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2-FIELD MODEL OF DARK ENERGY WITH CANONICAL AND NON-CANONICAL KINETIC TERMS

O. Sergijenko

Astronomical Observatory, Taras Shevchenko National University of Kyiv, Observatorna str. 3, Kyiv, 04053, Ukraine, *olga.sergijenko.astro@gmail.com*

ABSTRACT. For some parametrizations of the dark energy equation of state that varies in time there is transition from quintessence to phantom or vice versa at a certain redshift. Quintom – the 2-field model with 2 canonical kinetic terms (one with the "+" sign for quintessence and one with the "-" sign for phantom) and a potential $U(\phi,\xi)$ in Lagrangian – is one of the most popular scalar field models allowing for such behavior. We generalize quintom to include the tachyonic kinetic term along with the classical one. For such a model we obtain the expressions for energy density and pressure. For the spatially flat, homogeneous and isotropic Universe with Friedmann-Robertson-Walker metric of 4-space we derive the equations of motion for the fields. We discuss in detail the reconstruction of the scalar fields potential $U(\phi,\xi)$. Such a reconstruction cannot be done unambiguously, so we consider 3 simplest forms of $U(\phi,\xi)$: the product of $\Phi(\phi)$ and $\Xi(\xi)$, the sum of $\Phi(\phi)$ and $\Xi(\xi)$ and the sum of $\Phi(\phi)$ and $\Xi(\xi)$ to the κth power. The second additional assumption that should be made is about the dependence of either kinetic term X_ϕ or X_ξ on the scale factor a. For each case we obtain the reconstructed potentials in the parametric form. If it is possible to invert dependences of the fields ϕ and ξ on the scale factor a and obtain the analytical expressions for $a(\phi)$ and $a(\xi)$ then we can find the potentials $U(\phi,\xi)$ in explicit form. From the obtained explicit expressions it is clear that they are not suitable for practical use for the multicomponent cosmological models with realistic parametrizations of the dark energy equation of state crossing -1. On the other hand, the parametric dependences which define the potential $U(\phi,\xi)$ are suitable for multicomponent cosmological models and all parametrizations of the dark energy equation of state.

Keywords: Cosmology: dark energy.

АБСТРАКТ. Для ряду параметризацій рівняння стану темної енергії, що змінюється з часом, на деякому червоному зміщенні є можливим перехід від квінтесенції до фантома або навпаки. Квінтом – 2-польова модель з 2 канонічними кінетичними членами (один зі знаком "+" для квінтесенції та один зі знаком "-" для фантома) та потенціал $U(\phi,\xi)$ в лагранжіані – є однією з найпопулярніших скалярно-польових моделей, що дозволяють таку поведінку. Ми пропонуємо узагальнення квінтома, що включає тахіонний кінетичний член поряд з класичним. Для такої моделі ми отримуємо вирази для густини енергії та тиску темної енергії. Для просторово плоского однорідного ізотропного Всесвіту з метрикою 4-простору Фрідмана-Робертсона-Уокера ми виводимо рівняння руху для полів. Ми детально обговорюємо реконструкцію потенціалу скалярних полів $U(\phi,\xi)$. Така реконструкція не може бути зроблена однозначно, отже, ми розглядаємо 3 найпростіші форми $U(\phi,\xi)$: добуток $\Phi(\phi)$ та $\Xi(\xi)$, сума $\Phi(\phi)$ та $\Xi(\xi)$ та сума $\Phi(\phi)$ та $\Xi(\xi)$ у степені κ. Друге додаткове припущення, яке необхідно зробити, стосується залежності кінетичного члена X_ϕ або X_ξ від масштабного множника a. Для кожної з комбінацій цих 2 припущень ми отримуємо реконструйовані потенціали в параметричній формі. В тих випадках, коли можливо обернути залежності полів ϕ та ξ від масштабного множника а та отримати аналітичні вирази для $a(\phi)$ та $a(\xi)$, ми можемо знайти потенціали $U(\phi,\xi)$ в явній формі. З отриманих явних виразів очевидно, що вони не підходять для практичного використання для багатокомпонентних космологічних моделей з реалістичними параметризаціями рівняння стану темної енергії, що переходять через -1. З іншого боку, параметричні залежності, що визначають потенціал $U(\phi,\xi)$, є придатними для багатокомпонентних космологічних моделей та всіх параметризацій рівняння стану темної енергії.

Ключові слова: Космологія: темна енергія.

1. Introduction

20 years ago the accelerated expansion of the Universe was discovered. The cosmological constant in Einstein equations (equation of state parameter $w=-1$)

is the simplest explanation of it. The dark energy in form of a scalar field is the most popular alternative to Λ. Its equation of state parameter can either be constant or vary in time. Dark energy with $w > -1$ is called quintessence, with $w < -1$ – phantom. For several widely used parametrizations of $w(z)$, e.g. Chevallier & Polarski (2001) and Linder (2003) (CPL), Komatsu et al. (2009) (WMAP5), at a certain redshift there is transition from quintessence to phantom or vice versa. Such behavior is forbidden for a single minimally coupled scalar field, as it was shown for the first time by Vikman A. (2005) (see also Easson D.A. & Vikman A. (2016)). However, crossing of the phantom divide is possible in the cases of kinetic gravity braiding (Deffayet C. et al. (2010)), sound speed vanishing in phantom domain (Creminelli P. et al. (2009)) or nonminimal couplings (Amendola L. (2000), Pettorino V. & Baccigalupi C. (2008)).

The most popular scalar field model allowing for $w = -1$ crossing is quintom proposed by Feng B. et al. (2005). It is the 2-field model with 2 canonical kinetic terms – one with the "+" sign for quintessence and one with the "-" sign for phantom – and a potential $U(\phi, \xi)$ in Lagrangian.

Quintom can be generalized to include a non-canonical kinetic term. The simplest physically motivated Lagrangian with the non-canonical kinetic term is the tachyon one. So, we propose the 2-field model of dark energy with classical and tachyonic kinetic terms.

2. 2-field model with classical and tachyonic kinetic terms

We consider the spatially flat, homogeneous and isotropic Universe with Friedmann-Robertson-Walker (FRW) metric of 4-space

$$ds^2 = g_{ij}dx^i dx^j = a^2(\eta)(d\eta^2 - \delta_{\alpha\beta}dx^\alpha dx^\beta) \quad (1)$$

(here $i, j = 0, 1, 2, 3$, $\alpha, \beta = 1, 2, 3$, a is the scale factor, η is the conformal time and $c = 1$). The Universe is filled with relativistic (radiation, neutrinos), non-relativistic (baryons, dark matter) matter and dark energy. The latter is modeled by 2 scalar fields with the Lagrangian:

$$L = -X_\phi - U(\phi, \xi)\sqrt{1 - 2X_\xi}, \quad (2)$$

where

$$X_\phi = \frac{1}{2}\phi_{,i}\phi^{,i} = \frac{\dot{\phi}^2}{2},$$
$$X_\xi = \frac{1}{2}\xi_{,i}\xi^{,i} = \frac{\dot{\xi}^2}{2}$$

are the kinetic terms. This Lagrangian is classical (Klein-Gordon) with respect to the field ϕ which corresponds to phantom and tachyon (Dirac-Born-Infeld) with respect to the field ξ which corresponds to quintessence.

The energy density and pressure for such a model are as follows:

$$\rho_{de} = -X_\phi + \frac{U(\phi, \xi)}{\sqrt{1 - 2X_\xi}}, \quad (3)$$
$$p_{de} = -X_\phi - U(\phi, \xi)\sqrt{1 - 2X_\xi}. \quad (4)$$

The dark energy equation of state (EoS) parameter is defined as $w(a) = p_{de}/\rho_{de}$.

For the metric (1) the Lagrangian (2) yields the following equations of motion:

$$\ddot{\phi} + 2aH\dot{\phi} - \frac{\partial U}{\partial \phi}a^2\sqrt{1 - \frac{\dot{\xi}^2}{a^2}} = 0, \quad (5)$$

$$\ddot{\xi} + 2aH\dot{\xi} - 3aH\dot{\xi}\frac{\dot{\xi}^2}{a^2} + \frac{1}{U}\left(\frac{\partial U}{\partial \phi}\dot{\phi}\dot{\xi} + a^2\frac{\partial U}{\partial \xi}\right)\left(1 - \frac{\dot{\xi}^2}{a^2}\right) = 0. \quad (6)$$

Here a dot denotes the derivative with respect to η and $H \equiv \dot{a}/a$ is the Hubble parameter.

From (3)-(4) it is clear that reconstruction of the potential $U(\phi, \xi)$ cannot be done unambiguously and requires additional assumptions. First of all, we should choose a form of $U(\phi, \xi)$. We restrict our consideration to 3 simplest ansatzes from Andrianov et al. (2008):

- $U(\phi, \xi) = \Phi(\phi)\Xi(\xi)$,
- $U(\phi, \xi) = \Phi(\phi) + \Xi(\xi)$ and
- $U(\phi, \xi) = [\Phi(\phi) + \Xi(\xi)]^\kappa,\ \kappa = const.$

Secondly, we should assume either X_ϕ or X_ξ to be a known function of the scale factor. Then for $X_\xi = \alpha(a)$ we get:

$$U(a) = \frac{1}{2}\frac{1 - w}{1 - \alpha}\sqrt{1 - 2\alpha}\rho, \quad (7)$$
$$X_\phi(a) = -\frac{1}{2}\frac{1 - 2\alpha + w}{1 - \alpha}\rho \quad (8)$$

and for $X_\phi = \beta(a)$:

$$U(a) = \sqrt{-(\rho w + \beta)(\rho + \beta)}, \quad (9)$$
$$X_\xi(a) = \frac{1}{2}\frac{\rho(1 + w) + 2\beta}{\rho + \beta}. \quad (10)$$

Dependences of the fields ϕ and ξ on the scale factor a are determined from $X_\xi = \alpha$ and (8):

$$\xi(a) = \pm\int \frac{da}{aH}\sqrt{2\alpha}, \quad (11)$$

$$\phi(a) = \pm\int \frac{da}{aH}\sqrt{-\frac{1}{2}\frac{1 - 2\alpha + w}{1 - \alpha}\rho} \quad (12)$$

or $X_\phi = \beta$ and (10):

$$\phi(a) = \pm\int \frac{da}{aH}\sqrt{2\beta}, \qquad (13)$$

$$\xi(a) = \pm\int \frac{da}{aH}\sqrt{\frac{1}{2}\frac{\rho(1+w)+2\beta}{\rho+\beta}}. \qquad (14)$$

These expressions together with either (7) or (9) define $U(\phi)$ in the parametric form. This allows us to reconstruct the potential even if the integrals in (11)-(14) cannot be solved analytically.

3. Reconstructed potentials in an explicit form

If it is possible to invert the analytical dependences (11)-(12) or (13)-(14) then we can obtain the explicit expressions for potentials (as it has been done for single-field models in e.g. Sergijenko & Novosyadlyj (2008), Novosyadlyj & Sergijenko (2009)). For $U(\phi,\xi) = \Phi(\phi)\Xi(\xi)$, $X_\xi = \alpha$ the potential is reconstructed as:

$$U(\phi,\xi) = \exp\left\{\pm\int d\phi\left[\frac{1}{a}\sqrt{-\frac{(1-\alpha)(1-2\alpha+w)}{\rho}}\right.\right.$$
$$\times\frac{1}{(1-w)(1-1-2\alpha)}\left(\frac{\dot{w}-2\dot{\alpha}}{1-2\alpha+w}+\frac{\dot{\alpha}}{1-\alpha}\right.$$
$$+3aH(1-w))\Big](a(\phi)) \mp \int d\xi\left[\frac{1}{a}\frac{\sqrt{2\alpha}}{1-2\alpha}\left(\frac{1}{2}\frac{\dot{\alpha}}{\alpha}\right.\right.$$
$$+\frac{\dot{\alpha}}{1-\alpha}\frac{1-2\alpha+w}{1-w}+\frac{\dot{w}-2\dot{\alpha}}{1-w}$$
$$+3aH(2-4\alpha+w))\Big](a(\xi))\Big\},$$

for $U(\phi,\xi) = \Phi(\phi)\Xi(\xi)$, $X_\phi = \beta$ it is as follows:

$$U(\phi,\xi) = \exp\left\{\pm\int d\phi\left[\frac{1}{a}\frac{\sqrt{2\beta}}{\rho w+\beta}\left(3aH+\frac{1}{2}\frac{\dot{\beta}}{\beta}\right)\right]\right.$$
$$\times(a(\phi)) \pm \int d\xi\left[\frac{1}{2a}\frac{\sqrt{(\rho(1+w)+2\beta)(\rho+\beta)}}{\rho w+\beta}\right.$$
$$\times\left(\frac{\rho(\dot{w}-3aH(1+w)^2)+2\dot{\beta}}{\rho(1+w)+2\beta}\right.$$
$$\left.\left.\left.+3\frac{aH(1-w)\rho-6aH\beta-\dot{\beta}}{\rho+\beta}\right)\right](a(\xi))\right\}.$$

For $U(\phi,\xi) = \Phi(\phi)+\Xi(\xi)$, $X_\xi = \alpha$ we get:

$$U(\phi,\xi) = \pm\int d\phi\left[\frac{1}{2a}\sqrt{-\frac{1-2\alpha+w}{(1-\alpha)(1-2\alpha)}\rho}\right.$$
$$\left.\times\left(\frac{\dot{w}-2\dot{\alpha}}{1-2\alpha+w}+\frac{\dot{\alpha}}{1-\alpha}+3aH(1-w)\right)\right](a(\phi))$$
$$\mp\int d\xi\left[\frac{1}{2a}\sqrt{\frac{2\alpha}{1-2\alpha}\frac{1-w}{1-\alpha}\rho}\left(\frac{1}{2}\frac{\dot{\alpha}}{\alpha}+\frac{\dot{\alpha}}{1-\alpha}\right.\right.$$
$$\times\frac{1-2\alpha+w}{1-w}+\frac{\dot{w}-2\dot{\alpha}}{1-w}+3aH$$
$$\times(2-4\alpha+w))\Big](a(\xi)),$$

while for $U(\phi,\xi) = \Phi(\phi)+\Xi(\xi)$, $X_\phi = \beta$:

$$U(\phi,\xi) = \pm\int d\phi\left[\frac{1}{a}\sqrt{2\beta}\sqrt{-\frac{\rho+\beta}{\rho w+\beta}}\left(3aH\right.\right.$$
$$\left.\left.+\frac{1}{2}\frac{\dot{\beta}}{\beta}\right)\right](a(\phi)) \pm \int d\xi\left[\frac{1}{2a}\sqrt{-\frac{\rho(1+w)+2\beta}{\rho w+\beta}}\right.$$
$$\times(\rho+\beta)\left(\frac{\rho(\dot{w}-3aH(1+w)^2)+2\dot{\beta}}{\rho(1+w)+2\beta}\right.$$
$$\left.\left.+3\frac{aH(1-w)\rho-6aH\beta-\dot{\beta}}{\rho+\beta}\right)\right](a(\xi)).$$

In the case of $U(\phi,\xi) = [\Phi(\phi)+\Xi(\xi)]^\kappa$, $\kappa = const$, $X_\xi = \alpha$ the potential reads:

$$U(\phi,\xi) = \frac{1}{(2\kappa)^\kappa}\left\{\pm\int d\phi\left[\frac{1}{a}\sqrt{-(1-2\alpha+w)}\rho^{\frac{1}{\kappa}-\frac{1}{2}}\right.\right.$$
$$\times(1-w)^{\frac{1-\kappa}{\kappa}}(1-\alpha)^{\frac{1}{2}-\frac{1}{\kappa}}(1-2\alpha)^{\frac{1}{2\kappa}-1}\left(\frac{\dot{w}-2\dot{\alpha}}{1-2\alpha+w}\right.$$
$$\left.\left.+\frac{\dot{\alpha}}{1-\alpha}+3aH(1-w)\right)\right](a(\phi)) \mp \int d\xi\left[\frac{1}{a}\sqrt{2\alpha}\right.$$
$$\times(1-2\alpha)^{\frac{1}{2\kappa}-1}\left(\frac{1-w}{1-\alpha}\rho\right)^{\frac{1}{\kappa}}\left(\frac{1}{2}\frac{\dot{\alpha}}{\alpha}+\frac{\dot{\alpha}}{1-\alpha}\right.$$
$$\left.\left.\times\frac{1-2\alpha+w}{1-w}+\frac{\dot{w}-2\dot{\alpha}}{1-w}+3aH(2-4\alpha+w)\right)\right]$$
$$\times(a(\xi))\Big\}$$

and $U(\phi,\xi) = [\Phi(\phi)+\Xi(\xi)]^\kappa$, $\kappa = const$, $X_\phi = \beta$ yields:

$$U(\phi,\xi) = \frac{1}{\kappa^\kappa}\left\{\pm\int d\phi\left[\frac{1}{a}\sqrt{2\beta}(\rho+\beta)^{\frac{1}{2\kappa}}\right.\right.$$
$$\left.\times(-(\rho w+\beta))^{\frac{1}{2\kappa}-1}\left(3aH+\frac{1}{2}\frac{\dot{\beta}}{\beta}\right)\right](a(\phi))$$
$$\mp\int d\xi\left[\frac{1}{2a}\sqrt{\rho(1+w)+2\beta}(\rho+\beta)^{\frac{\kappa+1}{2\kappa}}\right.$$
$$\times(-(\rho w+\beta))^{\frac{1}{2\kappa}-1}\left(\frac{\rho(\dot{w}-3aH(1+w)^2)+2\dot{\beta}}{\rho(1+w)+2\beta}\right.$$
$$\left.\left.\left.+3\frac{aH(1-w)\rho-6aH\beta-\dot{\beta}}{\rho+\beta}\right)\right](a(\xi))\right\}.$$

It is clear that for the multicomponent cosmological models with realistic parametrizations of the dark energy EoS crossing −1 (CPL, WMAP5) these expressions are not suitable for practical use since even for $\alpha = const$ or $\beta = const$ the integrals cannot be solved analytically. Thus, to reconstruct the potentials for certain values of cosmological parameters and dependences $w(a)$ and $\alpha(a)$ or $\beta(a)$ we have to use the parametric dependences (7), (11), (12) or (9), (13), (14) which define the potential $U(\phi, \xi)$.

4. Conclusion

In the reconstruction of potential of the proposed 2-field model of dark energy with classical and tachyonic kinetic terms there are 2 ambiguities requiring additional assumptions: about the form of $U(\phi, \xi)$ and about the dependence of a kinetic term on the scale factor. The third ambiguity – a sign in front of the integrals in (11)-(14) – is less important since the potentials with "+" and "-" are symmetric with respect to $\phi = 0$ and $\xi = 0$. For 3 ansatzes for the form of $U(\phi, \xi)$ we obtained the reconstructed potentials in explicit form and in parametric one that is suitable for multicomponent cosmological models and all parametrizations of the dark energy equation of state.

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