot{\xi} = -\frac{c^5 l}{2\kappa \sqrt{\kappa}} \frac{d\xi}{d\Omega}, \quad (18)$$

$$P_{\varphi} = \frac{\partial L}{\partial \dot{\varphi}} = \frac{l}{cN} R^2 \dot{\varphi} = \frac{cl}{\sqrt{\kappa}} R^2 \frac{d\varphi}{d\Omega}. \quad (19)$$

Comparing the right-hand sides of the relations (15), (17), and (16), (19), we conclude that the dynamical system (17)-(19) has two integrals of motion in the CS

$$P_{\xi} = const, \quad P_{\varphi} = \frac{l}{c} q = const, \quad (20)$$

where the constant  $q$  is the electric charge of the system.

Further, let us note that the Legendre transform

$$NH = P_{\xi} \dot{\xi} + P_R \dot{R} + P_{\varphi} \dot{\varphi} - L \quad (21)$$

with (7) taken into account, leads to the Hamiltonian constraint

$$H = \frac{c}{2l} \left\{ -\frac{4\kappa}{c^4} P_{\xi} P_R + \frac{1}{R^2} P_{\varphi}^2 - \mu^2 \right\} \sim 0. \quad (22)$$

2.3 Classic solution in configuration space

Substituting the relations

$$P_{\xi} = \frac{\partial S}{\partial \xi}, \quad P_R = \frac{\partial S}{\partial R}, \quad P_{\varphi} = \frac{\partial S}{\partial \varphi} \quad (23)$$

into the Hamiltonian constraint (22) we get a Einstein-Hamilton-Jacobi equation

$$-\frac{4\kappa}{c^4} \frac{\partial S}{\partial \xi} \frac{\partial S}{\partial R} + \frac{1}{R^2} \left( \frac{\partial S}{\partial \varphi} \right)^2 = \mu^2. \quad (24)$$

This relation can be interpreted as the Hamilton-Jacobi equation for a free particle with mass  $\mu = lc/\sqrt{\kappa}$  moving along a geodesic in a CS with metric (10). Its solution has the form

$$S = \frac{l}{c} q\varphi + P_{\xi} \xi - \frac{l^2 c^6}{4\kappa^2 P_{\xi}} \left( \frac{\kappa q^2}{c^4 R} + R \right). \quad (25)$$

Equating the derivatives of  $S$  with respect to the constants  $q$  and  $P_{\xi}$  the new constants  $\xi_0$  and  $\varphi_0$ , we find the trajectories

$$\frac{\partial S}{\partial q} = \frac{l}{c} \varphi - \frac{l^2 c^2}{2\kappa P_{\xi}} \frac{q}{R} = \frac{l}{c} \varphi_0, \quad (26)$$

$$\frac{\partial S}{\partial P_\xi} = \xi + \frac{l^2 c^6}{4\kappa^2 P_\xi^2} \left( \frac{\kappa q^2}{c^4 R} + R \right) = \xi_0. \quad (27)$$

From the equation (26), assuming  $\varphi_0 = 0$ , we obtain

$$\varphi = \frac{lc^3}{2\kappa P_\xi} \frac{q}{R}. \quad (28)$$

Hence, taking into account the asymptotic condition  $\varphi = q/R$  for  $R \rightarrow \infty$ , we find the specific value of the momentum  $P_\xi$  for the considered trajectory

$$P_\xi = \frac{lc^3}{2\kappa}.$$

#### 2.4 Additional dynamic quantities and relations

From the formulas (19) and (20) it follows

$$\dot{\varphi} = \frac{Nc}{l} \frac{P_\varphi}{R^2} = N \frac{q}{R^2}.$$

In the general case, the quantity

$$Q = \frac{c}{l} P_\varphi = \frac{R^2}{N} \dot{\varphi} = \frac{c^2}{\sqrt{\kappa}} R^2 \frac{d\varphi}{d\Omega}, \quad (29)$$

we will be called the charge function of the system (Gladush 2018). For the trajectory under consideration,  $Q = q$ .

Let us turn to the equation (27). Taking into account (20), we can rewrite it as follows

$$\frac{1}{2} \left( \frac{c^2}{\kappa} R + \frac{4\kappa}{l^2 c^4} \xi P_\xi^2 \right) + \frac{P_\varphi^2}{2l^2 R} = \frac{2\kappa}{l^2 c^4} \xi_0 P_\xi^2 = m, \quad (30)$$

The left side of this expression is equal to the total field mass of the configuration (Berezin 1987, Gladush 2012, 2018) and, in the general case, determines the mass function

$$M_{tot} = \frac{1}{2} \left( \frac{c^2}{\kappa} R + \frac{4\kappa}{l^2 c^4} \xi P_\xi^2 \right) + \frac{P_\varphi^2}{2l^2 R} = m \quad (31)$$

of the SS configuration expressed in terms of the momenta  $P_\xi$  and  $P_\varphi$ . The value of  $M_{tot}$  is constant and equal to the total field mass  $m$  of the configuration. Therefore, we denoted a constant on the right-hand side of (30) by  $m$ .

We also present the Poisson brackets between the introduced additional dynamical quantities  $M_{tot}$ ,  $Q$  and the Hamilton function  $H$ :

$$\{M_{tot}, Q\} = \{H, Q\} = 0, \quad \{H, M_{tot}\} = \frac{2\kappa}{c^4 l^2} P_\xi H \sim 0.$$

Thus, we have two conservation laws for the geodesics:  $M_{tot} = m$  and  $Q = q$ . These relations allows us to find momenta and action in terms of field variables  $\{\xi, R, \varphi\}$  and conserved quantities  $\{q, m\}$ . The equation (31) leads to the expression for the momentum  $P_\xi$ :

$$P_\xi = \frac{lc^3}{2\kappa} \sqrt{\frac{R}{\xi} F_T}, \quad (32)$$

where

$$F_T = -1 + \frac{2\kappa m}{c^2 R} - \frac{\kappa q^2}{c^4 R^2}. \quad (33)$$

Further, from the constraint (22) and the relations (20) and (32), for the momentum  $P_R$  we find

$$P_R = \frac{lc^3}{2\kappa} \sqrt{\frac{\xi}{R F_T}} \left( \frac{\kappa q^2}{c^4 R^2} - 1 \right), \quad (34)$$

So, we have obtained expressions for the momenta  $P_\xi$  and  $P_R$ , as functions of the field variables  $\{\xi, R, \varphi\}$  and the conserved quantities  $\{m, q\}$ .

Using (32) and (34), one can find the action  $S$  as a function of field variables and conserved quantities. For this, we write down the action differential

$$dS = P_\xi d\xi + P_R dR + P_\varphi d\varphi = d\tilde{S} + \frac{lq}{c} d\varphi.$$

Hence it follows  $S = \tilde{S} + (lq/c)\varphi$ , besides

$$d\tilde{S} = P_\xi d\xi + P_R dR. \quad (35)$$

The integrability condition for this expression is

$$\frac{\partial P_\xi}{\partial R} = \frac{\partial P_R}{\partial \xi}.$$

From the formulas (32) and (34) we obtain

$$\frac{\partial P_R}{\partial \xi} = \frac{P_R}{2\xi}, \quad \frac{\partial P_\xi}{\partial \xi} = -\frac{P_\xi}{2\xi}. \quad (36)$$

Hence follows

$$P_R = 2\xi \frac{\partial P_R}{\partial \xi} = 2\xi \frac{\partial P_\xi}{\partial R}.$$

Taking into account these relations, we have

$$\begin{aligned} P_R dR &= 2\xi \frac{\partial P_\xi}{\partial R} dR = 2\xi \left( dP_\xi - \frac{\partial P_\xi}{\partial \xi} d\xi \right) = \\ &= 2\xi \left( dP_\xi + \frac{P_\xi}{2\xi} d\xi \right) = 2\xi dP_\xi + P_\xi d\xi, \end{aligned}$$

Substituting the found expression into the formula (35), we obtain

$$d\tilde{S} = P_\xi d\xi + 2\xi dP_\xi + P_\xi d\xi = d(2\xi P_\xi).$$

Hence we get the action

$$S = \tilde{S} + \frac{lq}{c} \varphi = 2\xi P_\xi + \frac{lq}{c} \varphi.$$

Taking into account (32), the action can be written as

$$S(\xi, R, \varphi; m, q) = S_g + S_q = \frac{lc^3}{\kappa} \sqrt{\xi R F_T} + \frac{l}{c} q \varphi. \quad (37)$$

The trajectories of motion in the CS are obtained by differentiating the action  $S$  with respect to  $m$  and  $q$  and equating the results with new constants:

$$\frac{\partial S}{\partial m} = la_m, \quad \frac{\partial S}{\partial q} = la_q.$$

From the first relation we obtain

$$c\sqrt{\frac{\xi}{RF}} = a_m,$$

whence follows

$$\frac{\xi}{R} = \frac{a_m^2}{c^2} F_T = \frac{a_m^2}{c^2} \left( -1 + \frac{2\kappa m}{c^2 R} - \frac{\kappa q^2}{c^4 R^2} \right). \quad (38)$$

From the second relation we find

$$-\frac{q}{cR} \sqrt{\frac{\xi}{RF_T}} + \frac{\varphi}{c} = a_q.$$

Here you can put  $a_q = 0$ . Then, taking into account (38), we have

$$\varphi = \frac{a_m}{c} \frac{q}{R}.$$

As a result, the metric (4) takes the form

$$ds^2 = \frac{c^2}{a_m^2 F_T} \left( \tilde{N} dx^0 \right)^2 - \frac{a_m^2}{c^2} F_T (dx^1)^2 - R^2 d\sigma^2.$$

After choosing new time  $T$  and radial  $r$  variables and imposing coordinate conditions

$$\tilde{N} dx^0 = \frac{a_m}{c} dR = a_m dT, \quad dr = \frac{a_m}{c} dx^1 \quad (39)$$

the metric takes the standard form of the Reissner-Nordstrom metric in the  $T$ -region

$$ds^2 = \frac{c^2}{F_T} dT^2 - F_T dr^2 - c^2 T^2 d\sigma^2.$$

Here we went beyond the CS and presented the solution in ST manifold

### 2.5. Quasi-Cartesian coordinates in configuration space

It turns out that the curvature tensor of the considered CS with the metric (10) is equal to zero and the CS is flat. Indeed, using the transformation of field functions

$$\xi = \eta \left( c\tau - x - \frac{y^2}{R} \right), \varphi = \frac{1}{\eta} \frac{y}{R}, R = \eta (c\tau + x), \quad (40)$$

where  $\eta = \sqrt{\kappa}/c^2$ , CS metric (10) reduced to Lorentzian form

$$d\Omega_0^2 = -c^2 d\tau^2 + dx^2 + dy^2 > 0. \quad (41)$$

and admits the motions group  $O(1,2)$ . We also see that the field function  $\tau$  has the meaning of the time coordinate in the CS.

In the new field variables, the Lagrangian (6) takes the form

$$L_0 = \frac{l}{2c} \left\{ \frac{1}{N} (-c^2 \dot{\tau}^2 + \dot{x}^2 + \dot{y}^2) + \frac{N}{\eta^2} \right\}. \quad (42)$$

From here we find the new momenta

$$P_\tau = \frac{\partial L}{\partial \dot{\tau}} = -\frac{cl}{N} \dot{\tau},$$

$$P_x = \frac{\partial L}{\partial \dot{x}} = \frac{l}{Nc} \dot{x}, \quad (43)$$

$$P_y = \frac{\partial L}{\partial \dot{y}} = \frac{l}{Nc} \dot{y}. \quad (44)$$

which are saved in the CS. The Hamiltonian constraint in the new variables has the form

$$H_0 = \frac{c}{2l} (P_0^2 - \mu^2) \sim 0, \quad (45)$$

where  $P_0^2 = -P_\tau^2/c^2 + P_x^2 + P_y^2$  is the square of momentum with respect to metric (41).

We see that the original field configuration in the new variables  $\{\tau, x, y\}$  with the Lagrangian (42) and the Hamiltonian (45) describes a free particle of mass  $\mu = lc/\sqrt{\kappa}$ , which moves in a three-dimensional CS with a pseudo-Euclidean metric (41).

### 3. Quantum model description in CS

The problem of studying the quantum states of the SS configuration of the gravitational and electromagnetic fields in the coordinates (40) is reduced to describing the states of the system in a three-dimensional pseudo-Euclidean space, taking into account the conservation laws  $M_{tot} = m$  and  $Q = q$ . Here, the quantum states are specified by the wave function  $\Psi(\tau, x, y)$  in a flat CS with the metric (41). The corresponding momentum operators in the coordinate representation are determined by the partial derivatives:

$$\hat{P}_\tau = -i\hbar \frac{\partial}{\partial \tau}, \quad \hat{P}_x = -i\hbar \frac{\partial}{\partial x}, \quad \hat{P}_y = -i\hbar \frac{\partial}{\partial y}. \quad (46)$$

The momentum square is associated with the operator

$$P_0^2 \rightarrow \hat{P}_0^2 = -\hbar^2 \Delta_0, \quad (47)$$

where

$$\Delta_0 = -\frac{1}{c^2} \frac{\partial^2}{\partial \tau^2} + \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \quad (48)$$

is the Laplace operator in CS with respect to the metric (41). The Hamiltonian (45) is associated with a quantum analog

$$H_0 \rightarrow \hat{H}_0 \Psi = -\frac{c}{2l} (\hbar^2 \Delta_0 + \mu^2) \Psi = 0. \quad (49)$$

Let us now turn to the initial configuration with the characteristic variables  $\{\xi, R, \varphi\}$  in the CS and the metric (10). The quantum state of the configuration under consideration is determined by the wave function  $\Psi(R, \xi, \varphi)$  given on the CS. To introduce a quantization procedure in curvilinear coordinates  $\{\xi, R, \varphi\}$ , similar

to (46)-(49), we can use the transition formulas (40) to transform these relations to the coordinates  $\{\xi, R, \varphi\}$  into CS. In fact, it is enough for this, in all formulas like (46)-(49), instead of partial derivatives, to use covariant derivatives with respect to the metric (10). So instead of (46) we have

$$\hat{P}_\xi = -i\hbar\nabla_\xi, \quad \hat{P}_R = -i\hbar\nabla_R, \quad \hat{P}_\varphi = -i\hbar\nabla_\varphi. \quad (50)$$

where  $\{\nabla_\xi, \nabla_R, \nabla_\varphi\}$  are covariant derivatives with respect to (10). Now, instead of (47), (48), we get

$$P^2 \rightarrow \hat{P}^2 = -\hbar^2\Delta, \quad (51)$$

were

$$\Delta = \nabla^a \nabla_b = \frac{1}{\sqrt{|G|}} \frac{\partial}{\partial q^a} \left( \sqrt{|G|} G^{ab} \frac{\partial \Psi}{\partial q^b} \right). \quad (52)$$

is operator Laplace-Beltrami. The quantities  $G_{ab}$  and  $G$  are defined in (10) and (12). In this case, we naturally come to the Laplace-Beltrami operator of the form:

$$\Delta\Psi = -\frac{2\kappa}{c^4} \frac{\partial^2 \Psi}{\partial \xi \partial R} - \frac{2\kappa}{c^4} \frac{1}{R} \frac{\partial}{\partial R} \left( R \frac{\partial \Psi}{\partial \xi} \right) + \frac{1}{R^2} \frac{\partial^2 \Psi}{\partial \varphi^2}.$$

Thus, in the relation (49) we must make the substitution  $\Delta_0 \rightarrow \Delta$ , and the Hamiltonian constraint  $H = 0$  (22) becomes the DeWitt equation of the form

$$\hat{H}\Psi = -\frac{c}{2l} \{ \hbar^2 \Delta + \mu^2 \} \Psi = 0. \quad (53)$$

One can check that the constructed operator  $\Delta$  is Hermitian in the natural measure (13) in CS.

If the law of conservation of total mass had not been fulfilled, then the quantization procedure could have been completed. Such a system could be interpreted classically as a free particle of mass  $\mu = lc/\sqrt{\kappa}$ , moving in a three-dimensional CS with a pseudo-Euclidean metric (41). In the transition to a quantum picture, we would get a plane wave. However, due to the presence conserved mass, when quantizing, we must consider our configuration as a superposition of plane waves, the overall contribution of which gives the total mass  $m$ .

Thus, you should take into account the total mass. To construct the Hermitian total mass operator, in CS with the volume element (13), it is sufficient to use the following ordering of operators:  $\xi P_\xi^2 \Rightarrow \hat{P}_\xi \xi \hat{P}_\xi$ . It turns out here that the momenta entering into the functions of charge (29) and mass (31) can be associated with partial derivatives:

$$\hat{P}_\xi = -i\hbar \frac{\partial}{\partial \xi}, \quad \hat{P}_\varphi = -i\hbar \frac{\partial}{\partial \varphi}. \quad (54)$$

Thus, the charge (29) and the total mass (31) correspond operators

$$\hat{Q} = \frac{c}{l} \hat{P}_\varphi = -i \frac{c\hbar}{l} \frac{\partial}{\partial \varphi}, \quad (55)$$

$$\hat{M} = \frac{1}{2l^2} \left( \frac{l^2 c^2}{\kappa} R - \frac{4\kappa\hbar^2}{c^4} \frac{\partial}{\partial \xi} \xi \frac{\partial}{\partial \xi} - \frac{\hbar^2}{R} \frac{\partial^2}{\partial \varphi^2} \right). \quad (56)$$

For the operators introduced, the commutation relations are satisfied

$$[\hat{H}, \hat{M}] = -\frac{2\kappa\hbar^2}{l^2 c^4} \frac{\partial}{\partial \xi} \hat{H} \sim 0, \quad [\hat{H}, \hat{Q}] = 0, \quad [\hat{Q}, \hat{M}] = 0.$$

Further, we construct states with a certain total mass and charge, that is, states corresponding to the eigenfunctions and eigenvalues of the operators of the total mass and charge

$$\hat{M}\Psi_m = m\Psi_m, \quad \hat{Q}\Psi_q = q\Psi_q. \quad (57)$$

In expanded form, we have  $-i(c\hbar/l)(\partial)/\partial\varphi\Psi_q = q\Psi_q$ ,

$$\left( \frac{l^2 c^2}{\kappa} R - \frac{4\kappa\hbar^2}{c^4} \frac{\partial}{\partial \xi} \xi \frac{\partial}{\partial \xi} - \frac{\hbar^2}{R} \frac{\partial^2}{\partial \varphi^2} \right) \Psi = 2l^2 m \Psi_m.$$

The first relation gives:  $\Psi_q = A e^{i(ql/c\hbar)\varphi}$ . Therefore, the general wave functions of the DeWitt and charge operators, as well as the operators of the total mass and charge, can be represented in the form

$$\Psi = \psi(\xi, R) e^{i(ql/c\hbar)\varphi}, \quad \Psi_m = \psi_m(\xi, R) e^{i(ql/c\hbar)\varphi}.$$

According to the formula  $\mu = cl/\sqrt{\kappa}$  and (53), (56) and (57)  $\psi$  and  $\psi_m$  satisfy the equations

$$\left\{ \frac{2\kappa\hbar^2}{c^4} \frac{\partial^2}{\partial \xi \partial R} + \frac{2\kappa\hbar^2}{c^4 R} \frac{\partial}{\partial R} R \frac{\partial}{\partial \xi} + \frac{l^2 q^2}{c^2 R^2} - \frac{c^2 l^2}{\kappa} \right\} \psi = 0 \quad (58)$$

$$\left( -\frac{4\kappa\hbar^2}{c^4} \frac{\partial}{\partial \xi} \xi \frac{\partial}{\partial \xi} + \frac{l^2 c^2}{\kappa} R + \frac{l^2 q^2}{c^2 R^2} \right) \psi_m = 2l^2 m \psi_m. \quad (59)$$

After introducing the Planck and dimensionless quantities according to the formulas

$$m_{pl}^2 = \frac{c\hbar}{\kappa}, \quad l_{pl}^2 = \frac{\hbar\kappa}{c^3}, \quad q_{pl} = m_{pl} \sqrt{\kappa} = \sqrt{c\hbar},$$

$$\nu = \frac{m}{m_{pl}}, \quad \sigma = \frac{q}{q_{pl}}, \quad x = \frac{\xi}{l_{pl}}, \quad y = \frac{R}{l_{pl}}, \quad \chi = \frac{l}{l_{pl}}$$

the system of equations (58) and (59) can be rewritten as follows

$$\frac{\partial^2 \psi}{\partial x \partial y} + \frac{1}{2y} \frac{\partial \psi}{\partial x} = \frac{\chi^2}{4} \left( 1 - \frac{\sigma^2}{y^2} \right) \psi, \quad (60)$$

$$x \frac{\partial^2 \psi_m}{\partial x^2} + \frac{\partial \psi_m}{\partial x} = \frac{\chi^2}{4} \left( y - 2\nu + \frac{\sigma^2}{y} \right) \psi_m. \quad (61)$$

Note that the equation (61) for the mass operator does not contain derivatives with respect to the variable  $y$ . Therefore, we consider it as an ordinary differential equation containing the variable  $y$  as a parameter. The regular on the horizon general solution of the equation (61) has the form

$$\psi = f(y) J_0 \left( \chi \sqrt{x \left( 2\nu - y - \frac{\sigma^2}{y} \right)} \right), \quad (62)$$

where  $J_0$  is Bessel functions of the first kind of order zero,  $f(y)$  is an arbitrary function.

Substituting the obtained solution into the DeWitt equation (60) we find  $f(y) = C/\sqrt{y}$ . Ultimately, the joint solution of the (60), (61) system has the form

$$\psi(x, y) = \frac{C}{\sqrt{y}} J_0 \left( \chi \sqrt{x \left( 2\nu - y - \frac{\sigma^2}{y} \right)} \right). \quad (63)$$

Returning to the original variables, we obtain the following general, regular on the horizon  $F_T = 0$ , solution

$$\Psi_{m,q}(\xi, R, \varphi) = C \sqrt{\frac{l_{pl}}{R}} J_0 \left( \frac{l}{l_{pl}^2} \sqrt{\xi R F_T} \right) e^{\frac{iq l}{\hbar} \varphi}, \quad (64)$$

where the function  $F_T$  is defined in (33). As a result, we get a model of a charged BH with a continuous spectrum of mass  $m$  and of charge  $q$ .

#### 4. Conclusions

Note that the coefficient  $N$  is not included in the wave function  $\Psi_{m,q}(\xi, T, \varphi)$  (64), which determines the amplitude of the configuration probability  $\{\xi, T, \varphi; m, q\}$ , that is, points  $\{\xi, T, \varphi\} \in \text{CS}$  for observables  $\{m, q\}$ . Thus, here the wave function is determined in the CS and sets the state of the BH in the CS.

We also mention an interesting connection between the classical action (37) and the wave function (64). For this we turn to the decomposition of  $S$  (37) into two components  $S_g$  and  $S_q$ . The components written out are coinciding with the arguments of the Bessel function and exponent in (64). Substituting these values into (64), we obtain

$$\Psi_{m,q}(\xi, R, \varphi) = C \sqrt{\frac{l_{pl}}{R}} J_0 \left( \frac{S_g}{\hbar} \right) e^{i \frac{S_q}{\hbar}}.$$

The state vector of a charged BH is expressed through the components  $S_g$  and  $S_q$  of the classical action.

We see that  $S_g$  is contained in the argument of the Bessel function and  $S_q$  is in the exponent argument.

#### References

- Schulz, B., Review of quantum gravity. ArXiv 1409.7977v1 [gr-qc] 29 Sep 2014.
- Gibbons, G. W., Hawking, S. W. Euclidean Quantum Gravity, World Scientific Publishing. 1993.
- Gladush V. D., Holovko M. H.: 2018, *Space, time and fund. interect.*, **2**, 28 (in Russian).
- Gladush V. D.: 2019, *J. Phys and Electron.*, **27(1)**, 3.
- Landau L. D., Lifshic E. M. Teoreticheskaya fizika. M.: Nauka, 1988. T. 1. 216 p.
- Barbour J., Foster B., Murchadha N.: 2002, *Clas. and Quant. Grav.*, **19**, 3217.
- Kiefer C. Quantum Gravity. New York: Oxford University Press, 2007. 375 p.
- Anderson E. arXiv:1111.1472 [gr-qc].
- Berezin V. A., Kuzmin V. A., Tkachev I. I.: 1987, *JETP.*, **93**, 4(10), 1159 (in Russian).
- Gladush V. D., Petrusenko A, I.: 2012, *Space, time and fund. interect.*, **1**, 48 (in Russian).