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GLUEBALLS

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ABSTRACT. Glueball and oddball resonances lying on the pomeron/odderon trajectories (Chew-Frautchi plot) with threshold and asymptotic behaviour required by analyticity and unitarity are predicted. While the parameters of meson and baryon trajectories can be determined both from the scattering data and from the particles spectra, this is not so for the pomeron (and odderon) trajectory, known only from fits to scattering data only.

The main idea in our approach is in use of a nonlinear complex Regge trajectory for the pomeron satisfying the requirements of the analytic S-matrix theory yet fitting the data. The crucial task, on which glueball (and oddball) predictions are based is the correct fit of the pomeron (and odderon) trajectory to the data.

The basic sub-process is pomeron-pomeron scattering, producing glueballs lying on the direct-channel pomeron trajectory. Glueball в “towers”, called reggeized Breit-Wigner resonances, lie on this trajectory. Crucial for the identification of these states is knowledge of the non-linear complex trajectory, interpolating between negative and positive values of its argument. Its parameters are found trom to the scattering data. While the real part of the trajectory is almost linear, the recovery of the imaginary part, determining the widths of predicted glueballs, is an important ingredient in our approach.

Oddballs, resonances made of three gluons have the same right of existence as glueballs made of two gluons. Oddballs are expected to lie on the odderon trajectory exactly in the same way as glueballs lie on the pomeron trajectory.

The trajectory introduced in this paper can be applied also to studies or ordinary meson and baryon spectra (Chew-Frautchi plot).

The parameters of the pomeron and odderon trajectories are fitted to the data on high-energy elastic proton-proton scattering. The fitted trajectories are extrapolated to the resonance region to predict masses and widths of glueballs and oddballs. Appended by unitary symmetry, the Chew-Frautchi plot remains a powerful tool to classify hadrons. The proposed trajectory opens a new avenue of research in hadron spectroscopy (Chew-Frautchi plot), applicable both to ordinary mesons and baryons as well as to glueballs lying on the pomeron and odderon trajectories.

Keywords: Regge trajectories, resonances, pomeron, odderon, glueball, meson, baryon.

АНОТАЦІЯ. Передбачено резонанси глюболів та одболів, що лежать на траєкторії померона (оддерона, графік Чю-Фраучі) з порогом та асимптотичною поведінкою, що задовільняють вимогам аналітичністі та унітарністі. В той час, як параметры мезонних та баріонних траекоторій можна вижначити як з даних про розсіяння, так і зі спектра часнинок, це неможливо для траекторій померона (та оддерона), про які ми маємо інформацію лише з даних про розсіяння.

Основною нашею ідеєю є застосуваня нелінійних комплексних траекторій Редже, що задовільяють вимогам аналітичної теорії $S-$ матриці і при цьому описують дані. Критичним є згода з експериментаьними даними.

Базовим є під-процес розсіяння померонів з продукуваням глюболів, що лежать на траекотрії померона в прямому каналі. Стовпчики глюболів, які ми називаємо реджезованними резонансами Брейта-Вігнера, лежать на цій траекторії. Критичним для ідентифікації цих станів є знання нелінійною компексної траекторії, яка інтерполює між негативними та позитивними значеннями аргумента. В той час, як реальна частина траекторії майже лінійна, знаходження її уявної частини, яка дає визначає ширини глюболів, є важливою оригінальною частиною нашого підходу.

Одболи - резонанси, які складаються з трьох глюонів, мають таке саме право на існування, як і глюболи, що складаються з двох глюболів. Ми прогнозуємо, що одболи лежать на траекотрії одерона так само, як глюболи - на траекторії померона.

Траекторії запропоновані в даній роботі можна застосувати також до дослідження спектрів звичайних мезонів та баріонів.

Параметри траекторії померона та оддерона прив'язані до даних високоенергетичного пружного протон-протонного розсіювання. Підігнані траєкторії екстрапольовані в область резонансів для передбачення мас і ширин глюболів та одболів. Разом з унітарною симетрією, графік Чю-Фраучі є потужним інструментом у класифікації адронів.

Ключові слова: Траекторії Редже, резонанси, померон, оддерон, глюбол, мезон, баріон.

1. Introduction

Regge trajectories $\alpha(t)$ connect the scattering region, $t < 0$ with particle spectroscopy, $t > 0$. In this way they realize crossing symmetry and anticipate duality: dynamics of two kinematically disconnected regions are interrelated. The behaviour of trajectories both in the scattering and particle region is close to linear. This observation, combined with the properties of dual models and hadron strings resulted in a prejudice of the linearity of Regge trajectories. Appended by unitary symmetry, the Chew-Frautchi plot remains a powerful tool to classify hadrons.

Unitarity imposes (Barut et al., 1962) a severe constraint on the threshold behavior of the trajectories:

$$\Im\alpha(t)_{t\to t_0} \sim (t-t_0)^{\alpha(t_0)+1/2}, \tag{1}$$

while asymptotically the trajectories are constrained by (Bugrij et al., 1973)

$$\left|\frac{\alpha(t)}{\sqrt{t}\ln t}\right|_{t\to\infty} \le \text{const.} \tag{2}$$

The above asymptotic constrain can be still lowered to a logarithm by imposing (Jenkovszky, 1987 and earlier references) wide-angle power behaviour for the amplitude. While the parameters of meson and baryon trajectories can be determined both from the scattering data and from the particles spectra, this is not true for the pomeron (and odderon) trajectory, known only from fits to scattering data only (negative values of its argument). An obvious task is to extrapolate the pomeron trajectory from negative to positive values to predict glueball states at $J = 2, 4, ...$ non has been found. Given the nearly linear form of the pomeron trajectory, known from the fits to the (exponential) diffraction cone, little room is left for variations in the region of particles ($t > 0$.)

We continue the lines of research initiated in (Fiore et al., 2016) and (Fiore et al., 2018) in which an analytic pomeron trajectory was used to calculate the pomeron-pomeron cross section in central exclusive production measurable in proton proton scattering. The basic idea in that approach is in the use of a nonlinear complex Regge trajectory for the pomeron satisfying the requirements of the analytic S-matrix theory and fitting the data. Fits imply high-energy elastic proton-proton and/or proton-antiproton scattering (with the odderon in mind!).

The basic sub-process is pomeron-pomeron scattering, producing glueballs lying on the direct-channel pomeron trajectory, with a triple pomeron vertex. Glueball вЂњtowersвЂќ, i.e. excited glueball states, called reggeized Breit-Wigner resonances, lie on this trajectory. Crucial for the identification of these states is knowledge of the nonlinear complex trajectory, interpolating between negative and to positive values of its argument. Its parameters are fitted to the scattering data. While the real part of the trajectory is almost linear, the recovery of the imaginary part, determining the widths of predicted glueballs, is a highly non-trivial problem.

2. Simple analytic Regge trajectory

What is the simplest ansatz for a Regge trajectory satisfying the following constrains: a) threshold behavior imposed by unitarity, Eq. (1), b) asymptotic behavior constrained by Eq. (2), c) yet compatible with the nearly linear behavior in the resonance region (Chew-Frautchi plot)? Attempts and explicit examples can be found in a number of papers, see e.g. (Fiore et al., 2016 and 2018) and (Szanyi., 2017) and earlier references therein.

The trajectory:

$$\alpha(t) = \frac{1+\delta+\alpha_1 t}{1+\alpha_2\big(\sqrt{t_0-t}-\sqrt{t_0}\big)}. \tag{3}$$

where $t_0 = 4m_\pi^2$ for pomeron and $t_0 = 9m_\pi^2$ for odderon and $\delta, \alpha_1, \alpha_2$ are adjustable parameters, to be fitted to scattering ($t < 0$) data with the obvious constrains: $\alpha(0) \approx 1.08$ and $\alpha'(0) \approx 0.3$ (in case of the pomeron trajectory). Trajectory Eq. (3) has square-root asymptotic behavior, in accord with the requirements of the analytic S-matrix theory.

With the parameters fitted in the scattering region, we continue trajectory Eq. (3) to positive values of t. When approaching the branch cut at $t = t_0$ one has to chose the right Riemann sheet, For $t > t_0$ trajectory Eq. (3) may be rewritten as

$$\alpha(t) = \frac{1+\delta+\alpha_1 t}{1-\alpha_2(i\sqrt{t-t_0}+\sqrt{t_0})}, \tag{4}$$

with the sign "minus" in front of α_2, according to the definition of the physical sheet.

For $t >> t_0$, $|\alpha(t)| \to \frac{\alpha_1}{\alpha_2}\sqrt{|t|}$. For $t > t_0$ (on the upper edge of the cut), $Im\alpha > 0$.

The intercept is $\alpha(0) = 1 + \delta$ and the slope at $t = 0$ is

$$\alpha'(0) = \alpha_1 + \alpha_2 \frac{1+\delta}{2\sqrt{t_0}}. \quad (5)$$

To anticipate subsequent fits and discussions, note that the presence of the light threshold $t_0 = 4m_\pi^2$ (required by unitarity and the observed "break" in the data) results in the increasing, compared with the "standard" value of about 0.25 GeV^{-2}, slope.

The crucial task, on which glueball (and oddball) predictions are based is the correct fit of the pomeron (and odderon) trajectory to the data. Credibility of such predictions depend on the reliability of these fits. The most direct and reliable ways to fit the pomeron (and odderon) trajectory are those to high-energy elastic nucleon scattering, of which the LHC data on proton-proton scattering, dominated by pomeron exchange are the best. The contribution of secondary trajectories here is negligible.

The situation, however is not that simple. The smooth, nearly exponential small-$|t|$ part of the cone ($|t| \leq 0.3$ before the dip) at the LHC is too short to fit the trajectory and provide its reliable interpolation to large positive values, where glueball are expected. Fits in this limited interval are under control and the small deviation from and exponential of the cone, can be parametrized by a pomeron (and odderon) exchange within the simple Donnachie-Landshoff model (Donnachie et al., 2002), see next Section, where the resulting trajectory inherits the curvature, called "break" seen both at the ISR and LHC near $t \approx -0.1$ GeV^2.

3. Glueballs and Oddballs

3.1. Pomeron/odderon trajectories in Central Exclusive Production

Central exclusive diffractive (CED) production continues to attract the attention of both theorists and experimentalists, see *e.g.* Refs.(Fiore et al.,2018, Ewerz et al., 2019) and references therein. This interest is triggered by LHC's high energies, where even the sub-energies at equal partition are sufficient to neglect the contribution from secondary Regge trajectories and consequently CED can be considered as a gluon factory producing exotic particles such as glueballs.

In single-diffraction dissociation or single dissociation (SD) one of the incoming protons dissociates, in double-diffraction dissociation or double dissociation (DD) both protons dissociate, and in central diffraction (CD) or double-Pomeron exchange (DPE) neither proton dissociates. These processes are listed below,

$$\begin{aligned} \text{SD} \quad & pp \to p^*p \\ \text{or} \quad & pp \to pp^* \\ \text{DD} \quad & pp \to p^*p^* \\ \text{CD (DPE)} \quad & pp \to pXp, \end{aligned}$$

where p^* represents a diffractively dissociated proton and X denotes a central system, consisting of meson/glueball resonances. Schematic diagrams are shown in Fig. 1.

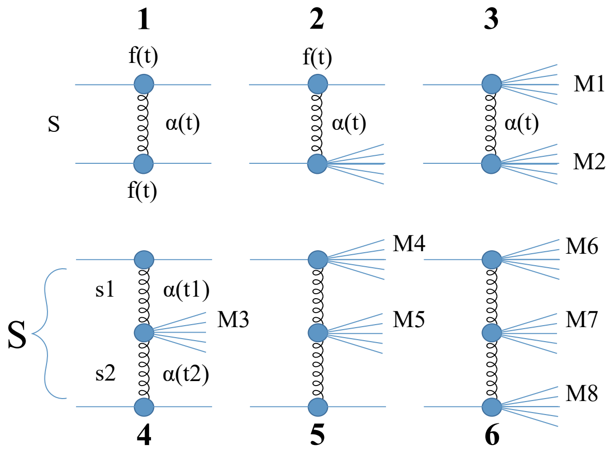

Figure 1: Regge-pole factorization.

The basic sub-process is pomeron-pomeron scattering, producing glueballs lying on the direct-channel pomeron trajectory, with a triple pomeron vertex. Glueball вЂњtowersвЂќ, *i.e.* excited glueball states, called reggeized Breit-Wigner resonances, lie on this trajectory. Crucial for the identification of these states is knowledge of the non-linear complex trajectory, interpolating between negative and positive values of its argument. Its parameters are fitted to the scattering data. While the real part of the trajectory is almost linear, the recovery of the imaginary part, determining the widths of predicted glueballs, is a highly non-trivial problem.

In the present paper, by introducing a new model of the Regge trajectories, both for the pomeron and odderon, we continue studies along the lines of Ref. (Fiore et al., 2016 and 2018) . We first fit the parameters of those trajectories to high-energy elastic scattering data then extrapolate the fitted new trajectories to the particle region to predict the masses and widths of the glueballs and oddballs lying respectively on the pomeron and odderon trajectories.

3.2. Scattering amplitude, cross sections, resonances

In (Fiore et al., 2018) the resonances contribution to pomeron-pomeron (PP) cross section was calculated from the imaginary part of the amplitude by use of the optical theorem

$$\sigma_t^{PP}(M^2) = \Im m\, A(M^2, t=0) = \tag{6}$$

$$= a \sum_{i=f,P} \sum_J \frac{[f_i(0)]^{J+2}\, \Im m\, \alpha_i(M^2)}{(J - \Re e\, \alpha_i(M^2))^2 + (\Im m\, \alpha_i(M^2))^2}, \tag{7}$$

In this Section we concentrate on the pomeron. In this case Eq. (7) reduces to

$$\sigma_t^{PP}(M^2) = a \sum_J \frac{k^{J+2}\, \Im m\, \alpha(M^2)}{(J - \Re e\, \alpha(M^2))^2 + (\Im m\, \alpha(M^2))^2}, \tag{8}$$

where $k = f_i(0)$, and, for simplicity here we set $k = 1$.

We start by comparing the resulting glueball spectra in two ways: first we plot the real and imaginary parts of the trajectory (Chew-Frautchi plot) and calculate the resonances' widths by using the relation (see: e.g. Eq. (18) in R.Fiore et al. 0404021)

$$\Gamma(s = M^2) = \frac{2\Im\alpha(s)}{|\alpha'(s)|}, \tag{9}$$

where $\alpha'(s) = dRe\alpha(\sqrt{s})/d\sqrt{s}$.

3.3. Regge-pole fits to high-energy elastic scattering data

High-energy elastic proton-proton and proton-antiproton scattering, including ISR and LHC energies was successfully fitted with non-linear pomeron trajectories in a number of paper, see (Jenkovszky et al., 2018) and references therein. Since here we are interested in the parametrization of the pomeron (and odderon) trajectories, dominating the LHC energy region, we concentrate on the LHC data, where secondary trajectories can be completely ignored in the near forward direction.

While at lower energies, e.g. at the ISR, the diffraction cone shows almost perfect exponential behavior corresponding to a linear pomeron trajectory in a wide span of $0 < -t < 1.3$ GeV^2, violated only by the "break" near $t \approx -0.1$ GeV^2, at the LCH it is almost immediately followed by another structure, namely by the dip at $t \approx -0.6$ GeV^2. The dynamic of the dip (diffraction minimum) has been treated fully and successfully (Szanyi, Bence et al, 2019), however those details are irrelevant to the behavior of the pomeron trajectory in the resonance (positive s) region and expected glueballs there, that depend largely on the imaginary part of the trajectory and basically on the threshold singularity in Eq. (3).

In Fig. 2 we show a fit to the low-$|t|$ elastic proton-proton differential cross section data (The TOTEM collaboration, 2018) at 13 TeV with a simple model:

$$A_P(s,t) = a_P e^{b_P t} e^{-i\pi\alpha_P(t)/2} (s/s_{0P})^{\alpha_P(t)}, \tag{10}$$

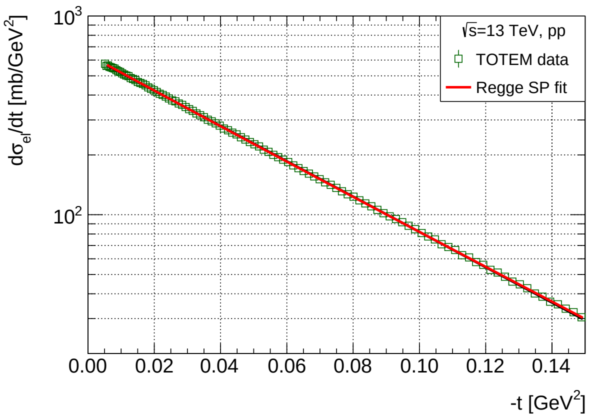

Figure 2: *pp* differential cross section at 13 TeV fitted to the model: Eq. (10) and trajectory Eq. (3)

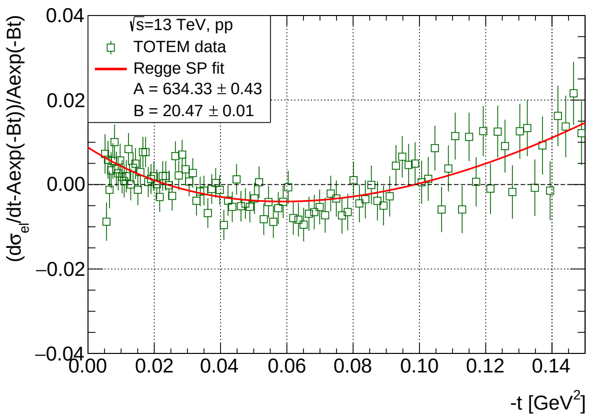

Figure 3: Normalized *pp* differential cross section at 13 TeV fitted to the model Eq. (10) with trajectory Eq. (3)

where $\alpha_P(t)$ is given by Eq. (3) (changing variable s to variable t). We used the norm:

$$\frac{d\sigma}{dt} = \frac{\pi}{s^2}|A_P(s,t)|^2. \tag{11}$$

Fig. 3 shows the normalized form of the differential cross section (used by TOTEM (The TOTEM collaboration, 2018)) illustrating the low-$|t|$ "break" phenomenon (Jenkovszky et al., 2018) related to the non-linear square root term in the pomeron trajectory. However, it should be also noted that the "break" may result from the two-pion threshold both in the trajectory and the non-exponential residue, as discussed in (Jenkovszky et al., 2018).

Fits with the SP model is quite simple. However, they are not sensitive to the odderon and thus are not suitable to predict oddballs. A more advanced, DP model was considered in (Szanyi et al., 2019) with the result shown in the figure below.

3.4. Glueball and Oddball spectroscopy

The non-linear trajectory can be applied to glueballs and oddballs (Szanyi et al., 2019). The real and imaginary part of the pomeron trajectory obtained by the SP (simple model of scattering amplitude) and DP (double-pole model) fits to high energy elastic scattering data are shown in Fig. 4 and Fig. 5. Fig. 4 shows also the predicted glueballs (with their widths) lying on the on the pomeron trajectory. The slope of trajectory is shown in Fig. 6.

The predicted pomeron component of the PP total cross section with the calculated ratios of neighboring resonances' widths both SP and DP case are shown in Fig. 7.

Table 1: Fitted parameters of the pomeron and odderon trajectories, see (Szanyi et al., 2019).

	Pomeron
a	1.08009 ± 0.00005
b [GeV^{-2}]	0.2980 ± 0.0021
c [GeV^{-1}]	0.02467 ± 0.00128

Table 2: Fitted parameters of the pomeron and odderon trajectories in the DP model, (Szanyi et al., 2019).

	Pomeron	Odderon
a	1.04592 ± 0.00005	1.6131 ± 0.0020
b [GeV^{-2}]	0.3042 ± 0.0009	0.1987 ± 0.0010
c [GeV^{-1}]	0.05880 ± 0.00046	0.08483 ± 0.00155

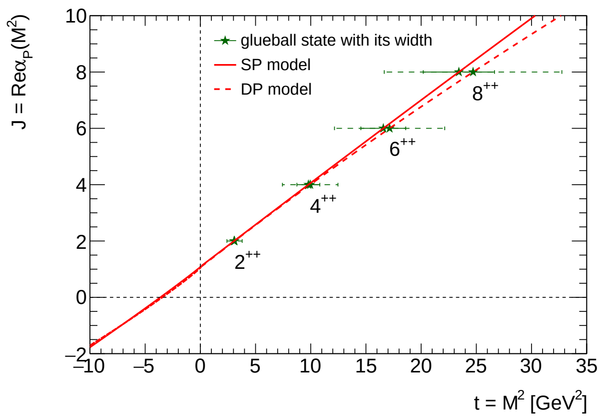

Figure 4: Real part of the pomeron trajectory Eq. (3) both for the SP and DP models as functions of t. The widths of resonances (glueballs) are shown as horizontal error bars.

Oddballs, resonances made of three gluons have the same right of existence as glueballs made of two gluons. Oddballs are expected to lie on the odderon trajectory

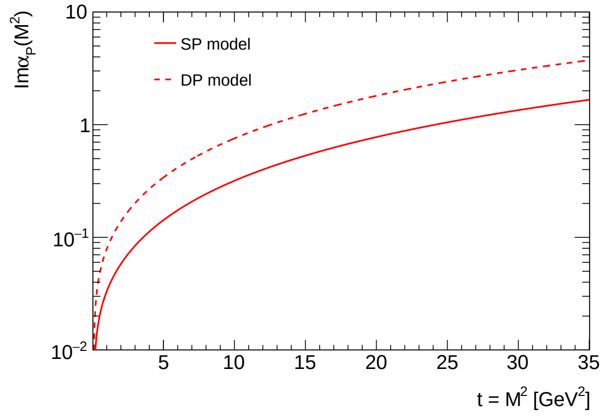

Figure 5: Imaginary part of the pomeron trajectory Eq. (3) both for SP and DP models as functions of t.

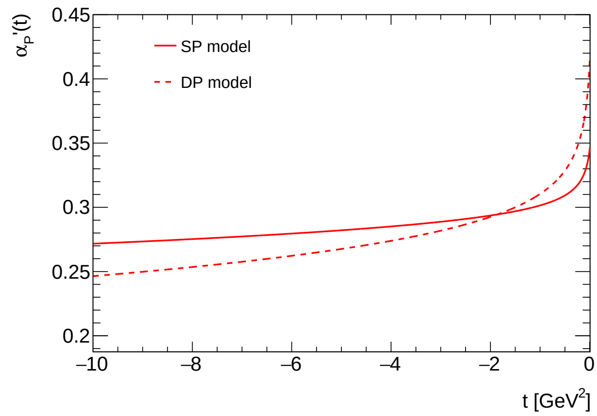

Figure 6: Slope of the pomeron trajectory Eq. (3) both for SP and DP models as functions of t.

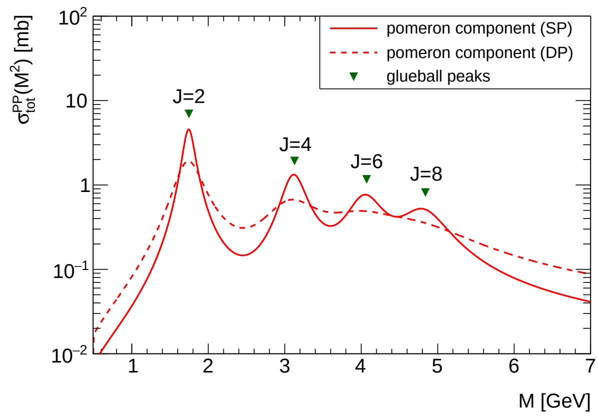

Figure 7: The pomeron component in PP total cross section for two different amplitude models.

exactly in the same way as glueballs lie on the pomeron trajectory.

The real part of the odderon trajectory are shown in Figs. 8. The predicted oddballs (with their widths)

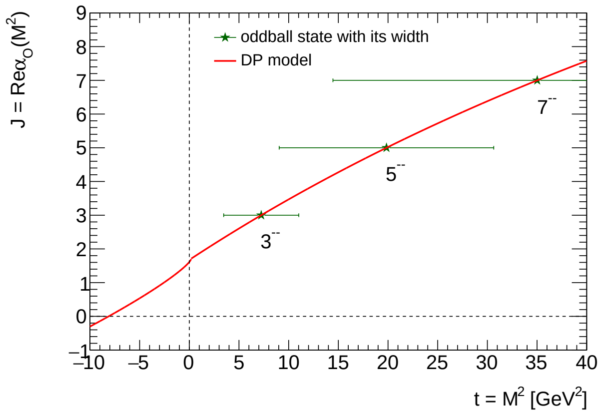

Figure 8: Real part of the odderon trajectory Eq. (3) as function of t. The widths of resonances (oddballs) are shown as horizontal error bars.

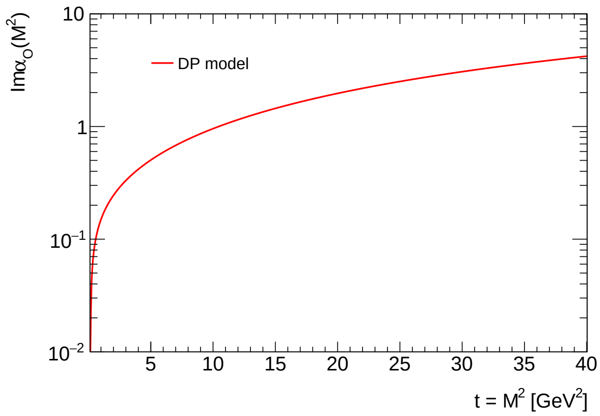

Figure 9: Imaginary part odderon of the trajectory Eq. (3) both for SP and DP models as functions of t.

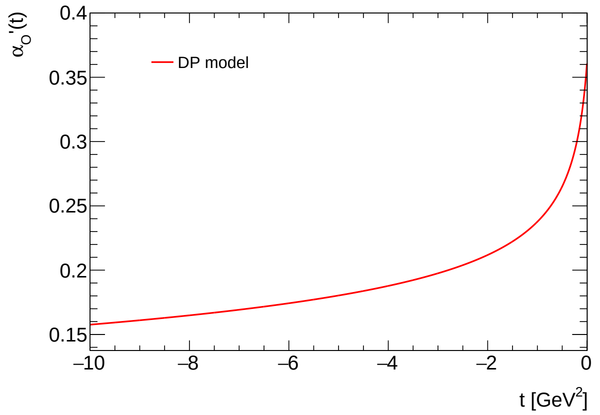

Figure 10: Slope of the odderon trajectory Eq. (3) both for SP and DP models as functions of t.

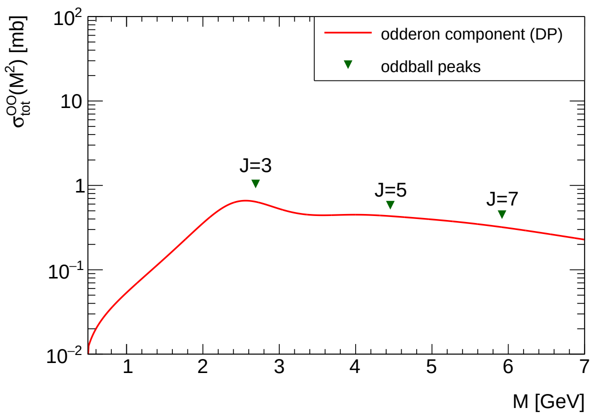

Figure 11: The odderon component in the total cross section for two different models.

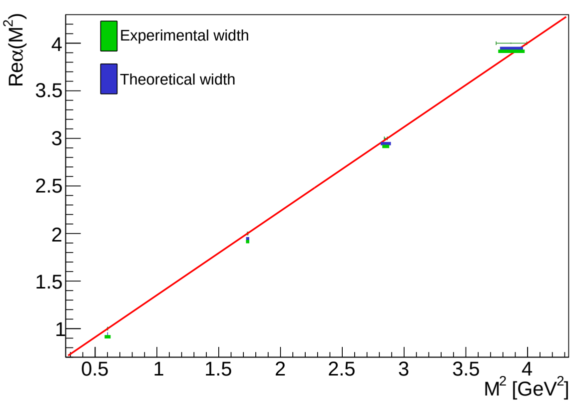

Figure 12: Fit of the non-linear *rho* trajectory. Here $s_0 = 4m_\pi^2$.

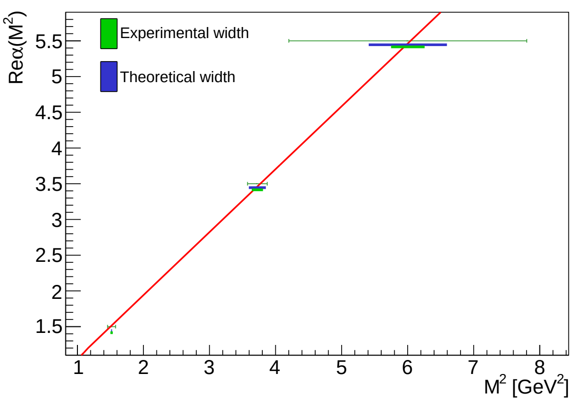

Figure 13: Fit of the Δ-trajectory trajectory, $s_0 = (m_\pi + m_p)^2$.

lying on the on the odderon trajectory are also shown.

4. Mesons and baryons

The trajectory introduced in this paper can be applied also to studies or ordinary meson and baryon spectra (Chew-Frautchi plot). Figs 12 and 13. The theoretical (calculated using non-linear trajectory) and experimental widths are shown as red and green lines, respectively. Note that the mesons and baryon‘s widths data is available, which is opposite to the pomeron case, which allows making more precise fits.

Data about mesons and baryons were taken from *M. Tanabashi et al. (Particle Data Group), Phys. Rev. D 98, 030001 (2018) and 2019 update.*

Table 3: Values of the parameters of the meson and baryon trajectories, calculated from corresponding fits.

	$\rho - mesons$	$\Delta - baryons$
a	0.466118 ± 0.001389	0.167111 ± 0.116851
b	0.880401 ± 0.002208	0.858974 ± 0.057756
c	0.017517 ± 0.000869	0.027311 ± 0.004087

5. Conclusions

Trajectory as Eq. (3) opens a new avenue in hadron spectroscopy (Chew-Frautchi plot), applicable both to ordinary mesons and baryons as well as to glueballs lying on the pomeron and odderon trajectories. Work in this direction is in progress. Appended by unitary symmetry, the Chew-Frautchi plot will remains a powerful tool in hadron spectroscopy.

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