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MODEL OF GRAVITATIONAL EFFECTS IN SATURN'S RINGS

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ABSTRACT. In this work, we consider one possible gravitational interaction in a system consisting of Saturn, its satellites and rings. The structure of rings can be changed because of such interactions leading to some sculpting of rings. In consequence of not trivial gravitational effects, various perturbations are observed on the edges of rings. We use methods of celestial mechanics to describe a system containing the planet, two satellites and the ring placed between them. In such a system, gravitational forces lead to running waves that deform the ring. These effects are peculiar to Saturn's F ring. The description of these perturbations needs more difficult physical model. In particular, to find the change in the ring's edge it's necessary to go to the non-inertial reference frame. To describe some effects we expect to employ methods of nonlinear physics to build the model.

АНОТАЦІЯ. Завдяки місії Кассіні за останні роки було отримано багато даних про планету Сатурн та її кільця. Було виявлено, що система кілець Сатурна та його супутників має безліч прикладів прояву гравітаційних взаємодій. У даній роботі ми розглядаємо одну з можливих взаємодій в гравітаційному полі системи. Структура кілець може змінюватись залежно від прояву сил гравітації, що призводить до нерівномірного розподілу речовини в кільцях. Відомо, що частинки на границях кілець можуть відчувати збурення внаслідок проходження поряд супутників. Ми розглядаємо систему, що складається з планети, й кільця, що розташоване між двох супутників. Траєкторії частинок у такій системі описуються методами небесної механіки. Гравітаційні сили в системі можуть призводити до збурень у вигляді біжучих хвиль. Ця картина притаманна кільцю F планети Сатурн. Для опису цих збурень, треба перейти до неінерціальної системи відліку, пов'язаної з частинкою у кільці, щоб побачити зміну траєкторії. У подальшому ми збираємось описувати деякі ефекти, що можуть виникнути у системі, за допомогою методів нелінійної фізики.

Keywords: Saturn's rings, gravitation, rotating reference frame.

1. Introduction

After the end of the Cassini Mission in 2017 scientists are still processing and analyzing obtained images of Saturn's and it rings. One of the main conclusions based on obtained data is that a structure of Saturn's rings is fairly difficult. Scientists assume complex models of gravitational interactions in order to explain the heterogeneity in the distribution of rings matter. For example, the gap between rings A and B is explained by gravitational resonance 2:1 of A ring with the Mimas and named Cassini Division. Other interesting interactions due to orbital resonance are considered in work (Araujo et al., 2015). The satellites Janus and Epimetheus are influenced by orbital resonance 7:6 (Proco et al., 1987) and impact on the edge of the A ring. Thereby, there are perturbations in the form of waves on the ring's edge. Such waves have a three-dimensional structure and fade out with extending deep into the ring. Janus and Epimetheus are located on the close orbits, and when the one overtakes another, their orbits switch (every 4.2 years). In (Fig. 1) the change of orbits is demonstrated in dependence on a period of time. Appeared gravitational

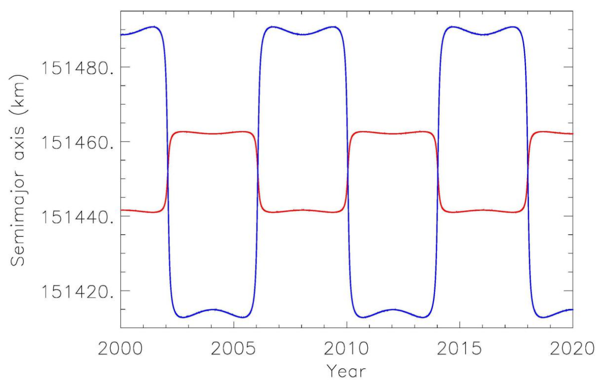

Figure 1: Semi-major axis for Janis (red) and Epimetheus (blue) as a function of time between 2000 and 2020 years, from a numerical integration, fitted to Cassini data (Cooper et al., 2015)

force due to such a switching causes perturbations on the edge. These perturbations are small in comparing with the size of the ring and do not change its overall structure. It is assumed that waves fade out enough quickly then we can speak about particles bonds in the ring.

In this work, we concern with another problem. Consider the thin ring between two moons in orbital resonance. This situation is realized for an F ring of Saturn surrounded by Prometheus and Pandora. The F ring is far from the main rings system, therefore is not influenced by them. The F ring can have dynamical changing in structure (Fig. 2) due to a satellites impact.

Figure 2: In the left corner is Pandora, and in the right is Prometheus, acquired by the Cassini spacecraft [solarsystem.nasa.gov]

2. Physical model

Each ring surrounding Saturn consists of dust and ice particles that move on stable orbits. If the outer gravitational field of Saturn is supplemented by a small perturb additive (it can be caused by distant satellite or tidal forces of the planet) then the search of orbital parameters will be more difficult. One of the ways to describe such a motion is presented in (Burns, 1976). If the ring is located between orbits of two satellites moving in gravitational resonance, the variable in time and localized in space gravitational field, that could be not assumed as small, acts on the particles. In result, the strong perturbations, distorting the ring, are observed. To describe the motion of the particle in the ring in such a case, we suggest using the Newtonian approach: the gravitational force, acting on the particle, consists of force sourced by the planet, and perturbation forces sourced by satellites. To describe the latter, we introduce disturbing potential. The technique of introducing such potential is presented in (Shu et al., 1984; Renner et al., 2006) Obtained equations, generally speaking, describe perturbed motion and is a case of N-body problem.

For numerical simulation of the motion of such a gravitational system one should use the following equations:

$$m_i \frac{\mathrm{d}^2 \overrightarrow{r_i}}{\mathrm{d}t^2} = G \sum_{i \neq j}^{N} \frac{m_i m_j (\overrightarrow{r_i} - \overrightarrow{r_j})}{|\overrightarrow{r_i} - \overrightarrow{r_j}|^3} \tag{1}$$

The system of equations (1) with initial conditions describe the N-body problem completely. Unfortunately, this system is not applicable for the analytical study and can be time-consuming for the numerical simulation. Here we propose an alternative approach that is lack of the mentioned disadvantages. We will study an orbit of a separate particle in the non-inertial frame that rotates around the central mass with the angular velocity of the ring. In such a frame non-perturbed particle should rest, while perturbed particles oscillate about their rest points. Moreover while the amplitude of such oscillation evidently much smaller the ring radius, it can be comparable with the distance between satellites and ring, producing non-trivial and non-linear dynamics. For two particles an equation has a form:

$$\frac{\mathrm{d}^2 \overrightarrow{r}}{\mathrm{d}t^2} = -\frac{GM}{|\overrightarrow{r}|^3} \overrightarrow{r} + 2 \left[\frac{\mathrm{d}\overrightarrow{r}}{\mathrm{d}t}, \overrightarrow{\Omega} \right] + \left[\overrightarrow{\Omega}, \left[\overrightarrow{r}, \overrightarrow{\Omega} \right] \right] \tag{2}$$

where $\overrightarrow{r}$ – is a radius vector directed from the particle with mass m, M – is a mass of moving particle in the new reference frame, $\overrightarrow{\Omega}$ – is an angular frequency vector of rotating reference frame.

3. Discussion

Besides, we propose to consider the classical task of celestial mechanics from the point of view nonlinear physics and to describe perturbation as non-small additive in the equation of motion in the rotating reference frame. In result, we aim to find an analogue with soliton solutions (Kosevich et al., 1978), that is a case in propagation nonlinear waves in a medium. This analogue could describe some dynamical effects, that can have a place in Saturn's rings.

4. Conclusions

We have proposed a physical model of gravitational interactions for the Saturn's F ring to describe the change of its structure. This approach should simplify the numerical simulation for the N-body problem and enables analytical studies. In the future using this physical model, we might describe the dynamical structure and the change of the separate particle orbit.

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References

Araujo N. et al.: 2019, *MNRAS*, **486**, 4, 5037.

Burns J. A.: 1976, *American Journal of Physics*, **44**, 944.

Cooper N.J. et al.: 2015, *AJ*, **149**, 27.

Kosevich A.M. & Kovalev A.S.: 1978, Kiev: Naukova Dumka, 304 p. [in Russian]

Proco C.C., Nicholson P.D.: 1987, *Icarus*, **72**, 437.

Renner S. & Sicardy B.: 2006, *Celestial Mechanics and Dynamical Astronomy*, **94**, 237.

Shu F.H.: 1984, in: *IAU Colloq. 75: Planetary Rings /eds. Greenberg R., Brahic A.*, 513.