DOI: http://dx.doi.org/10.18524/1810-4215.2017.30.114384

THE CHARACTERISTICS OF ELECTRON-NUCLEAR MODEL IN THE DEGENERATE DWARFS THEORY. EQUATION OF STATE

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ABSTRACT. Within the microscopic theory the ground state energy of spatially homogeneous degenerate relativistic subsystem of electrons in the field of stationary nuclei of l-th sorts with charges $z_1, \ldots, z_l$ was calculated. In the two- and three-particle electron correlations approximation the contributions of Coulomb interactions to the equation of model state at low temperatures was analyzed.

Key words: electron-nuclear model, correlation functions, energy of model at $T = 0K$, equation of state.

PACS 97.20Rp, 97.60Bw

1. Introduction

The theory of internal structure of cold dwarfs was developed by S. Chandrasekhar in the 40-th years of the XX century, and it was based on the equation of state of ideal relativistic electron gas at $T = 0K$ [1, 2]. Generalization of this theory followed in the next decades, when in the works of many authors were investigated effects of such important factors as axial rotation [3, 4], Coulomb interactions [5], incomplete degeneration of electron subsystem [6, 7], effects of magnetic fields [8, 9], effects of general relativity [4, 10], processes of neutronization [11], etc. Interpretation of all the diversity of dwarfs properties obtained from the observations of space observatories [12, 13] requires constructing a general theory, which also takes into account the effects of these factors, among which are competing.

The effect of the interactions which play an important role in determining the structure of dwarfs at different masses and luminosities, and especially for the case of massive cold dwarfs is least studied. Basing on Wigner-Seitz, Thomas-Fermi approximations and nonrelativistic random phase approximation corresponding to the approximate accounting of two-particle electron correlations, E. Salpeter [5] showed, that Coulomb interactions lead to a small decrease of pressure of ideal degenerate relativistic electron gas at $T = 0K$, what is still considered as the basis of S. Chandasekhar's theory [9].

Due to the high density ($\sim 10^5$ g/cm^3) a matter in the internal ranges of degenerate dwarfs has the metal electron structure with completely collectivized electrons, and the Fermi momentum is of the order m_0c. Therefore the electron subsystem is weakly non-ideal, allowing to use the perturbation theory with regard to interactions. We have used the reference system approach, which is renormalized perturbation theory, formulated in terms of the n-particle correlation functions of the ideal as well as the interacting relativistic electron gas [14, 15].

2. The general relations

We consider a spatially homogeneous in the macroscopic sense electrically neutral model, which consists of N_e electrons and nuclei of l-th sorts (N_1 nuclei of charge $z_1, \ldots, N_l$ nuclei of charge z_l) in the volume V in thermodynamic limit $N_e,\ V \to \infty,\ N_e/V = const$ at the temperatures much lower than the temperature of the electron subsystem degeneration. The Hamiltonian of this model is

$$\hat{H} = \hat{H}_0 + \hat{V}_{ee} + \sum_{i=1}^{l} \hat{V}_{en}^{i} + \hat{V}_{nn}, \tag{1}$$

in which

$$\hat{H}_0 = \sum_{\mathbf{k},s} E_k \, a^{+}_{\mathbf{k},s} a_{\mathbf{k},s} \tag{2}$$

is the Hamiltonian of free electrons ($E_k = \{(m_0c^2)^2 + \hbar^2k^2c^2\}^{1/2} - m_0c^2$),

$$\hat{V}_{ee} = (2V)^{-1} \sum_{\mathbf{q}\neq 0} V_q \sum_{\mathbf{k}_1,s_1} \sum_{\mathbf{k}_2,s_2} \times \\ \times a^{+}_{\mathbf{k}_1+\mathbf{q},s_1} a^{+}_{\mathbf{k}_2-\mathbf{q},s_2} a_{\mathbf{k}_2,s_2} a_{\mathbf{k}_1,s_1} \tag{3}$$

is the operator of electron interactions,

$$\hat{V}_{en}^{i} = -V^{-1} z_i \sum_{\mathbf{q}\neq 0} V_q \, S^{(i)}_{-\mathbf{q}} \, \hat{\rho}_{\mathbf{q}},\ \hat{\rho}_{\mathbf{q}} = \sum_{\mathbf{k},s} a^{+}_{\mathbf{k}+\mathbf{q},s} a_{\mathbf{k},s} \tag{4}$$

is the operator of electron interactions with i-th nuclear subsystem,

$$\hat{V}_{nn} = (2V)^{-1} \sum_{\mathbf{q}\neq 0} V_q \sum_{i,j=1}^{l} z_i z_j \{S_{\mathbf{q}}^{(i)} S_{-\mathbf{q}}^{(j)} - N_i \delta_{i,j}\} \quad (5)$$

is the sum of direct nuclear interactions. Here $V_q = 4\pi e^2/q^2$, $S_{\mathbf{q}}^{(i)} = \sum\limits_{j=1}^{N_i} \exp\left[i(\mathbf{q}, \mathbf{R}_j^i)\right]$ is the structure factor of i-th nuclear subsystem, $a_{\mathbf{k},s}^{+}, a_{\mathbf{k},s}$ is the generation and destruction operators of electrons in quantum states with the given vector $\mathbf{k}$ and the spin orientation s, $\mathbf{R}_j^i$ is the radius-vector of j-th nucleus with the charge z_i.

We have used the subsystem of free relativistic electrons as the basis for calculating energetic and structure characteristics of model of interacting degenerate relativistic electron gas. In its turn the interacting electron gas is the basis system for consideration the electron-nuclear interactions. Energy of model with the Hamiltonian (1) is represented as the expansion by the correlation functions of model of interacting electron gas, that is:

$$\begin{aligned} E = E_e + \frac{1}{2V} \sum_{\mathbf{q}\neq 0} V_q \sum_{i,j=1}^{l} z_i z_j \{S_{\mathbf{q}}^{(i)} S_{-\mathbf{q}}^{(j)} - N_i \delta_{i,j}\} - \\ - \sum_{n\geq 2} \{n! V^n\}^{-1} \sum_{i_1,i_2,\cdots,i_n=1}^{l} z_{i_1} z_{i_2} \cdots z_{i_n} \times \\ \times \sum_{\mathbf{q}_1,\mathbf{q}_1,\cdots,\mathbf{q}_n\neq 0} V_{q_1} \cdots V_{q_n} S_{-\mathbf{q}_1}^{(i_1)} \cdots S_{-\mathbf{q}_n}^{(i_n)} \times \\ \times \delta_{\mathbf{q}_1+\cdots+\mathbf{q}_n,0}\, \tilde{\mu}_n(\mathbf{q}_1 \cdots \mathbf{q}_n|0). \end{aligned} \quad (6)$$

Here E_e is the ground state energy of basis system of interacting relativistic electron gas, and $\tilde{\mu}_n(\mathbf{q}_1 \cdots \mathbf{q}_n|0)$ is the static limits of n-particle correlation functions of model of interacting electron gas. The modern theory of n-particle statics and dynamics correlation functions of the non-relativistic electron gas is developed in the work [14, 15] and based on the basis approach. One version of this approach is the so-called post RPA, in which the polarization operator is represented as

$$\begin{aligned} M_2(\mathbf{q}, -\mathbf{q}|0) = \tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|0) \Big[1 + \frac{V_q}{V} \times \\ \times \tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|0) G(q) \Big], \end{aligned} \quad (7)$$

where $G(q)$ is the static local field correction function.

This approach can be applied not only for weak non-ideal systems, but also for the systems with intermediate non-ideality [15]. In this approach

$$\mu_2(\mathbf{q}, -\mathbf{q}|0) = M_2(\mathbf{q}, -\mathbf{q}|0) \Big[1 + \frac{V_q}{V} M_2(\mathbf{q}, -\mathbf{q}|0) \Big]^{-1}. \quad (8)$$

Taking into account that the model we consider is non-ideal, for three-particle function we use the approximation RPA, in which

$$\begin{aligned} \mu_3(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3|0) = \tilde{\mu}_3^0(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3|0) \prod_{i=1}^{3} \varepsilon_{RPA}^{-1}(q_i), \\ \varepsilon_{RPA}(q) = 1 + \frac{V_q}{V} \tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|0). \end{aligned} \quad (9)$$

The functions $\tilde{\mu}_n(\mathbf{q}_1 \cdots \mathbf{q}_n|0)$ are the partial case of dynamic correlation functions $\tilde{\mu}_n(\mathbf{q}_1 \cdots \mathbf{q}_n|\nu_1, \cdots, \nu_n)$, which are the spectral representation of n-particle correlation functions, given in the coordinate space. For example, the binary distribution electron function $F_2(\mathbf{r})$ is connected with the function $\tilde{\mu}_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu)$ by expression

$$\begin{aligned} F_2(\mathbf{r}) = 1 + [\beta N_e (N_e - 1)]^{-1} \sum_{\nu} \sum_{\mathbf{q}\neq 0} \times \\ \times \tilde{\mu}_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu) \exp\left[i(\mathbf{q}, \mathbf{r})\right]. \end{aligned} \quad (10)$$

where $\nu = 2\pi n \beta^{-1}$; $n = 0; \pm 1; \pm 2; \cdots$ is the Bose-Matsubara frequency. Therefore the energy of basis system of interacting electron gas it is convenient to calculate by expression

$$E_e = E_0 + (2\beta V)^{-1} \sum_{\nu} \sum_{\mathbf{q}\neq 0} V_q \int_0^1 \tilde{\mu}_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu|\lambda) d\lambda, \quad (11)$$

where $\tilde{\mu}_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu|\lambda)$ is the dynamic two-particle correlation function of auxiliary model of electrons with the potential of interaction λV_q:

$$\begin{aligned} \tilde{\mu}_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu|\lambda) = M_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu|\lambda) \Big\{ 1 + \lambda \frac{V_q}{V} \times \\ \times M_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu|\lambda) \Big] \Big\}^{-1}, \\ M_2(\mathbf{q}, -\mathbf{q}|\nu, -\nu|\lambda) = \tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|\nu, -\nu) \Big[1 + \lambda \frac{V_q}{V} \times \\ \times \tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|\nu, -\nu) G(q, \nu) \Big]. \end{aligned} \quad (12)$$

It has been taken into account, that in the case of weak non-ideality of model the local field correction function does not depend on "coupling constant".

The static and dynamic correlation functions of non-relativistic ideal electron gas are well known. The analytical expression for $\tilde{\mu}_2^0(y, -y)$ was obtained in [16], the functions $\tilde{\mu}_3^0(\mathbf{q}_1, \mathbf{q}_2, -\mathbf{q}_1 - \mathbf{q}_2|0, 0, 0)$ and $\tilde{\mu}_4^0(\mathbf{q}_1, -\mathbf{q}_1, \mathbf{q}_2, -\mathbf{q}_2|0, \cdots, 0)$ were calculated in [17, 19]. The dynamic functions $\tilde{\mu}_3^0(y_1, y_2, -y_1 - y_2)$ and $\tilde{\mu}_4^0(y_1, -y_1, y_2, -y_2)$ were first calculated in [14]. Microscopic theory of the local field correction function, which is based on the summation of infinite series of convergent diagrams and created on the

functions $\tilde{\mu}_2^0(\mathbf{q},-\mathbf{q}|\nu,-\nu)$, $\tilde{\mu}_3^0(\mathbf{q}_1,\mathbf{q}_2,\mathbf{q}_3|\nu_1,\nu_2,\nu_3)$ and $\tilde{\mu}_4^0(\mathbf{q}_1,-\mathbf{q}_1,\mathbf{q}_2,-\mathbf{q}_2|\nu_1,-\nu_1,\nu_2,-\nu_2)$, are developed in the works [14, 20].

Correlation functions of relativistic interacting electron gas also satisfy the expressions (7)-(10), (12). In general they are similar to the relevant functions of non-relativistic model, but they have a significant dependence on the relativistic parameter $x = \hbar k_F (m_0 c)^{-1}$ ($k_F = (3\pi^2 N_e/V)^{1/3}$), and the calculation $\tilde{\mu}_n^0(\mathbf{q}_1,\cdots\mathbf{q}_n|0)$ at $n \geq 3$ is difficult, because the electron spectrum is not a quadratic function of the wave vector. The function $\tilde{\mu}_2^0(\mathbf{q},-\mathbf{q}|0)$ has the analytical representation [22]:

$$\tilde{\mu}_2^0(\mathbf{q},-\mathbf{q}|0) = 2\sum_{\mathbf{k},s} n_{\mathbf{k},s}\{E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}}\}^{-1} =$$

$$= \frac{3N_e}{m_0 c^2 x^2} J_2(q_*,0|x);$$

$$q_* x\, J_2(q_*,0|x) = \frac{2}{9}(R_+ - R_-)\left[1 + \frac{7}{4}x^2 - \frac{q_*^2}{8}\right] +$$

$$+ \frac{5}{72} q_* x (R_+ + R_-) + \frac{q_* x}{12} R_0 + \frac{1}{3} R_0^3 \times$$

$$\times \ln\left|\frac{R_+ - R_0}{R_- - R_0}\right| + \frac{1}{8} q_* \left(1 + \frac{q_*^2}{6}\right)\left\{2\ln|x + R_0| -\right.$$

$$\left. - \ln|(R_+ + x + q_*)(R_- + x - q_*)|\right\} + \frac{1}{6} S_q^3 \times \qquad (13)$$

$$\times \left\{\ln\left|\frac{1 + \frac{1}{2}q_*^2 + \frac{1}{2}x q_* + S_q R_+}{1 + \frac{1}{2}q_*^2 - \frac{1}{2}x q_* + S_q R_-}\right| -\right.$$

$$\left. - \ln\left|\frac{1 + \frac{1}{2}q_* x + S_q R_0}{1 - \frac{1}{2}q_* x + S_q R_0}\right| - 2\ln\left|\frac{x + \frac{1}{2}q_*}{x - \frac{1}{2}q_*}\right|\right\},$$

$$R_0 = (1 + x^2)^{1/2}, \quad S_q = \left(1 + \frac{1}{4}q_*^2\right)^{1/2},$$

$$R_\pm = [1 + (q_* \pm x)^2]^{1/2}, \quad q_* = \frac{|\mathbf{q}|x}{k_F}.$$

The behavior of the dimensionless factor $J_2(q_*,0|x)$ is illustrated in fig. 1. As in the non-relativistic case, the function $J_2(q_*,0)$ has a weak logarithmic peculiarity of the type $\left(x - \frac{1}{2}q_*\right)\ln\left|x - \frac{1}{2}q_*\right|$ in the vicinity to $q_* = 2x$ ($|\mathbf{q}| = 2k_F$) and the asymptotics:

$$\tilde{\mu}_2^0(\mathbf{q},-\mathbf{q}|0) \Rightarrow \begin{cases} \dfrac{3N_e(1+x^2)^{1/2}}{m_0 c^2 x^2} + \cdots & \text{by } q \to 0; \\ \dfrac{2N_e}{m_0 c^2 q_*} + \cdots & \text{by } q \to \infty. \end{cases} \qquad (14)$$

The function $\tilde{\mu}_3^0(\mathbf{q},-\mathbf{q},0|0)$, which is the partial case of the static three-particle function at $\mathbf{q}_2 = -\mathbf{q}_1$, has the simple analytical representation:

$$\tilde{\mu}_3^0(\mathbf{q},-\mathbf{q},0|0) = \frac{3N_e}{(m_0 c^2 x^2)^2} J_3(q,0|x),$$

$$J_3(q,0|x) = \frac{R_0}{\tilde{q}}\left\{\tilde{R}_+ - \tilde{R}_- + R_0 \ln\left|\frac{\tilde{R}_+ - R_0}{\tilde{R}_- - R_0}\right|\right\}, \qquad (15)$$

$$\tilde{R}_\pm = [1 + x^2(1 \pm \tilde{q})^2]^{1/2}, \quad R_0 = (1 + x^2)^{1/2}.$$

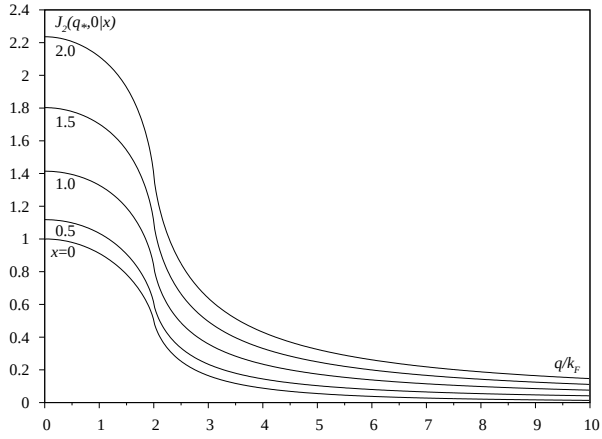

Figure 1: Dependence of the function $J_2(q_*,0|x)$ on the wave vector $\mathbf{q}$ at different values of the relativistic parameter.

In the formula (15) "non-relativistic" scale was used for the wave vector ($\tilde{q} \equiv |\mathbf{q}|/k_F$). In the long-wave limit

$$\lim_{\mathbf{q}\to 0} \tilde{\mu}_3^0(\mathbf{q},-\mathbf{q},0|0) \Rightarrow \frac{3N_e}{(m_0 c^2 x^2)^2}(1 + 2x^2). \qquad (16)$$

Dependence of dimensionless factor $J_3(q,0|x)$ on the wave vector and the relativistic parameter is illustrated in fig. 2. As in non-relativistic case, the function (15) has a logarithmic peculiarity at $q = 2k_F$.

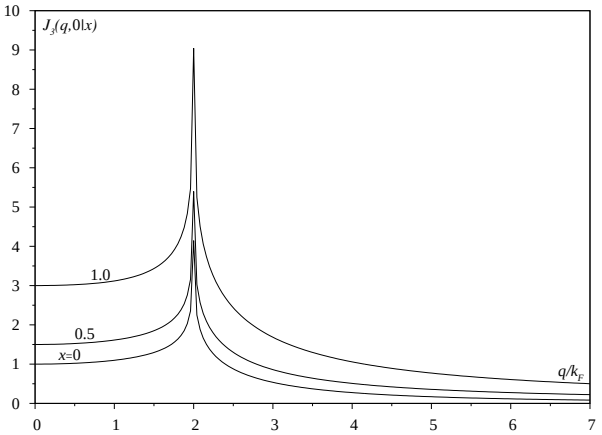

Figure 2: Dependence of the static function $J_3(q,0|x)$ on the wave vector $\mathbf{q}$ at different values of the relativistic parameter.

The formulae (13)-(15), and the fig. 1, 2 reveal the general property of correlation functions $\tilde{\mu}_n^0(y_1,\cdots,y_n)$ – steep decrease in the region of wave vectors ($|\mathbf{q}_i| > 2k_F$), providing convergence of integrals in the expand (6). In general for a rough estimate convergence of series (6) we consider a chemically homogeneous model ($z_i = z$, $z\sum_{i=1}^{l} N_i = N_n$), constrain

the integration for each independent vector $\mathbf{q}_i$ of the area $|\mathbf{q}_i| < 2k_F$, neglect screening interactions, replace the product of structural factors $S_{\mathbf{q}_1}, S_{\mathbf{q}_2} \cdots S_{\mathbf{q}_n}$ with N_n, and the functions $\tilde{\mu}_n^0(\mathbf{q}_1, \cdots \mathbf{q}_n|0)$ we replace with $3N_e(m_0c^2x^2)^{1-n}(1+x^2)^{\frac{1}{2}(n-1)}$, which approximately corresponds to the long-wave asymptotic. For the magnitude of n-th member of series (6) we obtain the estimate

$$N_e m_o c^2 z^{n-1} \alpha_0^n x^{2-n} (1+x^2)^{\frac{1}{2}(n-1)}, \tag{17}$$

where $\alpha_0 = e^2/\hbar c$ is the fine structure constant. Hence, the series (6) is expansion for dimensionless parameter $z\alpha_0$, which varies from 0,014 (helium dwarf) to 0,19 (iron dwarf). For the typical dwarfs, mainly consisting of nitrogen and oxygen, $z\alpha_0 \approx 0{,}1$. That is expansion parameter is a small value, which makes it possible to restrict ourselves to consideration of two- and three-electron correlations (we note, that correlation energy of basis system has the order α_0^2). Moreover, the three-electron function

$$\begin{aligned}
&\tilde{\mu}_3^0(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3|0) = \delta_{\mathbf{q}_1+\mathbf{q}_2+\mathbf{q}_3,0}\{\gamma_3(\mathbf{q}_1, -\mathbf{q}_2) + \\
&+ \gamma_3(\mathbf{q}_2, -\mathbf{q}_3) + \gamma_3(\mathbf{q}_3, -\mathbf{q}_1)\}, \\
&\gamma_3(\mathbf{q}_i, \mathbf{q}_j) = 2\sum_{\mathbf{k},s} n_{\mathbf{k},s}(\tilde{E}_{\mathbf{k}} + \tilde{E}_{\mathbf{k}+\mathbf{q}_i})(\tilde{E}_{\mathbf{k}} + \tilde{E}_{\mathbf{k}+\mathbf{q}_j}) \times \\
&\times (\hbar c)^{-4}\{2(\mathbf{k}, \mathbf{q}_i) + q_i^2\}^{-1}\{2(\mathbf{k}, \mathbf{q}_j) + q_j^2\}^{-1}; \\
&\tilde{E}_{\mathbf{k}} = E_{\mathbf{k}} + m_0c^2
\end{aligned} \tag{18}$$

can be calculated approximately, performing the substitution

$$\begin{aligned}
&\tilde{E}_{\mathbf{k}} + \tilde{E}_{\mathbf{k}+\mathbf{q}_i} \Rightarrow m_0c^2 C(\tilde{q}_i|\tilde{k}), \\
&C(\tilde{q}_i|\tilde{k}) = \{1 + x^2(\tilde{k}^2 + \tilde{q}_i^2)\}^{1/2} + \{1 + x^2\tilde{k}^2\}^{1/2},
\end{aligned} \tag{19}$$

which is asymptotically correct both at small and at large q_i. According to the Feynman identity [21]

$$\begin{aligned}
&\{2(\mathbf{k}, \mathbf{q}_i) + q_i^2\}^{-1}\{2(\mathbf{k}, \mathbf{q}_j) + q_j^2\}^{-1} = \\
&= \int_0^1 d\lambda\, F^{-2}(\mathbf{q}_i, \mathbf{q}_j|\mathbf{k}), \\
&F(\mathbf{q}_i, \mathbf{q}_j|\mathbf{k}) = \Omega_{ij} + 2(\mathbf{k}, \boldsymbol{\rho}_{ij}), \\
&\Omega_{ij} \equiv q_j^2 + \lambda(q_i^2 - q_j^2); \quad \boldsymbol{\rho}_{ij} = \lambda\mathbf{q}_i + (1-\lambda)\mathbf{q}_j.
\end{aligned} \tag{20}$$

Passing from the sum by vector $\mathbf{k}$ to integral, we use dimensionless variable $\tilde{k} = |\mathbf{k}|/k_F$, $\tilde{q}_i = |\mathbf{q}_i|/k_F$, and the spherical coordinate system, the Oz of which axis coincides with the vector $\boldsymbol{\rho}_{ij}$, we perform integration over the angular variables, reducing $\gamma_3(\mathbf{q}_i, \mathbf{q}_j)$ to one-dimensional integral:

$$\begin{aligned}
&\gamma_3(\mathbf{q}_i, \mathbf{q}_j) = \frac{3N_e}{4(m_0c^2x^2)^2}\int_0^1 dk\, C(q_i|k)\, C(q_j|k)\, f_{ij}(k), \\
&f_{ij}(k) = \frac{1}{\sqrt{-\delta(k)}}\ln\left|\frac{R_{ij} + (-\delta(k))^{-1/2}}{R_{ij} - (-\delta(k))^{-1/2}}\right| \text{ by } k < q_R; \\
&f_{ij}(k) = \frac{2}{\sqrt{\delta(k)}}\arctan\{\delta^{1/2}(k)R_{ij}^{-1}\} \qquad \text{by } k > q_R.
\end{aligned} \tag{21}$$

Here the following notations are was introduced:

$$\begin{aligned}
&R_{ij} \equiv R_{ij}(k) = 2(\mathbf{q}_i, \mathbf{q}_j) - \frac{q_i^2 q_j^2}{2k^2}; \\
&\delta(k) = \delta_{ij}(k) = \left(1 - \frac{q_R^2}{k^2}\right)\cdot 4q_i^2q_j^2(1 - t_{ij}^2); \\
&q_R = (\mathbf{q}_i - \mathbf{q}_j)^2\{4(1 - t_{ij}^2)\}^{-1};
\end{aligned} \tag{22}$$

$\delta(k)$ is the invariant of problem ($\delta_{12}(k) = \delta_{23}(k) = \delta_{31}(k)$), q_R is the radius of the circle, circumscribing the triangle constructed on the vectors $\mathbf{q}_1, \mathbf{q}_2, -\mathbf{q}_1 - \mathbf{q}_2$; t_{ij} is a cosine of the angle between the vectors $\mathbf{q}_i, \mathbf{q}_j$. In the formulae (21), (22) the variables k and q_i, q_j are dimensionless (in unit k_F). We use numerical integration over the variable k in the formula (21).

Integration over the angular variables of the vector $\mathbf{k}$ in the expression for the dynamic two-particle correlation function

$$\tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|\nu, -\nu) = 2\sum_{\mathbf{k},s} n_{\mathbf{k},s}\frac{E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}}}{(E_{\mathbf{k}+\mathbf{q}} - E_{\mathbf{k}})^2 + \nu^2} \tag{23}$$

allows us to present it in the form of one-dimensionless integral:

$$\begin{aligned}
&\tilde{\mu}_2^0(\mathbf{q}, -\mathbf{q}|\nu, -\nu) = \frac{3N_e}{m_0c^2x^2}J_2(q_*, \tilde{\nu}|x), \\
&J_2(q_*, \tilde{\nu}|x) = (2xq_*)^{-1}\sum_s \int_0^\infty dk_* k_* n_{k_*,s} A(k_*|q_*, \tilde{\nu}), \\
&A(k_*|q_*, \tilde{\nu}) = \sum_{\sigma=\pm1}\sigma\{[1 + (k_* + \sigma q_*)^2]^{1/2} - \\
&- \tilde{\nu}\arctan[\tilde{\nu}^{-1}\eta_\sigma(k_*, q_*)] + \frac{1}{2}(1 + k_*^2)^{1/2} \times \\
&\times \ln[\tilde{\nu}^2 + \eta_\sigma^2(k_*, q_*)]\}, \\
&\eta_\sigma(k_*, q_*) = [1 + (k_* + \sigma q_*)^2]^{1/2} - [1 + k_*^2]^{1/2}.
\end{aligned} \tag{24}$$

Here appear the dimensionless variables

$$k_* = \frac{x|\mathbf{k}|}{k_F},\ q_* = \frac{x|\mathbf{q}|}{k_F},\ \tilde{\nu} = \frac{\nu}{m_0c^2}. \tag{25}$$

Fig. 3 illustrates the dependence $J_2(q_*, \nu|x)$ on the wave vector and the relativistic parameter.

It is well known from the non-relativistic electron fluid theory [14], that the dynamic local field correction

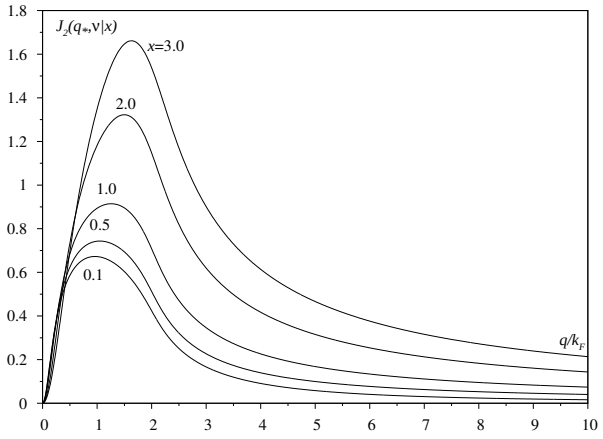

Figure 3: Dependence of the dynamic two-particle function $J_2(q_*,\nu|x)$ on the wave vector $\mathbf{q}$ at fixed values of the dimensionless frequency($\nu = 0.15m_0c^2x^2$) and different values of the relativistic parameter.

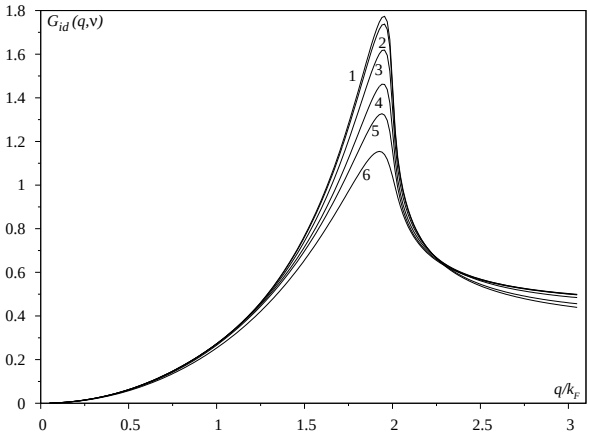

Figure 4: Dependence of the loclal field correction function $G_{id}(q,\nu)$ on the wave vector $\mathbf{q}$ and the relativistic parameter at the frequency $\nu = 0.01m_0c^2x^2$ (curve 1 – x=0.05; 2 – x=0.2; 3 – x=0.5; 4 – x=1.0; 5 – x=2.0; 6 – x=5.0).

function in the weakly non-ideal model is a universal function of the variable $y = (\mathbf{q},\nu)$, it does not depend on any parameters, and corresponds to the approximation

$$\begin{aligned} G_{id}(y) &= -(2\beta V_q)^{-1}\{\tilde{\mu}_2^0(y,-y)\}^{-2}\times \\ &\times \sum_{\mathbf{q}_1;\nu_1} V_{q_1}\tilde{\mu}_4^0(y,-y,y_1,-y_1), \\ \mu_4^0(y,-y,y_1,-y_1) &= \beta^{-1}\sum_{\mathbf{k},s}\sum_{\nu_*} G_{\mathbf{k},s}(\nu_*)\times \\ &\times G_{\mathbf{k}-\mathbf{q},s}(\nu_*-\nu)\sum_{\sigma=\pm1} G_{\mathbf{k}-\sigma\mathbf{q}_1,s}(\nu_*-\sigma\nu_1)\times \\ &\times \{2G_{\mathbf{k},s}(\nu_*)+G_{\mathbf{k}+\mathbf{q}+\sigma\mathbf{q}_1,s}(\nu_*+\nu+\sigma\nu_1)\}, \end{aligned} \tag{26}$$

where $G_{\mathbf{k},s} = \{i\nu_* - E_{\mathbf{k}} + \mu\}^{-1}$ is the spectral representation of the one-electron Green's function of the ideal model, $\nu_* = \pi\beta^{-1}(2n+1)$ is its chemical potential, $\nu_* = \pi\beta^{-1}(2n+1)$ is the Fermi-Matsubara frequency ($n = 0;\pm1;\pm2;\cdots$). After summarizing over the frequencies ν_* and $\nu_* \pm \sigma\nu_1$ according to the rule

$$\beta^{-1}\sum_{\nu_*} G_{\mathbf{k},s}(\nu_*) = n_{\mathbf{k},s} = \{1+\exp[\beta(E_k-\mu)]\}^{-1}, \tag{27}$$

we get the next representation:

$$\begin{aligned} G_{id}(y) &= -V_q^{-1}\{\tilde{\mu}_2^0(y,-y)\}^{-2}\sum_s\sum_{\mathbf{k}_1,\mathbf{k}_2} n_{\mathbf{k}_1,s}n_{\mathbf{k}_2,s}\times \\ &\times\big[V(\mathbf{k}_1-\mathbf{k}_2)f^-_{\mathbf{k}_1,\mathbf{k}_2}(\mathbf{q},\nu) - V(\mathbf{k}_1+\mathbf{k}_2+\mathbf{q})f^+_{\mathbf{k}_1,\mathbf{k}_2}(\mathbf{q},\nu)\big], \end{aligned}$$

$$\begin{aligned} f^{\mp}_{\mathbf{k}_1,\mathbf{k}_2}(\mathbf{q},\nu) &= Re\{[i\nu+E_{\mathbf{k}_1}-E_{\mathbf{k}_1+\mathbf{q}}]^{-1}\mp \\ &\mp[\pm i\nu+E_{\mathbf{k}_2}-E_{\mathbf{k}_2+\mathbf{q}}]^{-1}\}^2. \end{aligned} \tag{28}$$

In the work [22] were done numerical calculations in this approximation of the static the dynamic local field correction functions for the interacting relativistic electron gas. The asymptotics of the function at small and large values of the wave number $|\mathbf{q}|$ matches the asymptotics of corrections of the non-relativistic theory. The deviation of these functions is observed near the maximum and depends on the value of the relativistic parameter, as is shown in fig. 4 (the limit $G_{id}(q,\nu)$ at $x \to 0$ corresponds to the non-relativistic correction).

3. Ground state energy of a model

For the calculation of the interacting electron gas energy by formulae (11), (12) we should consider, that $G_{id}(q,\nu)$ does not depend on the "coupling constant" λ. From the integral term of the formula (11) it is convenient to extract the contribution of first-order perturbation theory

$$\begin{aligned} E_{HF}(x) &= -(2\beta V)^{-1}\sum_{\mathbf{q}\neq0} V_q\sum_{\nu}\tilde{\mu}_2^0(\mathbf{q},-\mathbf{q}|\nu,-\nu) = \\ &= -(2V)^{-1}\sum_{\mathbf{q}\neq0} V_q\sum_{\mathbf{k},s} n_{\mathbf{k}+\mathbf{q}/2,s}n_{\mathbf{k}-\mathbf{q}/2,s}, \end{aligned} \tag{29}$$

that allows to present E_e in a traditional form

$$E_e = E_0(x) + E_{HF}(x) + E_{corr}(x), \tag{30}$$

where

$$E_{corr}(x) = Nmc^2\alpha_0^2\left\{\frac{1}{\alpha_0^2}\frac{3}{4\pi x^3}\right\}\int\limits_0^\infty d\nu_* \int\limits_0^\infty dq_* q_*^2 \times$$
$$\times\left\{\frac{1}{2}\ln[1+L_0(y)(1+G(y)L_0(y))] - L_0(y)\right\} +$$
$$+\begin{cases} \dfrac{1}{2\sqrt{-\Delta}}\ln\dfrac{1+\frac{1}{2}L_0(y)(1+\sqrt{-\Delta})}{1+\frac{1}{2}L_0(y)(1-\sqrt{-\Delta})} & \text{by } \Delta<0 \\ \dfrac{1}{\sqrt{\Delta}}\arctan\dfrac{\frac{1}{2}L_0(y)\sqrt{\Delta}}{1+\frac{1}{2}L_0(y)} & \text{by } \Delta>0 \\ \dfrac{2G(y)L_0(y)}{1+2G(y)L_0(y)} & \text{by } \Delta=0, \end{cases} \quad (31)$$
$$\Delta\equiv\Delta(y) = 4G(y)-1, \quad L_0(y) = \frac{4\alpha_0 x}{\pi q_*^2}J_2(q_*,\nu_*|x).$$

is the so-called correlation energy. In the units m_0c^2

$$E_e = N_e m_0 c^2\left\{\varepsilon_0(x) - \frac{3}{4\pi}\alpha_0 x + \alpha_0^2\,\varepsilon_c(x)\right\}. \quad (32)$$

Here

$$\varepsilon_0(x) = (2x)^{-3}\big\{3x(1+x^2)^{1/2}(1+2x^2) - 8x^3 - \\ -3\ln\left[x+(1+x^2)^{1/2}\right]\big\} \quad (33)$$

is the contribution of an ideal system per one electron, $-3\alpha_0 x(4\pi)^{-1}$ is the contribution of interactions in the Hartree-Fock approximation, $\alpha_0^2\varepsilon_c(x)$ is the correlation energy. According to the results of numerical calculations $\varepsilon_c(x)$ can be approximated by the following expression:

$$\varepsilon_c(x) = -b_0\int\limits_0^x dt\,\frac{(b_1 a + t^{1/2})}{t^{3/2}+tb_1 a + b_2 t^{1/2}a^2 + b_3 a^3}\times \\ \times\frac{1+a_1 t + a_2 t}{1+td_0}, \quad (34)$$
$$a = (\alpha_0\eta)^{1/2};\ a_1 = 1{,}21954;\ a_2 = 1{,}33205;$$
$$d_0 = 1{,}18934;\ b_0 = 0{,}0621814;\ b_1 = 9{,}81379;$$
$$b_2 = 2{,}82214;\ b_3 = 0{,}73701.$$

At $a_1 = a_2 = d_0 = 0$ this expression matches the approximation [23] of the results of ground state energy calculation of non-relativistic electron fluid model $\varepsilon_c^{MC}(x)$ obtained using the Monte-Carlo method [24]. In the region $x \le 1$ the expression (34) is near to $\varepsilon_c^{MC}(x)$, and the deviation $\varepsilon_c(x)$ from $\varepsilon_c^{MC}(x)$ in $x > 1$ is caused by different asymptotics of these functions: $\varepsilon_c^{MC}(x) \to -b_0\ln x + \cdots$ by $x \gg 1$.

In order to calculate the contributions of electron-nuclear interactions in the products of structure factors in the formula (6) we have selected one-particle and two-particle sums by the coordinates of nuclei ignoring the three-nuclear effective interactions. In this approach

$$E \simeq E_e + E_{pol} + E_{conf}, \quad (35)$$

where E_{pol} is the polarization energy of electron fluid by nuclei, it does not depend on the structure of the nuclear subsystem,

$$E_{pol} = E_{pol}^{(2)} + E_{pol}^{(3)},$$
$$E_{pol}^{(2)} = -\frac{1}{2!}\sum_{i=1}^{l} z_i^2 N_i V^{-2}\sum_{\mathbf{q}} V_q^2\,\tilde{\mu}_2(\mathbf{q},-\mathbf{q}|0), \quad (36)$$
$$E_{pol}^{(3)} = -\frac{1}{3!}\sum_{i=1}^{l} z_i^3 N_i V^{-3}\sum_{\mathbf{q}_1,\mathbf{q}_2} V_{q_1}\times \\ \times V_{q_2}V_{-q_1-q_2}\,\tilde{\mu}_3(\mathbf{q}_1,\mathbf{q}_2,-\mathbf{q}_1-\mathbf{q}_2|0).$$

The configuration energy is determined by the structure of nuclear subsystem, and expressed through the effective two-particle potential of interactions of nuclei, which is formed by two- and three-electron correlations:

$$E_{conf} = \frac{1}{2!}\sum_{i=1}^{l}\frac{z_i^2}{V}\sum_{\mathbf{q}} V_q\left\{1-\frac{V_q}{V}\,\tilde{\mu}_2(\mathbf{q},-\mathbf{q}|0) - \right. \\ \left. -\frac{z_i}{V^2}\sum_{\mathbf{q}_1} V_{q_1}V_{-q-q_1}\tilde{\mu}_3(\mathbf{q},\mathbf{q}_1,-\mathbf{q}-\mathbf{q}_1|0)\right\}S_2^{(i)}(\mathbf{q},-\mathbf{q}) + \\ +\frac{1}{2!}\sum_{i\ne j=1}^{l}\frac{z_i z_j}{V}\sum_{\mathbf{q}} V_q\left\{1-\frac{V_q}{V}\,\tilde{\mu}_2(\mathbf{q},-\mathbf{q}|0) - \frac{1}{2V^2}\times\right. \quad (37) \\ \left.\times(z_i+z_j)\sum_{\mathbf{q}_1} V_{q_1}V_{-q-q_1}\,\tilde{\mu}_3(\mathbf{q},\mathbf{q}_1,-\mathbf{q}-\mathbf{q}_1|0)\right\}S_{\mathbf{q}}^{(i)}S_{-\mathbf{q}}^{(j)},$$
$$S_2^{(i)}(\mathbf{q},-\mathbf{q}) = \sum_{j_1\ne j_2=1}^{N_i}\exp\big\{i(\mathbf{q},\mathbf{R}_{j_1}-\mathbf{R}_{j_2})\big\}.$$

Let us rewrite the component $E_{pol}^{(2)}$, calculated in the local field approximation, in the form

$$E_{pol}^{(2)} = N_e m_0 c^2\frac{\langle z^2\rangle}{\langle z\rangle}\alpha_0^{3/2}\varepsilon_{pol}^{(2)}(x), \quad (38)$$

where the dimensionless function $\varepsilon_{pol}^{(2)}(x)$ is of the same order as $\varepsilon_c(x)$, and $\langle z^n\rangle = \left\{\sum\limits_{i=1}^{l} N_i\right\}^{-1}\sum\limits_{i=1}^{l} z_i^n N_i$. The function $\varepsilon_{pol}^{(2)}(x)$ can be approximated as

$$\varepsilon_{pol}^{(2)}(x) = -\int\limits_0^x\frac{c_0+c_1 t+c_2 t^2+c_3 t^3}{1+d_1 t+d_2 t^2+d_3 t^3}\,dt; \\ c_0 = 4{,}06151;\ c_1 = 32{,}6118;\ c_2 = -43{,}6587; \quad (39) \\ c_3 = 104{,}13;\ d_1 = 73{,}8252;\ d_2 = -67{,}1028; \\ d_3 = 189{,}781.$$

As is shown in fig. 5, $\varepsilon^{(2)}_{pol}(x)$ has the linear asymptotics at $x \gg 1$, as well as $\varepsilon_c(x)$. However, the polarization energy $E^{(2)}_{pol}$ exceeds the correlation energy of basis system by about $\langle z\rangle\alpha_0^{-1/2} \approx 10\langle z\rangle$ times, and for $\langle z\rangle \sim 10$ it is comparable with E_{HF}.

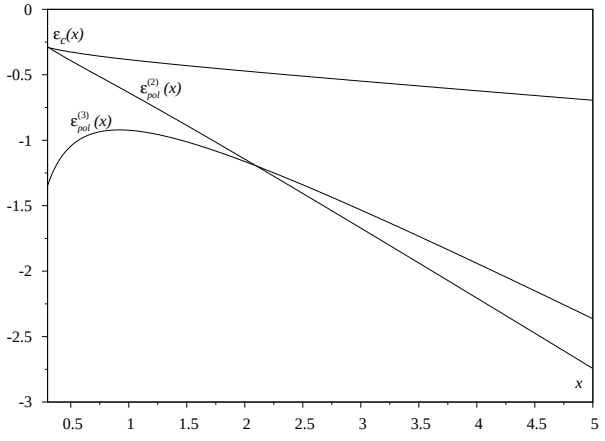

Figure 5: Dependence of functions $\varepsilon^{(2)}_{pol}(x)$, $10\cdot\varepsilon^{(3)}_{pol}(x)$ and $\varepsilon_c(x)$ on the relativistic parameter.

The contribution of three-particle correlations in the polarization energy is represented in the form

$$E^{(3)}_{pol} = N_e m_0 c^2 \frac{\langle z^3\rangle}{\langle z\rangle}\alpha_0^{5/2}\varepsilon^{(3)}_{pol}(x). \tag{40}$$

In the region $x > 1$ the ratio $\varepsilon^{(3)}_{pol}(x) \approx 0{,}1\varepsilon^{(2)}_{pol}(x)$ is satisfied. At sufficiently large values of nuclei charges $E^{(3)}_{pol}$ is not less than the correlation energy of basis system: at $\langle z\rangle \geq 6$ the contribution $E^{(3)}_{pol}$ is close to the correlation energy, at $\langle z\rangle \geq 12$ it exceeds the correlation energy by 5 times, and at $\langle z\rangle = 26$ – more than by 20 times. The result of numerical calculation of $\varepsilon^{(3)}_{pol}(x)$ is approximated by the expression

$$\begin{aligned}
&\varepsilon^{(3)}_{pol}(x) = -ax - c_0\int\limits_x^\infty \frac{1 + c_1/t + c_2 t}{1 + d_1 t + d_2 t^2 + d_3 t^3}\,dt,\\
&a = 0{,}0450;\ c_0 = 0{,}12607;\ c_1 = -0.93695;\\
&c_2 = 78{,}8552;\ d_1 = -23{,}2602;\ d_2 = 114{,}5030;\\
&d_3 = 164{,}060.
\end{aligned} \tag{41}$$

From the formulae (38), (40) it follows, that $E^{(3)}_{pol}/E^{(2)}_{pol} \sim 0{,}1z\alpha_0$, and it determines the order of three-electron correlations contribution function.

We have calculated the configuration energy in coordinate representation by introducing the effective two-nuclear potentials. In the two-particle correlations approximation they are determined by the expressions

$$\begin{aligned}
&V_2^{i_1,i_2}(\mathbf{R}^{(i_1)}_{j_1} - \mathbf{R}^{(i_2)}_{j_2}) = V^{-1}\sum_{\mathbf{q}} V_2(q)\times\\
&\times\exp\left\{i(\mathbf{q}, \mathbf{R}^{(i_1)}_{j_1} - \mathbf{R}^{(i_2)}_{j_2})\right\},\\
&V_2(q) = V_q\left\{1 - \frac{V_q}{V}\tilde{\mu}_2(\mathbf{q}, -\mathbf{q})\right\}.
\end{aligned} \tag{42}$$

In the formula (42) the sum of the vector $\mathbf{q}$ includes a component with $\mathbf{q} = 0$. We have done our calculation in the frame of model with two-sorts of nuclei. Therefore

$$\begin{aligned}
&E^{(2)}_{conf} = \frac{1}{V}\sum_{\mathbf{q}\neq 0} V_2(q)\left\{\frac{1}{2}\sum_{i=1,2} S^{(i)}_2(\mathbf{q}, -\mathbf{q}) z_i^2 + z_1 z_2\times\right.\\
&\left.\times S^{(1)}_{\mathbf{q}} S^{(2)}_{-\mathbf{q}}\right\} = \frac{1}{2}\sum_{i=1,2} z_i^2 \sum_{j_1\neq j_2=1} V_2(\mathbf{R}^{(i)}_{j_1} - \mathbf{R}^{(i)}_{j_2}) + z_1 z_2\times\\
&\times\sum_{j_1=1}^{N_1}\sum_{j_2=1}^{N_2} V_2(\mathbf{R}^{(1)}_{j_1} - \mathbf{R}^{(2)}_{j_2}) - \frac{1}{2}N_e^2\lim_{\mathbf{q}\to 0}\left\{V^{-1}V_2(q)\right\}.
\end{aligned} \tag{43}$$

To simplify the calculation of the lattice sum we adopt a simple model of nuclei distribution in the lattice, namely

$$\mathcal{N}^{(i)}_j = \mathcal{N}_j\left\{\sum_{i=1}^{l} N_i\right\}^{-1} N_i, \tag{44}$$

where $\mathcal{N}_j$ is the number of all knots on the j-th coordination sphere, and $\mathcal{N}^{(i)}_j$ is the number of the knots, occupied by nuclei with charge z_i. In this model

$$\begin{aligned}
&E^{(2)}_{conf} = \frac{N_e}{2}\langle z\rangle\sum_{j\geq 1}\mathcal{N}_j V_2(R_j) -\\
&-\frac{N_e m_0 c^2}{6}\left\{\frac{x^2}{(1+x^2)^{1/2}} - \frac{x\alpha_0}{\pi}\right\}
\end{aligned} \tag{45}$$

where R_j is the radius of the j-th coordination sphere.

The effective two-particle potential is screened, and at small and medium distances between nuclei it is close to the expression

$$V(R) = \frac{e^2}{R}\exp\{-R/R_0\}, \tag{46}$$

and the screened radius

$$R_0 = \frac{\sqrt{\pi}}{2}\alpha_0^{1/2} a_B\{x^{1/2}(1+x^2)^{1/4}\}^{-1} \tag{47}$$

has the order $0{,}1a_B$ (where $a_B = \hbar^2/m_0 e^2$ is the Bohr radius). At the large distances $V_2(R)$ oscillates, but with small amplitude,

$$V_2(R) \approx \frac{e^2}{a_B}\left(\frac{R_0}{2xR}\right)^3\cos(2xR/R_0). \tag{48}$$

The configuration energy for the simple cubic lattice of nuclei is calculated numerically and can be represented as

$$E^{(2)}_{conf} = N_e m_0 c^2 \langle z\rangle^{2/3} \alpha_0 \varepsilon^{(2)}_L(x|\langle z\rangle), \tag{49}$$

approximating dimensionless factor by expression

$$\varepsilon^{(2)}_L(x|\langle z\rangle) = -\int\limits_0^x \frac{a_1 + t a_2 + t^2 a_3}{1 + t a_4 + t^2 a_5 + t^3 a_6} t\, dt, \tag{50}$$

where all the coefficients $a_1, \cdots a_6$ are the functions of $\langle z\rangle$, that is

$$a_i(\langle z\rangle) = \frac{a_{i_0} + \langle z\rangle a_{i_1} + \langle z\rangle^2 a_{i_2}}{a_{i_3} + \langle z\rangle a_{i_4} + \langle z\rangle^2 a_{i_5}}. \tag{51}$$

Similar to the formula (42) we have calculated the correction to the effective two-nuclear potential by the three-electron correlations

$$V^{(3)}_2(R) = -V^3 \sum_{\mathbf{q}} V_q \sum_{\mathbf{q_1}} V_{q_1} V_{-q-q_1} \times \\ \times \tilde{\mu}_3(\mathbf{q}, \mathbf{q}_1, -\mathbf{q} - \mathbf{q}_1|0) \exp\{i(\mathbf{q}, \mathbf{R})\}, \tag{52}$$

where the sum over the vector $\mathbf{q}$ includes the compo-

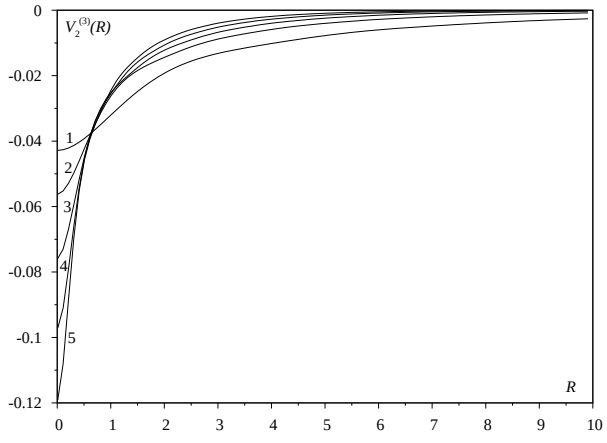

Figure 6: The effective potential of interactions $V^{(3)}_2(R)$ at different values of the relativistic parameter (curve 1 – $x = 1.0$, 2 – $x = 2.0$, 3 – $x = 3.0$, 4 – $x = 4.0$, 5 – $x = 5.0$).

nent $\mathbf{q} = 0$. As shown in fig. 6, $V^{(3)}_2(R)$ is the weak attracting potential of the type of quantum package screening potential, which is close to the expression

$$V^{(3)}_2(R) = -\frac{e^2}{R}\alpha_0^2\, A(x) \left\{1 - \exp\left[-\frac{R}{R_0}\gamma(x)\right]\right\} \times \\ \times \exp\{-R/R_0\}. \tag{53}$$

Contribution to the configuration energy of model by the three-particle correlations in the model (44) takes the form, similar to the formula (45):

$$E^{(3)}_{conf} = \frac{1}{2} N_e \langle z^2\rangle \sum_{j\geq 1} \mathcal{N}_j V^{(3)}_2(R_j) + \\ + \frac{4}{3\pi^2} N_e \alpha_0^2 \langle z\rangle m_0 c^2 (1 + x^2)^{-1/2} I_2(x), \tag{54}$$

$$I_2(x) = \int\limits_0^\infty \frac{dq}{q^2 \varepsilon^2(q)} J_3(q);\ J_3(q) = \frac{(m_0 c^2 x^2)^2}{3N_e} \tilde{\mu}^0_3(\mathbf{q}, -\mathbf{q}, 0|0);$$

This contribution is calculated for the simple cubic lattice of nuclei and represented

$$E^{(3)}_{conf} = N_e m_0 c^2 \alpha_0^2 \langle z^2\rangle \varepsilon^{(3)}_L(x|\langle z\rangle). \tag{55}$$

At sufficiently large nuclei charges $\langle z\rangle$ and $x \geq 2$ the function $\varepsilon^{(3)}_L(x|\langle z\rangle) \sim 0{,}1\varepsilon^{(2)}_L(x|\langle z\rangle)$, but it has a positive sign. It is approximated by the expression

$$\varepsilon^{(3)}_L(x|\langle z\rangle) = -a + xb - \int\limits_x^\infty \frac{c_0 + \frac{c_1}{y} + c_2 y + c_3 y^2}{1 + a_1 y + a_2 y^2 + a_3 y^3}\, dy. \tag{56}$$

4. Equation of state of model at $T = 0K$

For the well known dependence of model energy on the relativistic parameter we calculate the equation of state of cold degenerate matter using the expression

$$P(x) = \frac{dE}{dV} = \frac{x^4}{N_e}\left(\frac{m_0 c}{\hbar}\right)^3 (3\pi^2)^{-1} \frac{dE}{dx}. \tag{57}$$

Within accepted approximation in case of two-sorts of nuclei

$$P(x) = \frac{\pi m_0^4 c^5}{3h^3}\{\mathcal{F}(x) + f_2(x) + f_3(x)\}. \tag{58}$$

Here

$$\mathcal{F}(x) = x(2x^2 - 3)(1 + x^2)^{1/2} + 3\ln\left[x + (1 + x^2)^{1/2}\right] \tag{59}$$

is the contribution of the ideal degenerate relativistic spatially homogeneous electron gas;

$$f_2(x) = -2\alpha_0 x^4 \left\{\frac{1}{\pi} - \frac{4}{3}\frac{d}{dx}\left(\langle z\rangle^{2/3}\varepsilon^{(2)}_L(x|\langle z\rangle) + \right.\right. \\ \left.\left. + \frac{\langle z^2\rangle}{\langle z\rangle}\alpha_0^{1/2}\varepsilon^{(2)}_{pol}(x) + \alpha_0\varepsilon_c(x)\right)\right\} \tag{60}$$

is the contribution of Coulomb interactions in the two-electron correlations approximation;

$$f_3(x) = 8\alpha_0^2 x^4 \frac{d}{dx}\left\{\langle z^2\rangle\varepsilon^{(3)}_L(x|\langle z\rangle) + \frac{\langle z^3\rangle}{\langle z\rangle}\alpha_0^{1/2}\varepsilon^{(3)}_{pol}(x)\right\} \tag{61}$$

Table 1: Dependence of functions $\mathcal{F}(x)\cdot x^{-4}$, $f_2(x)\cdot x^{-4}$ and $f_3(x)\cdot x^{-4}$ on the relativistic parameter x according to the formulae (59), (60), (61).

x	$\mathcal{F}(x)\cdot x^{-4}$	$f_2(x)\cdot x^{-4}$				$f_3(x)\cdot x^{-4}$			
		$z=2$	$z=6$	$z_1=7;\ z_2=8$	$z=12$	$z=2$	$z=6$	$z_1=7;\ z_2=8$	$z=12$
0.5	0.737488	−0.0251212	−0.08993	−0.113385	−0.208688	0.00151018	0.00503624	0.00724728	0.0123084
0.6	0.857456	−0.0249004	−0.0898357	−0.113389	−0.209376	0.00144201	0.0045685	0.00653182	0.0107583
0.7	0.966234	−0.0247338	−0.0896419	−0.113228	−0.209565	0.00140653	0.00431288	0.00610532	0.00988929
0.8	1.06412	−0.0246166	−0.0894277	−0.11301	−0.209485	0.00138662	0.00415774	0.00582077	0.00935182
0.9	1.15175	−0.0245372	−0.0892269	−0.112785	−0.209267	0.00137469	0.00405367	0.00561505	0.00899025
1.0	1.22991	−0.0244844	−0.089051	−0.112575	−0.208985	0.00136685	0.00397618	0.00545736	0.00872662
1.1	1.29949	−0.0244498	−0.0889018	−0.11239	−0.208681	0.00136085	0.00391243	0.00533111	0.00851882
1.2	1.36139	−0.0244276	−0.0887769	−0.112229	−0.20838	0.00135532	0.00385556	0.00522669	0.00834323
1.3	1.41647	−0.0244137	−0.088673	−0.11209	−0.208093	0.00134944	0.00380207	0.00513813	0.00818666
1.4	1.46551	−0.0244055	−0.0885864	−0.111972	−0.207827	0.00134276	0.0037504	0.00506157	0.008042
1.5	1.50924	−0.0244011	−0.0885141	−0.111872	−0.207584	0.00133514	0.00370008	0.00499435	0.00790579
1.6	1.5483	−0.0243994	−0.0884535	−0.111786	−0.207364	0.00132663	0.00365119	0.00493463	0.00777667
1.7	1.58327	−0.0243994	−0.0884024	−0.111712	−0.207166	0.00131746	0.00360403	0.00488104	0.00765433
1.8	1.61463	−0.0244006	−0.0883592	−0.111649	−0.206988	0.00130788	0.00355898	0.00483256	0.00753897
1.9	1.64282	−0.0244027	−0.0883224	−0.111594	−0.206828	0.0012982	0.00351632	0.00478842	0.00743084
2.0	1.66822	−0.0244053	−0.0882909	−0.111547	−0.206686	0.00128866	0.00347628	0.00474798	0.00733015
2.5	1.7636	−0.0244211	−0.0881871	−0.111386	−0.206164	0.00124826	0.00331614	0.00458706	0.00693448
3.0	1.82417	−0.0244366	−0.0881333	−0.111299	−0.205853	0.00122235	0.00321213	0.00447214	0.00668381
3.5	1.86463	−0.0244497	−0.0881032	−0.111248	−0.20566	0.00120675	0.00314507	0.00438579	0.00652551
4.0	1.89283	−0.0244606	−0.0880853	−0.111216	−0.205535	0.00119715	0.0031007	0.00431866	0.00642247
4.5	1.91319	−0.0244697	−0.0880743	−0.111195	−0.205451	0.00119099	0.0030703	0.00426516	0.0063528
5.0	1.92833	−0.0244773	−0.0880673	−0.111182	−0.205393	0.00118685	0.00304877	0.00422167	0.00630395

is the three-particle electron correlations.

In the region $x \geq 1$ all contributions (with the exception of $\varepsilon_L^{(3)}(x|\langle z\rangle)$) to the model energy caused by interactions are negative monotonically decreasing functions of the relativistic parameter. The corrections $f_2(x)$, $f_3(x)$ are negative and decrease the pressure. In the two-electron correlations approximation the equation of state (58) numerically is very close to the result of Salpeter [5].

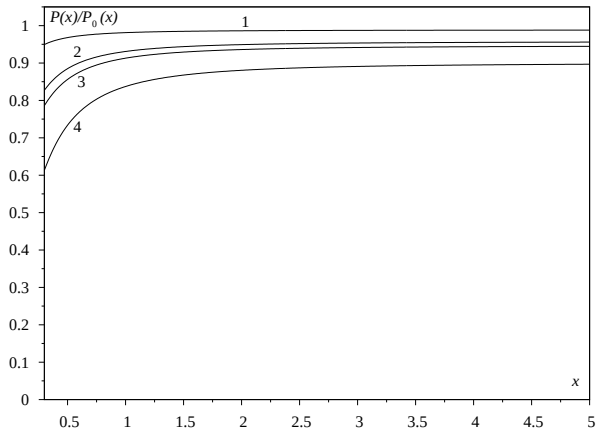

Figure 7: The ratio of pressure with Coloumb interactions $P(x)$ to the pressure of the ideal relativistic electron gas $P_0(x)$ as function of the relativistic parameter and nuclear charge (curve 1 – $z_1 = z_2 = 2$, 2 – $z_1 = z_2 = 6$, 3 – $z_1 = 7$; $z_2 = 8$; 4 – $z_1 = z_2 = 12$).

In table 1 is shown the dependence of terms $\mathcal{F}(x)$, $f_2(x)$, $f_3(x)$ on the relativistic parameter for the helium dwarf model ($z_1 = z_2 = 2$), carbon ($z_1 = z_2 = 6$), nitrogen-oxygen ($z_1 = 7$, $z_2 = 8$; $N_1 = N_2$) and magnesium ($z_1 = z_2 = 12$). The relative decrease of pressure by the interactions $\{\mathcal{F}(x) + f_2(x) + f_3(x)\}\mathcal{F}^{-1}(x)$ for the same models is illustrated in fig. 7.

5. Conclusions

Within the reference system approach, which was adapted for the description of degenerate relativistic electron subsystem, the energy of ground state of electron-nuclear model, as well as equation of state of the model, have been calclulated in a wide range of the relativistic parameter at absolute zero temperature. As it is shown in our calculation, the contributions of Coloumb interactions to the energy of ground state and pressure, caused by two-electron correlations are determinant and increase with increasing the nuclear charge. The contributions, caused by three-electron correlations are much smaller, but they exceed the contribution of correlation energy of electron fluid, especially at large values of nuclear charge.

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